LUNAR LANDSCAPE

The Earth as seen from a Crater of the Moon (p. 323).
From a painting by Howard Russell Butler, N.A., in the
American Museum of Natural History, New York

ASTRONOMY

A REVISION OF
YOUNG'S MANUAL OF ASTRONOMY

I
THE SOLAR SYSTEM

BY

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PREFACE

This revision of Professor Young's work has been undertaken by his successors at Princeton. After so long an interval, extensive changes have been required by the progress of the science; the book has been practically rewritten, and inevitably increased considerably in length. The new order of presentation of the material, in which the data secured by the methods of geometrical and dynamical astronomy are presented (so far as they relate to the solar system) before the introduction of the spectroscope and of astrophysical problems, has been adopted as a result of many years' experience in lecturing.

The scope of the new work is somewhat more extensive than that of the former *Manual* and intermediate between this and the *General Astronomy*. The liberal use of small type, in dealing with the more difficult or less important topics, may serve as a guide to the teacher who desires to give a shorter course.

The preliminary knowledge assumed on the part of the student involves only the elements of mathematics and physics. Use of the calculus has been completely avoided, and the physical principles which underlie spectroscopy and its applications have been explained in full (except for such matters as the theory of prisms and gratings, which are accessible in many textbooks).

Special attention has been paid to astrophysics, and recent developments have been rather fully treated. Some of these may be unfamiliar, but their inclusion has been deliberate. The relations, for example, which connect the absolute magnitude, radius, and temperature of an ideal star (perfect radiator) are actually much simpler than those connecting right ascension and declination with altitude and azimuth, and, in the writers' opinion, fully as worthy of attention, even in an elementary course. The derivation of these relations has been given in full (in small type), in this case and in some similar cases, because it is not readily accessible elsewhere, — but these sections may be omitted by the elementary student.

In dealing with such subjects as the constitution and evolution of the stars (concerning which theories are in a state of very active flux) the attempt has been made to present the situation as it appears to the writers at the "epoch" of completion of the manuscript, in 1926. Considerable changes may be necessary within a few years.

The whole manuscript of the book has been read by at least two of the authors, and usually by all three, and every effort has been made to avoid errors of statement and to obtain the best available values of numerical data. It cannot be hoped, however, that such efforts have been wholly successful, and information concerning any errors which may be detected will be welcomed. Professor Young's statements concerning the earlier history of the science have usually been accepted without fresh investigation.

New material for illustrations has been generously supplied by numerous friends in this country and abroad. Acknowledgments are gratefully made to the directors, and to many other members of the staffs, at the observatories at Greenwich, Heidelberg, and Victoria (British Columbia); also at the Mt. Wilson, Lowell, Lick, Yerkes, Harvard, and Yale observatories; to the United States Navy, the Carnegie Institution, the American Museum of Natural History, and the *Scientific American*; likewise to the late Professor Barnard, to Professors Stebbins and Slocum, Dr. Benjamin Boss, Mr. Donald B. MacMillan, Mr. D. M. Barringer, and Mr. Howard Russell Butler.

Special acknowledgments are made in connection with the individual illustrations. Acknowledgments are also due to Messrs. Arthur Fairley and Theodore Dunham, Jr., for the preparation of many line drawings, and to Miss Henrietta Young for valuable assistance in the preparation of the manuscript and proof.

The authors wish to thank Messrs. Ginn and Company for their ready and careful coöperation in the printing and in the making of the numerous cuts.

HENRY NORRIS RUSSELL
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JOHN QUINCY STEWART

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INTRODUCTION

1. **Astronomy,** as is indicated by the Greek roots of the word (*ἄστρον, νόμος*), is the science which treats of the heavenly bodies. It considers (1) their motions, both real and apparent, and the laws which govern those motions; (2) their forms, dimensions, masses, and surface features; (3) their nature, constitution, and physical condition; (4) the effects which they produce upon one another by their attractions and radiations; (5) their probable past history and future development.

As we look up at night we see in all directions the countless stars, and, conspicuous among them and looking like stars, though very different in their real nature, are scattered a few planets. Here and there appear faintly shining clouds of light, — the Milky Way, nebulæ, and possibly a comet. Most striking of all, if it happens to be in the heavens at the time, though really the most insignificant of all, is the moon. By day the sun alone is visible, flooding the air with its light and hiding the other objects from the unaided eye, but not all of them from the telescope.

The bodies thus seen from the earth are the heavenly bodies. The first great advance of modern science was the recognition that the earth itself should be counted among these. The earth, like most of the others, is a globe, whirling on its axis and moving swiftly through space, although on its surface we are wholly unconscious of the motion because of its perfect steadiness. Most of the heavenly bodies are so far away that their motions can be detected only by careful observation.

2. The Heavenly Bodies. The heavenly bodies may be classified as follows: First there is the solar system, composed of the *sun*, the *planets* which revolve around it (of which the earth is one), and the *satellites* which attend the planets in their motion. The moon thus accompanies the earth. The distances between these objects are great in comparison with the size of the earth; and the sun, which rules them all, is a body of gigantic magnitude. The solar system also includes the *comets* and the *meteors*, which move in orbits of a shape different from that of the planetary orbits and are bodies of a very different character.

Second come the *stars*, at distances from us immensely greater even than those which separate the planets. The stars are suns, — bodies like our own sun in nature, and, like it, self-luminous, whereas the planets and their satellites shine only by reflected sunlight. The telescope reveals millions of stars invisible to the naked eye, and every notable increase of power in our instruments brings out millions more. In many instances the stars are grouped in remarkable *clusters*. No telescope yet made or likely to be made is sufficiently powerful to reveal planetary systems, even if such exist, around stars other than our sun.

Finally, there are *nebulæ*, which are cloudlike masses of matter of almost inconceivable magnitude. Most of these are faintly luminous, but some are dark and are evident only because they conceal the stars which lie behind them. Many of them belong to the region of the stars, but it has recently been discovered that some are vast clusters of stars at distances so great that light from them may take a million years to reach us.

It is a remarkable fact that examples of all these classes of heavenly bodies can be seen with the unaided eye, although the telescope is required to reveal the real character of some of them.

3. Methods of Investigation. Since the phenomena with which astronomy deals are not subject to human control, it is of necessity an observational science rather than an experimental one. Opportunities for certain types of astronomical observations occur so rarely that they must be exploited to the utmost when they do occur. Moreover, knowledge of the heavenly bodies must be based almost exclusively on what studies can be made of the

feeble light that reaches the earth through the depths of space. Starlight is a very precious thing to the astronomer, and he spares neither labor nor cost in devising instruments for collecting and analyzing it. His quantitative observations have attained a precision, both in direct measurement and in the detection of hidden sources of error, that is rarely equaled in any other science. In the discussion and interpretation of his observations he continually employs mathematical analysis, often of the most advanced type, and utilizes freely the latest results and conclusions of physics and chemistry.

4. Relation of Astronomy to other Sciences. Thus there is no sharp boundary between astronomy and the other physical sciences. It is so intimately related to physics that it is often quite impracticable to say whether a given piece of work should be regarded as belonging more to one science than to the other. Its relation to mathematics is almost as close. In problems concerning the formation and constitution of the earth, astronomy overlaps the field of geology; and in questions dealing with the structure of atoms it comes in touch with chemistry.

Points of contact with the natural sciences are fewer; but in discussions of the length of time during which the earth has been habitable, and the possible habitability of other planets, astronomy meets biology, while in the consideration of those errors of observation which are personal to the observer it utilizes principles of physiology and psychology.

5. Branches of Astronomy. Astronomy is conventionally divided into several branches.

(1) *Practical astronomy* deals with the field of observation, — the design and use of astronomical instruments, the methods of observing, the elimination of errors, and the deduction of the data employed in other branches of astronomy. It is quite as much an art as it is a science.

(2) *Astronomy of position*, or *astrometry*, treats of the geometrical relations of the heavenly bodies, their positions, distances, dimensions, and surface markings, and their real and apparent motions. A subdivision, *spherical astronomy*, deals with their apparent positions and motions on the background of the sky (celestial sphere).

(3) *Celestial mechanics* is the astronomical application of the principles of mechanics, which describe the motions of material bodies acted on by forces. It deals principally with the motions of the planets and the moon. It is sometimes called gravitational astronomy, because, with few exceptions, gravitation is the only force sensibly concerned in the motions of the heavenly bodies.

Work in all the fields just outlined is involved in the *determination of the orbits* of the heavenly bodies, and in the *calculation of ephemerides* predicting their motions and their positions as seen from the earth.

(4) *Astrophysics* treats of the physical characteristics of the heavenly bodies, their brightness and spectroscopic peculiarities, their temperature and radiation, the nature and condition of their atmospheres, surfaces, and interiors, and all phenomena which indicate or depend upon their physical condition. This, though the youngest, is the most active branch of the science, and bids fair to outgrow all the others put together. Among its principal subdivisions may be mentioned *spectroscopy* and *photometry*.

(5) All branches of the science, and especially astrophysics, aid in the attack on the great central and still unsolved problem of *cosmogony*, — the origin and evolution of the stars, the sun, and the planets, including the earth.

(6) *Descriptive astronomy* is merely the orderly statement of astronomical facts and principles.

(7) *Nautical astronomy* includes so much of spherical and practical astronomy as is required by the navigator.

6. Practical Utility. Astronomy, although bearing less directly upon the material interests of life than many other sciences, is nevertheless of great practical importance.

It is by means of astronomy that the latitude and longitude of points upon the earth's surface are determined, and by such determinations alone is it possible to conduct extensive navigation. Moreover, all the operations of surveying upon a large scale depend more or less upon astronomical observations.

The same is true of operations which require an accurate knowledge and observance of time, for our fundamental timekeeper is the daily revolution of the heavens as determined by the astronomer's transit instrument.

The methods of nearly all such economically valuable observations, however, were discovered long ago. The end and object of present-day astronomy is chiefly knowledge pursued for its own sake. This no more needs defense than the pursuit of art; yet the most abstract astronomical investigations may in time have practical value, through their influence on the other sciences. Thus, astronomical studies undertaken for the sake of pure knowledge led, in the seventeenth century, to the discovery of the laws of dynamics and to the invention of the calculus, and so helped to lay the foundation for all later advances in physics and engineering. Very recently astronomy has had much to do with the development and testing of the important physical theory of relativity.

7. Value in Education. The student of astronomy, therefore, must expect his chief profit to be intellectual. Exactness of thought and expression are enforced by the precise character of many of the necessary discussions, and the taking of astronomical observations is a particularly instructive training in carefulness and accuracy. The solution of problems apparently hopeless cannot fail to impress the mind; the spectacle of simple law working out the most far-reaching results stimulates the imagination; and the beauty and grandeur of the subjects presented gratify the poetic sense.

For the advanced student there is no field in which it is possible sooner to get out to the front line of scientific advance and to learn how fresh territory is being won in a very active sector. Indeed, the number of objects that will repay observation is so great, and the opportunities for elementary calculation are so considerable, that undergraduate students, and amateurs without university training, have made and are making genuine contributions to the advance of astronomical knowledge.

8. Public Interest. Astronomy is the oldest of the sciences. Such striking phenomena as the rising and setting of the sun and moon, and the phases of the latter, must have been recognized as regular in the infancy of the race. Some of the earliest of all existing records relate to astronomical subjects, such as eclipses and the positions of the planets; and they are often of great value to the historian, since their dates can be accurately calculated.

The names of the planets, of the days of the week, and of the constellations still preserve to us an ancient mythology.

Until modern times it was believed that human affairs of every kind — the welfare of nations and the life history of individuals — were controlled, or at least prefigured, by the motions of the stars and planets, and that from the study of the heavens it was possible to predict futurity. The *pseudo-science of astrology*, based upon this belief, supplied the motive that led to many of the astronomical observations of the ancients. As modern chemistry had its origin in alchemy, so astrology was a progenitor of astronomy, and it is remarkable how persistent a hold this baseless delusion still retains upon the ignorant.

Far as astronomy has advanced today, almost all of its conclusions, and many of the methods by which they are reached, are explainable to the average person and excite a lively general interest. The enormous reaches of space and time thus opened up, and the revelation of bodies invisible to the eye but actually far exceeding in magnitude not only the little earth but the enormous sun, are examples of topics which stir the popular imagination. This interest is reflected in the liberal provision of endowment for further research.

For example, the greatest telescope in the world, at the Mt. Wilson Observatory, near Pasadena, and the second greatest, at the Dominion Astrophysical Observatory at Victoria, British Columbia, both belong to institutions maintained purely for astronomical research, one by private endowment, the other by the Canadian government. The support of astronomical investigation by universities is also generous.

9. Philosophical Value. Since astronomy is a physical science, it can give no direct answer to problems of philosophy. Thus, while it can tell that a star is larger than a man, it cannot decide which possesses the greater worth.

Nevertheless, the realization which this science brings of the tremendous extent of the material universe in space and time, and of its essential unity, in that the same types of matter and the same natural laws are found everywhere, is of great significance. Though our own planet thus appears as an insignificant speck, it is yet likely to be habitable for millions of years to come. The

appreciation of these facts cannot fail to possess an important influence in determining the attitude of the contemplative student toward such problems of philosophy as man's obligations to future generations, his place in the universe, and his relation to the Power which is behind it.

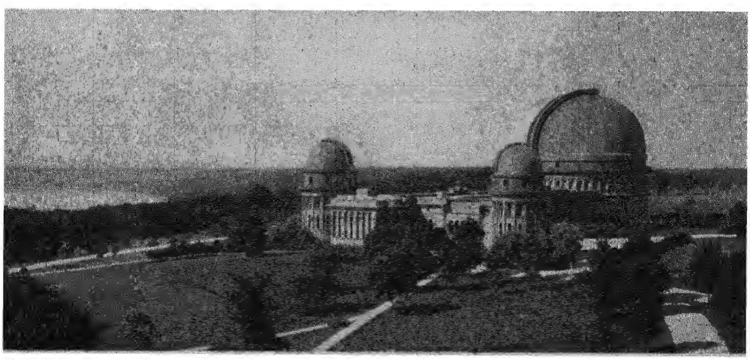
REFERENCES

Reference is made, at the end of each chapter, to a short list of books which contain fuller accounts of the matter treated in the chapter; for current material and the details of specific investigations the student must consult the periodicals and the observatory publications. The most important periodicals, and the abbreviations of their titles usually employed, are:

The Astronomical Journal (*A. J.*), dealing mainly with the astronomy of position.
The Astrophysical Journal (*Ap. J.*), which includes all the Mt. Wilson *Contributions*.
Publications of the Astronomical Society of the Pacific (*A. S. P.*).
Popular Astronomy (*P. A.*), largely for the amateur.
Monthly Notices of the Royal Astronomical Society (*M. N.*), containing the work of English astronomers in all branches of the science.
Astronomische Nachrichten (*A. N.*), the principal astronomical periodical on the continent of Europe.
Vierteljahrsschrift der Astronomischen Gesellschaft (*V. J. S.*), which contains observatory reports, and statistics of planets, comets, and variable stars.
Bulletin of the Astronomical Institutes of the Netherlands (*B. A. N.*), in English, published jointly by the Dutch observatories.

Summaries of the work done at most observatories appear as articles in the periodicals, but longer accounts, and in some cases short communications as well, appear in observatory publications. A few of the most important are:

Harvard Observatory: Annals, Circulars, and Bulletins.
Lick Observatory: Publications and Bulletins.
Dominion Astrophysical Observatory: Publications.
Potsdam Astrophysikalisches Observatorium: Publikationen.



Yerkes Observatory

CHAPTER I

ASTRONOMICAL SYSTEMS OF MEASUREMENT

THE CELESTIAL SPHERE • SYSTEMS OF COÖRDINATES AND THEIR TRANSFORMATION • TIME • THE CELESTIAL GLOBE

Astronomy, like all the other sciences, has a terminology of its own and uses technical terms in the description of its facts and phenomena. It is desirable that the student should be introduced to many of them at the very outset.

10. The Celestial Sphere.[1] The simplest form of astronomical observation is to note the relative positions of the stars in the sky. The line from the observer to the object at which he is looking is called a *line of sight.* It is difficult to make a picture on paper, or even in the mind, representing satisfactorily the *lines of sight* radiating in all directions from the point of observation. The best way is to draw a sphere about this point and mark the spot where each line cuts the sphere (Fig. 1). The distance on the sphere between two such spots then becomes a convenient equivalent of the difference in direction of two radiating lines. It has been found convenient to imagine the sphere as so enormous that the whole material universe of stars and planets lies in its center like a few grains of sand in the middle of the dome of the Capitol. Although the *celestial sphere* is thus conceived as indefinitely large, it is practically necessary, in making pictures of it, to represent it as if it were seen from the outside.[2] Any two parallel lines will pierce the surface of the sphere in two points that are indistinguishable to an observer at its infinitely distant center. Thus the axis of the earth and all lines parallel to it pierce the heavens at one point, the celestial pole.

[1] The study of the celestial sphere and its circles is greatly aided by the use of a globe or an armillary sphere. Without some such apparatus it is rather difficult for a beginner to get clear ideas on the subject.

[2] Star maps, however, picture the sky as it is seen by the observer at the center of the sphere.

11. The apparent place of a heavenly body is simply the point where a line drawn from the observer through the body in question, and continued onward, pierces the celestial sphere. It depends solely upon the direction of the body and has nothing to do with its distance (Fig. 1).

The *apparent distance* between two stars is therefore simply a difference in direction, and the *apparent diameter* of the moon is the angular separation between lines of sight to diametrically opposite points of the moon's disk (Fig. 42). Obviously, angular units alone can properly be used in describing apparent distances in the sky. One cannot say correctly that the two stars known as the pointers are so many *feet* apart; their distance is approximately five *degrees*, — which is the length of the arc of the great circle on the celestial sphere connecting the two stars.

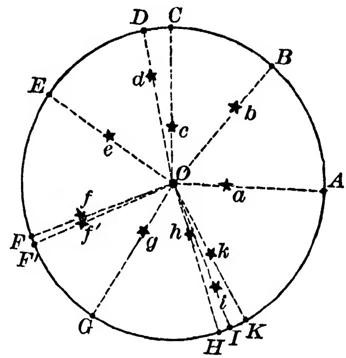


Fig. 1. Apparent Places of Stars on the Celestial Sphere

A, *B*, *C*, etc. are the apparent places of *a*, *b*, *c*, etc., the observer being at *O*. Objects that are nearly in line with each other, such as *h*, *i*, *k*, will appear close together in the sky, however great the real distance between them. The moon, for instance, often looks to us very near a star, which is always a great distance beyond

The student of astronomy should accustom himself as soon as possible to estimating celestial measures in angular units. A little practice soon makes it easy, although the beginner is likely to be embarrassed by the fact that the sky appears not as a true hemisphere but as a flattened vault, so that all estimates of angular distances for objects near the horizon are likely to be exaggerated. The moon when rising or setting looks to most persons much larger than when overhead, and the "Dipper bowl" when underneath the pole seems to cover a much larger area than when above it.

These illusions are directly traceable to the unconscious habit, developed from an early age, of interpreting apparent size by the aid of our familiarity with real size. This perspective adjustment is strongly developed in the horizontal plane, to which our experiences are largely confined, but is very imperfect in the vertical direction. It is worth remarking that a ship seen below an airplane looks much smaller than one seen on the horizon at the same distance.

POINTS AND CIRCLES OF REFERENCE AND SYSTEMS OF COÖRDINATES

In order to be able to describe the position of a heavenly body in the sky it is convenient to suppose the inner surface of the celestial sphere to be marked off by circles traced upon it, — imaginary circles, of course, like the equator, the meridians, and the parallels of latitude which serve as a system of reference for the latitude and longitude of a point on the surface of the earth.

Several distinct systems of such circles are made use of in astronomy, each of which has its own peculiar adaptation to its special purpose. It is evident that, to be practically useful, these circles must be so defined that their positions can be fixed by observation with as great precision as may be desirable.

A. System Depending on the Direction of Gravity at the Point where the Observer Stands

12. The Zenith and Nadir. If we suspend a plumb-line and imagine the line extended upward to the sky, it will pierce the celestial sphere at a point directly overhead known as the *astronomical zenith* or simply the *zenith*.

The *nadir* is the point opposite the zenith, directly under foot in the invisible part of the celestial sphere.

Both "zenith" and "nadir" are derived from the Arabic, as are many other astronomical terms introduced during the centuries when the Arabs were the chief cultivators of science.

As will be seen later (§ 143), the plumb-line does not point exactly to the center of the earth, because the earth rotates on its axis and is not strictly spherical. If an imaginary line be drawn from the center of the earth upward through the observer, and produced to the celestial sphere, it marks a different point, known as the *geocentric zenith*, which is never very far from the astronomical zenith but must not be confounded with it.

13. The Horizon. If now we imagine a great circle drawn completely around the celestial sphere halfway between the zenith and the nadir, and therefore 90° from each of them, it will be the *horizon*. Since the surface of still water is always perpendicular

to the direction of gravity, the horizon may also be defined as the great circle in which a plane tangent to a surface of still water at the place of observation cuts the celestial sphere.

The word "horizon" (from the Greek) means literally "the boundary," that is, the limit of the landscape, where sky meets earth or sea. This boundary line is known in astronomy as the *visible horizon.* On land it is irregular, but at sea it is practically a true circle, nearly coinciding with the horizon above defined.

14. Vertical Circles; the Meridian and the Prime Vertical. *Vertical circles* are great circles drawn from the zenith at right angles to the horizon, and therefore passing through the nadir also. Any point in the heavens has one of these circles passing through it.

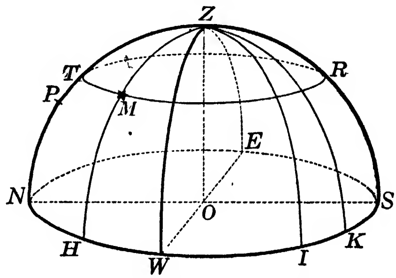


FIG. 2. The Horizon and Vertical Circles

O, the place of the observer; OZ, the observer's vertical; Z, the zenith; P, the pole; $SWNE$, the horizon; $SZPN$, the meridian; EZW, the prime vertical; M, some star; ZMH, arc of the star's vertical circle; TMR, the star's almucantar; angle SZM, or arc SWH, star's azimuth; arc HM, star's altitude; arc ZM, star's zenith distance

That particular vertical circle which passes north and south, through the pole (to be defined hereafter), is known as the *celestial meridian* and is the circle traced upon the celestial sphere by the plane of the terrestrial meridian upon which the observer is located. The vertical circle at right angles to the meridian is called the *prime vertical.* The points where the meridian intersects the horizon are the *north* and *south points*; and the *east* and *west points* are midway between them. These are known as the four *cardinal points.*

The *parallels of altitude*, or *almucantars*, are small circles of the celestial sphere parallel to the horizon, sometimes called *circles of equal altitude.*

15. Altitude and Zenith Distance. The *altitude* of a heavenly body is its angular elevation above the horizon, that is, the number of degrees between it and the horizon, measured on the vertical circle passing through the object. Referring to Fig. 2, the vertical circle ZMH passes through the body M.

The arc MH is the *altitude* of M, and the arc ZM (the complement of MH) is its *zenith distance*.

16. Azimuth. The *azimuth* of a heavenly body is the same as its "bearing" in surveying, — measured, however, from the true meridian and not from the magnetic. It may be defined as *the angle formed at the zenith between the meridian and the vertical circle which passes through the object*; or, what comes to the same thing, it is *the arc of the horizon measured westward from the south point to the foot of this circle.*

SZM (Fig. 2) is the azimuth of M, as is also the arc SWH, which measures this angle. Azimuth is reckoned by astronomers from the south point clear round through the west to the point of beginning, consequently the arc SWH rather than SEH is the azimuth of M, — about 130°.

Navigators now quite generally begin their reckoning at the north point and count in degrees through the east, but the old *points* are still retained on the compass cards. Surveyors often describe the bearing as so many degrees east or west of north or south. Thus, in Fig. 2 the bearing of M is N. 50° W., and the corresponding *course* of a ship would be described as 310° or, very nearly, NW$\frac{1}{2}$W.

B. System Depending upon the Direction of the Earth's Axis of Rotation

17. The Apparent Diurnal Rotation of the Heavens. If, on some clear evening in the early autumn, say about eight o'clock on the twenty-second of September, we face the north, we shall find the appearance of that part of the heavens directly before us substantially as shown in Fig. 3. In the north is the constellation of the Great Bear (Ursa Major), characterized by the conspicuous group of seven stars known as the Great Dipper, which lies with its handle sloping upward to the west. The two easternmost stars of the four which form its bowl are called the *pointers*, because they point to the *polestar*, — a solitary star not quite halfway from the horizon to the zenith (in the latitude of New York) and about as bright as the brighter of the two pointers. It is often called *Polaris*.

High up on the opposite side of the polestar from the Great Dipper, and at nearly the same distance, is an irregular zigzag

of five stars, each about as bright as the polestar itself. This is the constellation of Cassiopeia.

If now we watch these stars for only a few hours, we shall find that while all the configurations remain unaltered, their places in the sky are slowly changing. The Dipper slides downward toward the north, so that by eleven o'clock the pointers are

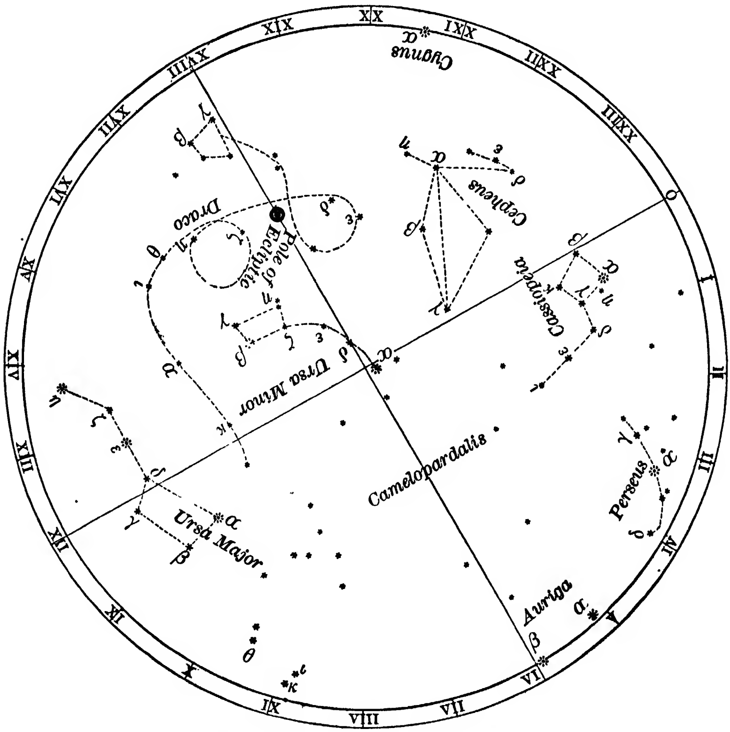


FIG. 3. The Northern Circumpolar Constellations

directly under Polaris. Cassiopeia still keeps opposite, however, rising toward the zenith; and if we were to continue to watch them all night, we should find that all the stars appear to be moving in circles around a point near the polestar, revolving in the opposite direction to the hands of a watch (as we look up toward the north) with a steady motion which takes them completely around once a day, or, to be exact, once in the *sidereal day*, which consists of $23^h\ 56^m\ 4^s.1$ of ordinary time. Thus the

stars behave just as if they were attached to the inner surface of a huge revolving sphere — the celestial sphere.

Instead of watching the stars with the eye we can reach the same result still better by photography. A camera is pointed up toward the polestar and remains firmly fixed while the stars, by their diurnal motion, impress their "trails" upon the plate (Fig. 4).

FIG. 4. Polar Star-Trails

During the exposure of nearly three hours each star has, by its diurnal motion, drawn a "trail" on the plate. The brighter stars make the heavier trails. All arcs are the same fraction of a complete circle, and they are all centered at the pole. The brightest trail — nearest but one to the pole — is that of Polaris

If instead of looking toward the north we now look southward, we shall find that there also the stars appear to move in the same kind of way. All that are not too near the polestar rise somewhere in the eastern horizon, ascend not vertically but obliquely to the meridian, and descend obliquely to their setting at points on the western horizon. The motion is always in an arc of a circle, called the star's *diurnal circle*, the size of which depends upon the star's distance from the pole. Moreover, all these arcs are strictly parallel.

The ancients accounted for these obvious facts by supposing the stars actually fixed upon a real material sphere, really turning daily in the manner indicated. According to this view, therefore, there must be upon the sphere two opposite, pivotal points which remain at rest, and these are the *poles*.

18. Definition of the Poles. The *celestial poles*, or the *poles of rotation*, may therefore be defined as those *two points in the sky where a star would have no diurnal motion*. The exact position of either pole may be determined with proper instruments by find-

ing the center of the small diurnal circle described by some star near it, as, for instance, the polestar.

Since the two poles are diametrically opposite in the sky, only one of them is usually visible from a given place; observers north of the equator see only the north pole, and vice versa in the southern hemisphere.

Knowing as we now do that the apparent revolution of the celestial sphere is due to the real rotation of the earth on its axis, we may also define the poles as the *two points where the earth's axis of rotation* (*or any set of lines parallel to it*), *produced indefinitely, would pierce the celestial sphere.*

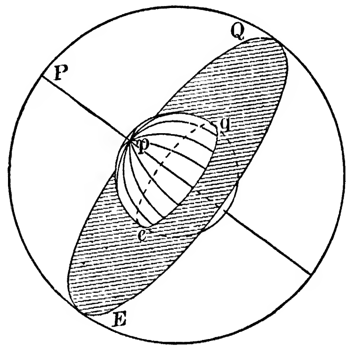


FIG. 5. The Plane of the Earth's Equator

When produced to cut the celestial sphere, this plane traces out the celestial equator

19. The Celestial Equator, and Hour-Circles. *The celestial equator is the great circle of the celestial sphere, drawn halfway between the poles* (and therefore everywhere 90° from each of them), *and is the great circle in which the plane of the earth's equator cuts the celestial sphere,* as illustrated in Fig. 5. Small circles drawn parallel to the celestial equator, like the parallels of latitude on the earth, are called *parallels of declination.* A star's parallel of declination is identical with its *diurnal circle.*

The great circles of the celestial sphere, which pass through the poles in the same way as the meridians on the earth, and which are therefore perpendicular to the celestial equator, are called *hour-circles.* Each star has its own hour-circle, which apparently moves with it. *That particular hour-circle which at any moment passes through the zenith of the observer coincides with the celestial meridian,* already defined.

20. Declination and Hour Angle. *The declination of a star is its distance in degrees north or south of the celestial equator,* + *if north and* − *if south.* It corresponds closely to the *latitude* of a place on the earth's surface, but cannot be called *celestial latitude,* because the term has been preëmpted for an entirely different

quantity, to be defined later (§ 23). The *polar distance* is the complement of the declination.

The *hour angle* of a star at any moment *is the angle at the pole between the celestial meridian and the hour-circle of the star;* or it is the *arc of the equator measured westward from the celestial meridian to the foot of the hour-circle.* In Fig. 6, for the body X it is the angle QPX or the arc QY.

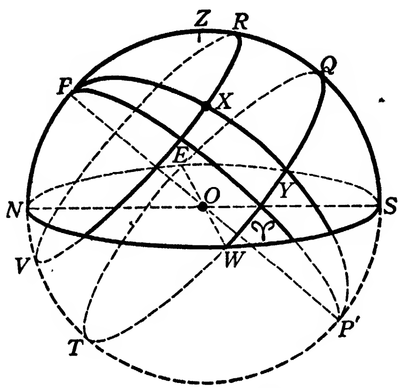


Fig. 6. Circles of the Celestial Sphere

O, place of the observer; Z, his zenith; $SENW$, the horizon; POP', the axis of the celestial sphere; P and P', the two celestial poles; $EQWT$, the celestial equator; ♈, the vernal equinox, or "first of Aries"; $P♈P'$, the equinoctial colure; X, some star; PXP', the star's hour-circle; YX, the star's declination; PX, the star's north-polar distance; angle RPX = arc QY, the star's hour angle; angle ♈PX = arc ♈Y, star's right ascension; sidereal time at the moment = angle QP♈ = arc Q♈

This angle or arc may of course be measured like any other, in degrees; but since it depends upon the time which has elapsed since the body was last on the meridian, it is more usual to measure it in hours, minutes, and seconds of time. The *hour* is then equivalent to 1/24 of a circumference, or 15°, and the *minute* and *second* of *time* to 15′ and 15″ of arc respectively. Thus, an hour angle of $4^h\ 2^m\ 3^s$ equals 60° 30′ 45″.

The position of the body X (Fig. 6) is, then, perfectly defined by saying that its *declination* is + 25° and its *hour angle* 40°. The hour angle of a star to the east of the meridian may be regarded as negative. Thus, the hour angle $21^h\ 20^m = -\ 2^h\ 40^m$.

21. The Ecliptic, Equinoxes, Solstices, and Colures. The sun, moon, and planets, though apparently carried by the diurnal revolution of the celestial sphere, are not, like the stars, apparently fixed upon it, but move over its surface like glowworms creeping on a whirling globe. In the course of a year, as will be explained later (§ 157), the sun makes a complete circuit of the heavens, traveling among the stars in a great circle called the *ecliptic.*

The ecliptic cuts the celestial equator in two opposite points at an angle of about $23\frac{1}{2}°$. These points are the *equinoxes.* The *vernal equinox*, or *first of Aries* (symbol ♈), *is the point where the sun crosses from the south to the north side of the equator*, on or about the twenty-first of March. The other is the *autumnal equinox.*

The angle at which the ecliptic and equator intersect is called the *obliquity of the ecliptic.*

The summer and winter *solstices* are points on the ecliptic, midway between the two equinoxes and 90° from each, where the sun attains its extreme declination of $+23\frac{1}{2}°$ and $-23\frac{1}{2}°$ — in summer and winter respectively, in the northern hemisphere.

The hour-circles drawn from the celestial pole through the equinoxes and solstices are called the *equinoctial* and the *solstitial colure* respectively.

Neglecting for the present the gradual effect of precession (§ 166), these points and circles are fixed with reference to the stars, and form a framework by which the places of celestial objects may be conveniently defined and catalogued.

No conspicuous star marks the position of the vernal equinox, but a line drawn from the polestar through β Cassiopeiæ and α Andromedæ, and continued another 30°, will strike very near it.

22. Right Ascension. The *right ascension* of a star may now be defined as *the angle made at the celestial pole between the hour-circle of the star and the hour-circle which passes through the vernal equinox* (called the *equinoctial colure*), or as *the arc of the celestial equator intercepted between the vernal equinox and the point where the star's hour-circle cuts the equator.* Right ascension is reckoned always *eastward* from the equinox, completely around the circle, and may be expressed either in degrees or in time units. A star one degree *west* of the equinox has a right ascension of 359°, or $23^h\ 56^m$.

Evidently the diurnal motion does not affect the right ascension of a star, but, like the declination, it remains practically unchanged for years.

C. Other Systems of Coördinates

23. Celestial Latitude and Longitude. The ancient astronomers confined their observations mostly to the sun, moon, and planets, which are never far from the ecliptic, and for this reason the *ecliptic* (which is simply the trace of the plane of the earth's orbit upon the celestial sphere) was for them a more convenient circle of reference than the equator. According to their terminology, *latitude (celestial) is the angular distance of a heavenly body north or south of the ecliptic, and longitude (celestial) is the arc of the ecliptic intercepted between the vernal equinox* (♈) *and the foot of a great circle drawn from the pole of the ecliptic, to the ecliptic, through the object.* Longitude, like right ascension, is always reckoned *eastward* from the equinox. These coördinates are still the most convenient to use in calculations dealing with the orbits or motions of the planets and the moon.

The *poles of the ecliptic* are the points 90° distant from the ecliptic. The position of the north ecliptic pole is shown in Fig. 7. It is on the solstitial colure, and its distance from the celestial pole is equal to the obliquity of the ecliptic. It is therefore in declination $+66\frac{1}{2}°$ and right ascension 18^h. It is marked by no conspicuous star.

It is confusing for beginners that celestial latitude and longitude do not correspond with the terrestrial quantities that bear the same name. Care must be taken to observe the distinction.

24. The Zodiac and its Signs. A belt 18° wide (9° on each side of the ecliptic) is called the *zodiac*, or "zone of animals" (German, *Thierkreis*), all the constellations in it, excepting Libra, being figures of living creatures. It is taken of that particular width simply because the moon and the principal planets always keep within it. It is divided into the so-called "signs," each 30° in length, having the following names and symbols:

Spring	Aries	♈	Autumn	Libra	♎
	Taurus	♉		Scorpio	♏
	Gemini	♊		Sagittarius	♐
Summer	Cancer	♋	Winter	Capricornus	♑
	Leo	♌		Aquarius	♒
	Virgo	♍		Pisces	♓

The symbols are for the most part conventional pictures of the objects. The symbol for Aquarius is the Egyptian character for water. The origin of the signs for Leo, Capricornus, and Virgo is not clear.

25. Galactic Coördinates. One more system of coördinates — a very modern one — deserves mention. In this the *plane of the Milky Way* is taken as fundamental, and latitudes are measured from the *galactic equator*, which is its trace on the sky, while longitudes are reckoned along this circle, starting from its intersection with the celestial equator in $18^h\,40^m$ right ascension. These *galactic coördinates* are used in studies of the apparent distribution of the stars in space. The concentration of the stars toward this plane renders it far more fundamental than any of the others with which we have dealt; but since the Milky Way is a broad and ill-defined belt in the sky, the position of its central line cannot be precisely determined by observation, and galactic coördinates are therefore not suitable for the precise specification of the positions of the heavenly bodies.

The accepted position of the north galactic pole is $12^h\,40^m$ right ascension and 28° north declination, so that the galactic equator cuts the celestial equator at an angle of 62°.

26. Recapitulation. The *direction of gravity* at the point where the observer happens to stand determines his *zenith*, *nadir*, and *horizon*, the *almucantars*, or *parallels of altitude*, and all the *vertical circles*. One of the vertical circles, the *meridian*, is singled out from the rest by the circumstance that it *passes through the pole*, marking the *north* and *south points* where it cuts the horizon. *Altitude* and *azimuth* define the position of a body by reference to the horizon and meridian.

This set of points and circles shifts its position among the stars with every change in the place of the observer and with every moment of time.

In a similar way the *direction of the earth's axis*, which is independent of the observer's place on the earth, determines the *celestial pole*, the *equator*, the *parallels of declination*, and the *hour-circles*. Two of these hour-circles are singled out as reference lines. One of them is the hour-circle which at any moment passes through the zenith and coincides with the meridian, — a

purely local reference line. The other, the *equinoctial colure*, passes through the vernal equinox, a point chosen from its relation to the sun's annual motion.

Declination and *hour angle* define the place of a star with reference to the equator and *meridian*, while *declination* and *right ascension* refer it to the equator and *vernal equinox*. The latter pair of coördinates are not affected by the diurnal motion, and remain practically unchanged for years. They are the coördinates usually given in star catalogues and almanacs for the purpose of defining the position of stars and planets, *and they correspond closely to latitude and longitude on the earth*, by means of which geographical positions are designated.

The earth's orbit gives us the great circle of the sky known as the *ecliptic*, and *celestial latitude* and *longitude* define the position of a star with reference to the ecliptic and vernal equinox (♈). For most purposes this pair of coördinates is practically less convenient than right ascension and declination, but it came into use centuries earlier and has advantages in dealing with the planets and the moon. *Galactic latitudes and longitudes*, referred to the plane of the Milky Way, are sometimes used in dealing with the stars.

System	Primary Circle, how Determined	Primary Circle	Origin	Secondary Circle	Coördinates	Usual Symbol
A	Direction of gravity	Horizon	South point on horizon	Vertical circle of star	Azimuth Altitude	(A) (h)
B 1	Rotation of earth	Celestial equator	Foot of the meridian on the equator	Hour-circle of star	Hour angle Declination	(t) (δ)
B 2	Rotation of earth	Celestial equator	The vernal equinox (♈)	Hour-circle of star	Right ascension Declination	(α) (δ)
C	Plane of earth's orbit	Ecliptic	The vernal equinox (♈)	Secondary to ecliptic through star	Longitude Latitude	(λ) (β)
D	Plane of Milky Way	Galactic equator	Intersection with celestial equator	Secondary to Galaxy through star	Galactic longitude Galactic latitude	(G) (g)

27. The scheme given on page 20 presents in tabular form the relations of the five different systems to each other. In each case one of the two coördinates is measured along a *primary* great circle, from a point selected as the *origin* to a point where a *secondary* circle cuts it, drawn through the object perpendicular to the primary. The second coördinate is the angular distance of the object from the primary circle measured along this secondary.

Still other coördinate systems are possible. For example, a planet has an equator and equinox of its own, to which the positions of the heavenly bodies would naturally be referred by an observer on its surface. *Planetocentric* and *selenographic* coördinates of the earth and sun are of use to observers of the surface markings of Mars, Jupiter, and the moon, and are given in the *American Ephemeris*.

LATITUDE, LONGITUDE, AND TIME

28. Astronomical Latitude. Evidently the appearance of the heavens for any observer will be radically influenced by the distance between his zenith and the equator. This quantity is called the *astronomical latitude* and is defined as the angle between the observer's *vertical* and the plane of the equator. This is obviously equal to the *declination of the zenith* and, almost as obviously, to the *altitude of the pole*; for PN and ZQ (Fig. 7) may be obtained by subtracting PZ from PQ or ZN, each of which equals 90°. These fundamental relations cannot be too strongly emphasized.

Since the earth is not exactly round, the distance of a point from the earth's equator measured on the earth's surface is not an exact measure of the astronomical latitude (Fig. 8).

29. The Right Sphere. If the observer is situated at the earth's equator, that is, in latitude *zero*, the pole will be in his horizon, and the celestial equator will be a vertical circle, coinciding with the prime vertical (§ 14). All heavenly bodies will *rise and set vertically*, and their diurnal circles will all be bisected by the horizon, so that they will be twelve hours above and twelve hours below it; and the length of the night will always equal that of the day (neglecting refraction, § 114). This aspect of the heavens is called the *right sphere*.

30. The Parallel Sphere. If the observer is at the pole of the earth, where his latitude is 90°, the celestial pole will be at his zenith, and the equator will coincide with his horizon. If he is at the north pole, all the stars north of the celestial equator will remain permanently above the horizon, never rising or setting, but sailing around the sky on almucantars, or parallels of altitude; while the southern stars will never rise to view.

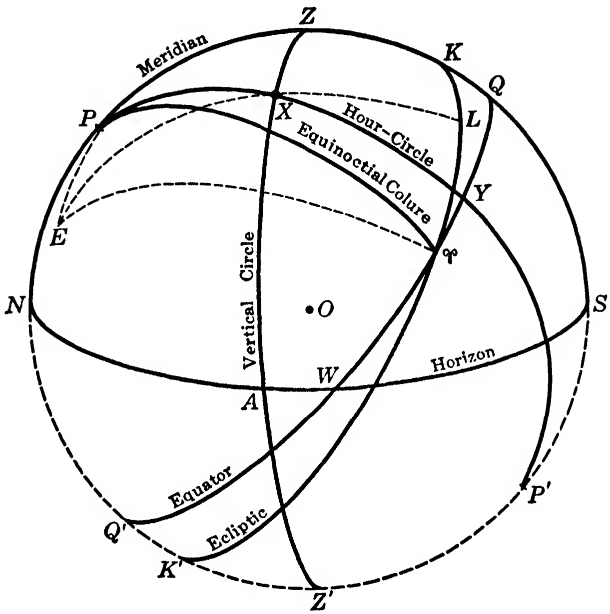


FIG. 7. Relation of the Different Coördinates

The figure shows how the coördinates are related to each other. The celestial sphere is represented as seen from a point *outside* it, on the west and a little above the plane of the horizon. O is the observer at the center of the sphere, OZ the observer's vertical. Z is the zenith, Z' the nadir, and P and P' are the *north* and *south celestial poles*. The circle $PZQSP'Z'N$ is the *meridian*. $NAWS$ is the *horizon*, intersecting the meridian in the *north* and *south* points N and S. $QYWQ'$ is the *equator*, intersecting the horizon at the *west point* W and the meridian at the "south point on the equator" Q. KLK' is the *ecliptic*, intersecting the equator at the *vernal equinox* ♈. E is the *north pole of the ecliptic*. The arc PE, or the angle K♈Q, is the *obliquity of the ecliptic*. X is some celestial object. $ZXAZ'$ is the *vertical circle* of the object. AX is its *altitude*, and XZ its *zenith distance*. Its *azimuth* is the angle SZX (measured also by the arc SA on the horizon). $PXYP'$ is the *hour-circle* of the object, YX its *declination*, and QPX its *hour angle* (which also equals the arc QY). ♈PY is its *right ascension* = arc ♈Y. The angle QP♈ (or the arc Q♈) is the *sidereal time* (§ 34), LX is the *celestial latitude* of the object, and ♈L (or the angle ♈EL) its *longitude*. P♈ is a quadrant of the equinoctial colure, and PE a portion of the solstitial colure. To avoid complication, the galactic coördinates are not shown

Since the sun and the moon move among the stars in such a way that half the time they are north of the equator and half the time south of it, they will be half the time above the horizon and half the time below it (again neglecting refraction). The moon will be visible for about a fortnight each month, and the sun for about six months each year.

It is worth noting that for an observer *exactly* at the north pole the definitions of meridian and azimuth break down, since at that point the zenith coincides with the pole. Facing in whichever direction he will, he is still looking directly *south*. If he changes his place a few steps, however, his zenith will move, and everything will become definite again.

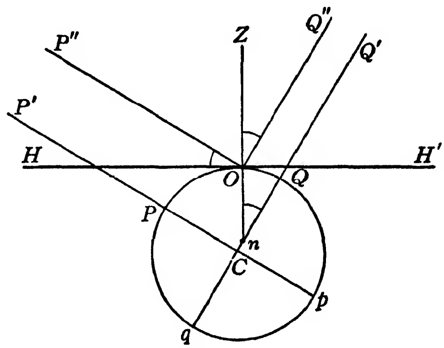


FIG. 8. Altitude of the Pole Equals the Observer's Latitude

This is a section of the earth through the observer's meridian. The observer is at O. His zenith is in the direction of Z, and his horizon is represented by HOH'. P'' lies in the direction of the pole, and the line OQ'' is in the plane of the celestial equator. The altitude of the pole, HOP'', equals the declination of the zenith, $Q''OZ$, which equals QnO, the astronomical latitude of the observer — On representing the direction of the nadir

31. The Oblique Sphere. At any station between the poles and the equator the pole will be elevated above the horizon, and the stars will rise and set in *oblique circles*, as shown in Fig. 9. Those whose distance from the elevated pole is less than PN (the latitude of the observer) will of course never set, remaining perpetually visible. The radius of this *circle of perpetual apparition*, as it is called (the shaded cap around P in the figure), is obviously just equal to the height of the pole, becoming larger as the latitude increases. On the other hand, stars within the same distance of the depressed pole will lie in the *circle of perpetual occultation* and will never rise above the horizon. A star exactly on the celestial equator will have its diurnal circle bisected by the horizon and will be above the horizon twelve hours. A star north of the equator, if the north pole is the elevated one, will have more than half its diurnal circle above the horizon and will be visible

for more than twelve hours each day, as, for instance, a star rising at *B* and setting at *B′*.

Whenever the sun is north of the equator, it will, in all north latitudes, rise at a point north of east, as *B* in the figure, and will continue to shine upon every vertical surface that faces the north, until, as it ascends, it crosses the prime vertical *EZW* at some point *V*.

In the latitude of New York the sun on the longest days of summer is south of the prime vertical only about eight hours of the whole fifteen during which it is above the horizon. During seven hours of the day it shines into north windows.

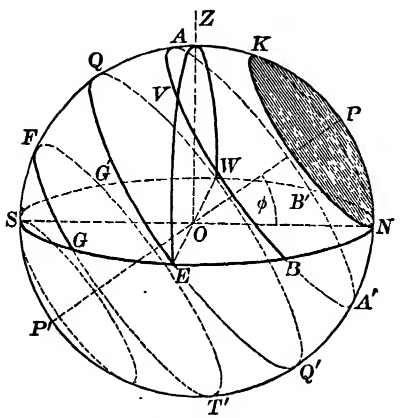


FIG. 9. The Oblique Sphere

32. The Midnight Sun. If the latitude of the observer is such that *PN* (Fig. 9) is greater than the sun's polar distance at the time when the sun is farthest north (about $66\frac{1}{2}°$), the sun will come into the circle of perpetual apparition and will make a complete circuit of the heavens without setting, until its polar distance again becomes greater than *PN*. This happens near the summer solstice at the North Cape and at *all stations within the arctic circle.* The "midnight sun" is illustrated in Fig. 67.

33. Measurement of Time. The definition of time raises difficult philosophical questions, but its *measurement* falls within the province of astronomy. In practice, time is always measured by the progress of some sort of *motion.* From the beginning the apparent diurnal rotation of the heavens has been accepted as the standard, and by this standard all mechanical measurers of time, such as clocks and watches, are regulated.

We may choose any object or point in the heavens to mark the beginning of the day by its passage across the meridian. The *measure of time* (or, briefly, the *time*) at any other instant will then be the hour angle of this object at that instant.

In astronomy three kinds of time are now recognized, *sidereal time*, *apparent solar time*, and *mean solar time*, the last being essentially the time of civil life and ordinary business, while the first is used exclusively for astronomical purposes.

34. Sidereal Time. The celestial object which determines sidereal time, by its position in the sky at any moment, is the *vernal equinox*, or *first of Aries*.

The local sidereal *day* begins at the moment when the first of Aries crosses the observer's meridian, and the *sidereal time* at any moment *is the hour angle of the vernal equinox.* It would be marked by a perfect clock so set and adjusted as to show *sidereal noon* ($0^h\ 0^m\ 0^s$) at each transit of the first of Aries.

The equinoctial point is, of course, invisible; but its position among the stars is always known, so that its hour angle at any moment can be determined by observing the stars. On account of the precession of the equinoxes (§ 166) the sidereal day thus defined is slightly shorter than it would be if defined by the transits of a point *fixed among the stars*. This difference amounts, on the average, to 1/120 of a second; but as the motion of the equinox is not uniform, not all sidereal days are of exactly the same length. The differences between them, however, are much smaller than the errors of the best clocks, and may be neglected.

35. Apparent Solar Time. Just as sidereal time is the hour angle of the vernal equinox, so *apparent solar time* at any moment is *the hour angle of the sun.* It is the *time shown by the sundial*, and its noon occurs at the moment when the sun's center crosses the meridian.

On account of the earth's orbital motion the sun appears to move eastward along the ecliptic, completing its circuit in a year. Each noon, therefore, it occupies a place among the stars about a degree farther east than it did the noon before, and so comes to the meridian about four minutes *later*, if time is reckoned by a *sidereal clock*. In other words, the solar day is about four minutes *longer* than the sidereal, the difference amounting to *exactly one day* each year, which contains $366\frac{1}{4}$ sidereal days.

But the sun's eastward motion is, for several reasons, not uniform, and the apparent solar days vary in length (§ 170). December 23, for instance, is about fifty-one seconds longer from

sundial noon to the next noon, by a sidereal clock, than September 16. Apparent solar time is therefore not a "uniformly increasing quantity" and is not satisfactory for scientific purposes. Indeed, it is so far from being so that special complicated mechanism would be required to make a clock or a watch indicate it.

36. Mean Solar Time. A *fictitious sun*, therefore, is imagined, which moves *uniformly eastward in the celestial equator*, completing its annual course in exactly the same time as that in which the actual sun makes the circuit of the *ecliptic*, and, on the average for the whole year, running as much behind the actual sun as ahead of it. This fictitious sun is the time-keeper for *mean solar time*. It is *mean noon* when it is on the meridian, and at any moment the *hour angle of the mean sun* is the mean time for that moment. All the mean solar days, therefore, are of exactly the same length and equal to the length of the *average* apparent solar day. The mean solar day is *longer* than the *sidereal* day by $3^m\ 55^s.91$ *mean solar* minutes and seconds, and the *sidereal* day *shorter* than the solar by $3^m\ 56^s.56$ *sidereal* minutes and seconds.

Sidereal time will not answer for business purposes, because its noon (the transit of the vernal equinox) occurs at all hours of the day and night in different seasons of the year; on September 22, for instance, it comes at midnight. Apparent solar time is unsatisfactory because of the variation in the length of its days and hours. Yet we have to live by the sun; its rising and setting, daylight and night, control our actions.

Mean solar time furnishes a satisfactory compromise. It has a time unit which is invariable,[1] and it can be kept by clocks and watches, while it agrees with sundial time closely enough for convenience. It is the time now used for all purposes except in some kinds of astronomical work, and, in particular, in defining the units employed in physics and other sciences.

The difference between apparent time and mean time (never amounting to more than about a quarter of an hour) is called the *equation of time* and will be discussed hereafter in connection with the earth's orbital motion (§ 169).

37. The Civil Day and the Astronomical Day. From January 1, 1925, astronomers have agreed to use the *civil* day, beginning at

[1] Except for a very minute change in the course of centuries (§ 136).

midnight and reckoned around through the whole twenty-four hours. Before this date an *astronomical* day was employed, which began twelve hours later. The reason was that astronomers are "night birds" and found it inconvenient to change dates at midnight, in the middle of their work. This must be borne in mind in referring to almanacs of 1924 or earlier; thus, 10 o'clock (10 A.M.) of Wednesday, February 27, *civil* reckoning, is Tuesday, February 26, 22 o'clock, by the old *astronomical* reckoning. *Civil time* is therefore obtained by *adding twelve hours to mean solar time* (which may change the day, the month, or even the year).

This is the present usage (1926) of the American and French nautical almanacs. The *British Nautical Almanac* of 1926 uses the designation "mean time" for what is here called "civil time." The French call Greenwich civil time *temps universel* or *temps civil de Greenwich*; the Germans, *Weltzeit*. It is greatly to be hoped that an international agreement may soon be reached. Meanwhile the meaning of the terms employed in each nautical almanac is clearly explained in the text.

38. Terrestrial Longitude. The longitude of a place on the earth may be defined as *the angle at the celestial pole between a standard meridian and the meridian of the place, or as the arc of the celestial equator intercepted between the two meridians.*

It would be equally permissible to measure this angle at the pole of the earth if it were not for the effects of deviations of the vertical (§ 142).

There is no inherent reason why one meridian should be chosen as a standard rather than another, and the decision must be arbitrary. The *meridian of Greenwich*, by international agreement, is now used in almost all cases.

This usage arose because Greenwich is the national observatory of England, and British ships naturally referred their longitudes to its meridian. The system was thus spread all over the world, and other countries have gradually adopted it, the advantages of having a single system overcoming local prejudices. From the definition of hour angle it follows that the difference between the hour angles of any celestial object, as seen from two places at the same instant, is equal to the difference of their longitudes, and therefore that *the longitude of any observer is*

equal to the difference between his local time and Greenwich time. It makes no difference what kind of time is used, so long as it is the same kind at both stations.

Longitude is usually expressed by geographers in degrees and by astronomers in hours, being reckoned in both directions from Greenwich, up to 180° or 12^h. West longitudes are considered positive.

39. Local and Standard Time. It was formerly customary to use only local time, each observer determining his own time by his own observations. But with the development of the telegraph and railway this became increasingly inconvenient, and now a system of standard time is almost everywhere in use. This greatly facilitates railway and telegraphic business, and makes it easy for everyone to keep accurate time, since signals can be sent daily from some observatory to every telegraph office and radio set.

In the United States and Canada, for example, there are five such standard times in use, — the Atlantic, Eastern, Central, Mountain, and Pacific, — which are respectively four, five, six, seven, and eight hours slow, compared with Greenwich time, the minutes and seconds being identical everywhere. The boundaries between time belts are irregular, the time usually changing at railroad division points.

Greenwich time is now adopted as the standard throughout western Europe; the standard time in central Europe is 1^h fast, compared with Greenwich time; in eastern Europe (except Russia), South Africa, and Egypt it is 2^h fast; in India, $5\frac{1}{2}^h$ fast; in Burma, $6\frac{1}{2}^h$ fast; in Western Australia, 8^h, and in New Zealand, $11\frac{1}{2}^h$ fast; in Hawaii, $10\frac{1}{2}^h$ slow; in southern Alaska, 9^h slow; in Brazil, from 3^h to 5^h slow, in three zones; etc.

The "daylight-saving time" now used in summer in many places is simply the standard time of the next zone to the eastward. It is adopted, between certain specified dates, to get the day's work over earlier in the afternoon.

In order to determine the standard time by observation it is only necessary to determine the local time by one of the methods given in Chapter III, and correct it according to the observer's longitude from Greenwich.

40. Where the Day Begins. If we imagine a traveler starting from Greenwich on Monday noon and journeying westward as swiftly as the earth turns to the east under his feet, he would, of course, keep the sun exactly on the meridian all day long and have continual noon. But what noon? It was Monday when he started, and when he gets back to London, twenty-four hours later, it is Tuesday noon there, although he has seen no intervening sunset. When does Monday noon become Tuesday noon? It is agreed among mariners *to make the change of date at the 180th meridian from Greenwich*, which passes over the Pacific, hardly anywhere touching land.

Ships crossing the line *from the east skip one day* in so doing. If it is Monday when a ship reaches the line, it becomes Tuesday when she passes it, the intervening twenty-four hours being dropped from the reckoning on the log-book. Vice versa, when a vessel crosses the line from the *western* side it counts the same day *twice over*, passing from Tuesday back to Monday and having to do Tuesday over again.

There is considerable irregularity in the date actually used on the different islands in the Pacific, as will be seen by looking at the so-called *date-line* as given in the Century Atlas of the World. Those islands which received their earliest European inhabitants via the Cape of Good Hope have adopted the Asiatic date, even if they really lie east of the 180th meridian, while those that were first approached from the American side have the American date. When Alaska was transferred from Russia to the United States, it was necessary to drop one day of the week from the official dates.

TRANSFORMATION OF COÖRDINATES

The almanac gives the right ascension and declination of the heavenly bodies. To point a telescope at a faint star we must have either the altitude and the azimuth, or the hour angle and the declination. The transformation in the latter case is very simple.

41. Hour Angle, Right Ascension, and Sidereal Time. From Fig. 7 and the definition of the quantities, taking into account the direction in which they are measured, we see that the ***hour angle of any point may be obtained by subtracting its right ascension from the sidereal time***; that is, $t = \theta - \alpha$ (another relation of fundamental

importance). In the application of this formula, negative hour angles or right ascensions may be avoided by adding 24 hours.

In the special case when the object is on the meridian, $t = 0$ and $\theta = \alpha$; that is, *when a body is on the meridian, its right ascension equals the local sidereal time.*

42. The Astronomical Triangle. Transformations between coördinate systems which depend on different fundamental planes involve spherical trigonometry. The passage from altitude and azimuth to hour angle and declination, for example, depends on the solution of the triangle PZX (pole-zenith-star), which is often called the *astronomical triangle* because very many problems, particularly of nautical astronomy, depend on it. The three sides and two of the angles are labeled in Fig. 10 with their values in the coördinates with which we are now familiar.[1] The angle at the star is called the *parallactic angle,* because it enters into the calculations of the effects of parallax and refraction upon the right ascension and declination of a body.

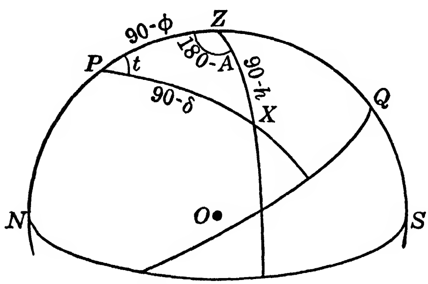


FIG. 10. The Astronomical Triangle

The general principles of spherical trigonometry are applicable to this triangle and give us formulæ of which the following are typical:

$$\sin \delta = \sin h \sin \phi - \cos h \cos \phi \cos A,$$
$$\cos \delta \cos t = \sin h \cos \phi + \cos h \sin \phi \cos A,$$
$$\cos \delta \sin t = \cos h \sin A.$$

With the aid of equations of this type, if any three elements of the triangle are known, the others may be found. Several important applications will be found in Chapter III, and further details may be found in Campbell's *Elements of Practical Astronomy.*

Transformations to ecliptic and galactic coördinates can be handled in a similar manner.

[1] In Fig. 10, δ is the declination, t the hour angle, and h the altitude of the star X; ϕ is the latitude of the observer at O.

43. Conversion of Time. It is very often necessary to convert the time at one place into the corresponding time at another, or the time measured in one system into that measured in a different one.

To find the time at one place when that at another is given, it is only necessary to *add or subtract the difference of longitude*, remembering that the time at the eastern station is always fast, compared with that at the western, unless the date-line intervenes. It makes no difference what kind of time is concerned, — sidereal, apparent, or mean, — so long as it is *measured in the same system at both stations* (§ 38).

A special case of this is the conversion of local mean time into standard time, which is accomplished by adding or subtracting the difference between the observer's longitude and that of the "standard time meridian." (This amounts to about -4^m at New York, the local mean time being 4^m faster than Eastern standard.)

To convert *apparent solar time into mean solar time* it is necessary to apply the *equation of time*, adding or subtracting as the case may be. At a given instant this correction is the same for all places, but it varies from day to day. Its exact value at the beginning of each Greenwich civil day is given in the *Nautical Almanac*, and the value at any other time may be found by interpolation. An approximate value of the equation of time (within half a minute or so) may be taken from Fig. 65 (p. 147); 12 hours must be added to the mean time to obtain the civil time.

44. Sidereal and Mean Time Intervals. The time divisions on the two systems are not of the same length. Since the tropical year (§ 175) contains 365.2422 mean solar days, and exactly one more sidereal day, it follows that the number of *sidereal* seconds in any time interval is equal to the number of *mean solar* seconds multiplied by $\frac{366.2422}{365.2422}$, that is, by 1.00273791. Hence, if I and I' are respectively the number of mean solar and sidereal seconds in any time interval, we have $I' = I + 0.00273791\, I$.

Conversely,

$$I = I' \times \frac{365.2422}{366.2422} = I' \times 0.99726957 = I' - 0.00273043\, I'.$$

Otherwise stated, the gain of sidereal upon mean time is $9^s.8565$ in one mean hour, and $9^s.8296$ in one sidereal hour. These quantities are the change in right ascension of the mean sun in 1 mean solar hour and 1 sidereal hour respectively.

The *American Ephemeris* gives at the end of the book two tables containing the values of the second terms of the two formulæ for every value of I and I' up to 24 hours, and the reduction of any sidereal *interval* to solar, or the reverse, is accomplished by simply adding or subtracting the tabular correction.

45. Conversion of Sidereal and Civil Time. In reducing a given *instant of time* from one system to the other we have usually to turn *standard* (civil) time into *local sidereal* time, or vice versa. The simplest way of doing this is by turning them into Greenwich time and back again. By adding the appropriate number of hours the standard time is converted into the *Greenwich civil time.* This is the mean time interval since the preceding Greenwich midnight, and may be converted into the *sidereal interval* since this moment by adding the correction taken from the proper almanac table. Now the *sidereal time at Greenwich mean midnight* (which is also the *right ascension of the mean sun* + 12 hours at this instant) is given for every day of the year in the *Nautical Almanac.* Adding the sidereal interval to this, we obtain the *Greenwich sidereal time.* Subtracting the observer's longitude, we have the *local sidereal time.* This process may be reversed step by step except that the correction to reduce the sidereal interval to a mean time interval is taken from the corresponding table in the almanac.

The *approximate* relation between sidereal time and civil time is very simple. Assuming that on September 22 the two times agree, after that day the sidereal time gains *two hours each month.* On October 22, therefore, the sidereal clock is two hours in advance; on November 22, four hours in advance; and so on. On account of the differing length of months this reckoning is slightly erroneous in some parts of the year, but is usually correct within four or five minutes. For the odd days the gain may be taken as four minutes daily.

46. Nautical Almanacs and Ephemerides. One of the absolute essentials for any observatory is the possession of tables giving the calculated positions of the sun, moon, and other heavenly bodies for the current year. Such tables, together with other data of importance in astronomical calculation, are contained in the Nautical Almanacs, which are prepared by a number of leading governments, issued three years in advance, and sold at a nominal

price. They contain *ephemerides* giving, besides other data, the right ascension and declination of the sun, moon, and planets at regular intervals of time, and also of a large number of "clock stars," which are observed for the determination of time. They also contain predictions of eclipses, occultations, and other phenomena. The work of computation (which is very heavy) is divided among the various offices by international agreement. The publication of those data which are of use only to astronomers is also divided among the various almanacs,[1] so that all are needed at an observatory.

A celestial globe will be of great assistance in studying the diurnal phenomena. By means of a globe it can be seen at once which stars never set, which ones never rise, and during what part of the twenty-four hours a heavenly body at a known declination is above or below the horizon.

47. The Celestial Globe. The celestial globe is a ball, usually of papier-mâché, upon which are drawn the circles of the celestial sphere and a map of the stars. It is mounted in a framework which represents the horizon and the meridian, in the manner shown in Fig. 11.

The *horizon*, HH' in the figure, is usually a wooden ring three or four inches wide, directly supported by the pedestal. It carries upon its upper surface, at the inner edge, a circle marked with degrees for measuring the azimuth of any heavenly body, and outside this the so-called zodiacal circles, which give the sun's longitude and the equation of time (§§ 36 and 169) for every day of the year.

The *meridian ring*, MM', is a circular ring of metal which carries the bearings of the axis on which the globe revolves. Things are, or ought to be, so arranged that the mathematical axis of the globe is exactly in the same plane as the graduated face of the ring, which is divided into degrees and fractions of a degree, with zero at the equator. The meridian ring fits into two notches in the horizon circle and is held underneath the globe by a support with a clamp, which enables us to fix it securely in any desired position, the mathematical center of the globe being precisely in the plane both of the meridian ring and of the horizon.

The *hour index* on the globe here figured is a pointer like the hour-hand of a clock, so attached to the meridian ring at the pole that it can be turned around the axis with stiffish friction, but will retain its position unchanged

[1] The most important are the *American Ephemeris and Nautical Almanac* (Superintendent of Documents, Government Printing Office, Washington, price $1.00), the *Nautical Almanac* (London), *Connaissance des Temps* (Paris), and *Berliner Jahrbuch.*

when the globe is made to turn under it. It points out the time on a small *time-circle*, graduated usually to hours and quarters, printed on the surface of the globe.

The *surface of the globe* is marked first with the celestial equator (§ 19) and next with the ecliptic (§ 21), which crosses the equator at an angle of $23\frac{1}{2}°$ (at X in the figure); each of these circles is divided into degrees and fractions. The *equinoctial* and *solstitial colures* (§ 21) also are always represented. As to the other circles, usage differs. The ordinary way at present is to mark the globe with twelve *hour-circles* 15° apart (the colures being two of them) and with *parallels of declination* 10° apart. On the surface of the globe are plotted the positions of the brighter stars and the outlines of the constellations.

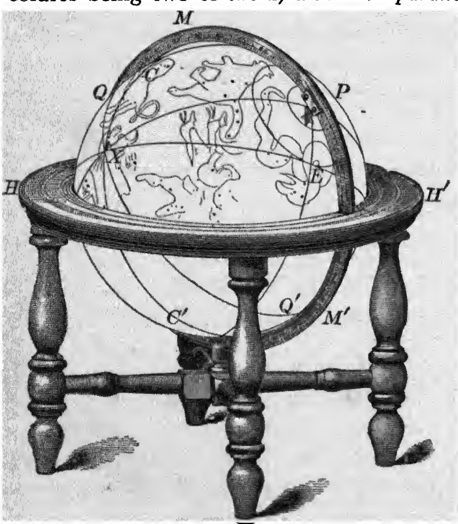


FIG. 11. The Celestial Globe

48. To rectify the globe, that is, to set it so as to show the aspect of the heavens at any given time,

(1) Elevate the north pole of the globe to an angle equal to the observer's latitude by means of the graduation on the meridian ring, and clamp the ring securely.

(2) Look up the day of the month on the horizon of the globe, and opposite to the day find, on the longitude circle, the sun's longitude for that day.

(3) On the ecliptic (on the surface of the globe) find the degree of longitude thus indicated, and bring it to the graduated face of the meridian ring by rotating the globe.

The globe is then set to correspond to (apparent) *noon* of the day in question. (It may be well to mark the place of the sun temporarily with a bit of moist paper applied at the proper place in the ecliptic; it can easily be wiped off after using.)

(4) Holding the globe fast, so as to keep the place of the sun on the meridian, turn the *hour index* until it shows on the graduated *time-circle* the local mean time of apparent noon, that is, $12^h \pm$ the equation of time given for the day on the horizon ring. (If standard time is used, the hour index must be set to the *standard* time of apparent noon.)

(5) Finally, turn the globe until the hour for which it is to be set is brought to the meridian, as indicated on the hour index. The globe will then show the true aspect of the heavens.

The positions of the moon and planets are not given by this operation, since they have no fixed places in the sky and therefore cannot be put upon the globe by the maker. If one wants them represented, he must look up their right ascensions and declinations for the day in some almanac and mark the places on the globe with bits of wax or paper.

All the problems involving transformation of coördinates may be solved very rapidly and with considerable accuracy by measurement on a celestial globe. It is only necessary to cut a strip of stiff paper and graduate this along one edge to correspond with the degrees on the equator of the globe. The distance in degrees between any two celestial objects may then be measured with this strip. By placing one end of it at the point representing the zenith, and carrying it down past any object to the horizon, the altitude of the object may be read off on the strip, and its azimuth on the horizon. Such measures, on an ordinary globe, are liable to errors of a degree or so.

EXERCISES

1. What point in the celestial sphere has both its right ascension and its declination zero? What are the celestial latitude and longitude of this point?

2. What are the hour angle and azimuth of the zenith?

3. At what points does the celestial equator cut the horizon?

4. What angle does the celestial equator make with the horizon at these points, as seen by an observer in latitude 40°? What if his latitude is 10°? 20°? 50°? 60°?

5. When the vernal equinox is rising on the eastern horizon, what angle does the ecliptic make with the horizon at that point for an observer in latitude 40°? what angle when it is setting?

6. What are the approximate right ascension and declination (α and δ) of the sun on March 21 and September 22?

7. On March 21, one hour after sunset, what would be the position of a star having a right ascension of seven hours and a declination of 40°, the observer being in latitude 40°?

8. If a star rises tonight at 10 o'clock, at what time (approximately) will it rise 30 days hence?

9. What are the longitude (λ) and latitude (β) of the sun when its right ascension is six hours? When its α is twelve hours?

10. What are the latitude and longitude of the north celestial pole?

11. Under what circumstances will the pole of the ecliptic be at an observer's zenith? Show that in this case $h = \beta$ and $A = 270 - \lambda$.

12. How much will a sidereal clock gain on a mean solar clock in 10 hours and 30 minutes of mean solar time? *Ans.* $1^m\ 43^s.5$.

13. How many times will the second-beats of a sidereal clock overtake those of a solar clock in a solar day if they start together? *Ans.* 236 times.

14. At what intervals of solar time do coincidences occur? *Ans.* $6^m\ 5^s.242$.

15. What is the approximate sidereal time on July 30 at 10 P.M.?

Solution according to second paragraph of section 45

July 22, noon, sid. time	$= 8^h\ 0^m$
8 days' gain	32
Sid. time at noon	$8^h\ 32^m$
10 hours = sid.	10 2
Sid. time at 10 P.M.	$18^h\ 34^m$

16. What is the approximate sidereal time on October 4 at 7 A.M.?

17. A ship leaving San Francisco on Tuesday morning, October 12, reaches Yokohama after a passage of exactly 16 days. On what day of the month and of the week does she arrive?

18. Returning, the same vessel leaves Yokohama on Saturday, November 6, and reaches San Francisco on Tuesday, November 23. How many days was she on the voyage?

19. Find the Greenwich civil time at local civil time $2^h\ 40^m$ A.M. in longitude 65° W; in longitude 57° E.

20. Interpolate in the almanac the α and δ of the sun and the equation of time at 3 P.M. Eastern standard time on February 2 of the current year.

21. What is the greatest possible number of Sundays in February?

Ans. 10, for the crew of a vessel making weekly sailings from Siberia to Alaska in a leap-year and leaving Siberia on Sunday, February 1. (One of Professor Young's favorite conundrums.)

NOTE. None of the above exercises require any calculation beyond simple arithmetic.

CHAPTER II

ASTRONOMICAL INSTRUMENTS

TELESCOPES AND THEIR ACCESSORIES AND MOUNTINGS · TIMEKEEPERS AND CHRONOGRAPHS · THE TRANSIT INSTRUMENT · THE MERIDIAN CIRCLE · THE MICROMETER · MEASUREMENT OF PHOTOGRAPHS · THE SEXTANT

Astronomical observations are of various kinds — the surface of a heavenly body is to be examined minutely; the position which it occupies at a given time, or the time at which it arrives at a given circle of the sky (usually the meridian), is to be ascertained; its brightness is to be measured or its spectrum investigated; a region of the sky is to be photographed and the relative positions of a number of stars determined from measurement of the plate. The telescope has been adapted to many purposes, and accessories have been devised for special problems.

49. The Telescope. The most important of all astronomical instruments is the telescope. Its principal uses are of three kinds: (1) to form an image of a heavenly body which may be observed with high magnifying power, measured with precision, or photographed; (2) to collect the light from a heavenly body and feed it into some other instrument, such as a photometer or spectroscope, with which the brightness, color, or composition of the light may be studied; and (3) to act as a pointer in making precise observations of the direction of a star.

Telescopes are of two kinds, refracting and reflecting. The former were invented first and are in more general use, but the largest instruments ever made are reflectors. The fundamental principle is the same in both: the large lens or the mirror of the instrument forms at its focus a *real image* of the object looked at, and this image is then examined and magnified by the eyepiece, which in principle is only a magnifying-glass.

In the form of telescope introduced by Galileo,[1] however, and still used as the opera-glass, the rays from the object-glass are

[1] See Grant's *History of Astronomy*, p. 514 ff., on the invention of the telescope.

intercepted by a concave lens, which performs the office of an eyepiece, *before* they meet at the focus to form the real image; but on account of the smallness of the field of view, and other objections, this form of telescope is never used when any considerable power is needed.

50. The Simple Refracting Telescope. This consists essentially (Fig. 12) of two convex lenses, one the object-glass A, of large size and long focus, the other the eyepiece B, of short focus, the two being set at a distance nearly equal to the sum of their focal lengths. Recalling the optical principles of the formation of images by lenses,[1] we see that if the instrument is pointed

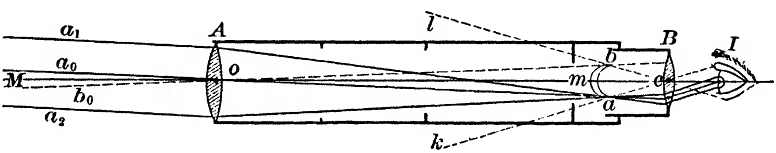


FIG. 12. The Simple Refracting Telescope

An inverted real image, ba, of the distant object is formed in the focal plane of the object-glass. The apparent diameter of the object, viewed with the naked eye, is angle $a_0ob_0 = boa$. By means of the eyepiece the diameter of the image is apparently enlarged to the angle bca. The ratio of angle bca to angle boa is the magnifying power. It equals the ratio of the distances of the image from the object-glass and the eyepiece, and hence the ratio of their focal lengths

toward the moon, for instance, all the rays that strike the object-glass from the top of the object will come to a focus at a, while those from the bottom will come to a focus at b, and similarly with rays from other points on the surface of the moon. There is formed in the *focal plane* of the object-glass a small inverted ***real image*** of the moon. If a photographic plate is inserted in the focal plane at ab and properly exposed, a picture of the object will be obtained.

The *size* of the picture will depend upon the apparent angular diameter of the object and the distance of the image ab from the object-glass, and is determined by the condition that, *as seen from point o* (the optical center of the object-glass), *the object and*

[1] In this explanation we use the approximate theory of lenses (in which their thickness is neglected), as given in the elementary textbooks. The more exact theory would require some slight modification in statements, but none of substantial importance.

its image subtend equal angles, since rays that pass through the point *o* suffer no sensible deviation.

If the focal length of the lens *A* is 10 feet, then the image of the moon formed by it will appear, when viewed from a distance of 10 feet, just as large as the moon itself; from a distance of 1 foot the image will, of course, appear ten times as large.

With such an object-glass, therefore, one can see the mountains of the moon and the satellites of Jupiter by removing the eyepiece and putting the eye in the line of the rays, at a distance of 10 or 12 inches from the eyepiece tube.

51. Magnifying Power. With the naked eye one cannot, unless near-sighted, see the image distinctly from a distance much less than 10 inches; but if a magnifying-lens of 1-inch focus is used, the image can be viewed from a distance of only an inch, and it will look ten times larger in diameter. The *magnifying power* is equal to the *quotient obtained by dividing the focal length of the object-glass by that of the eye-lens,* or, as a formula, $M = F/f$. The magnifying power of the telescope is changed at pleasure by simply changing the eyepiece (§ 59).

If, for example, the focal length of the object-glass be 4 feet and that of the eye-lens $\frac{1}{4}$ inch, then

$$M = 48 \div \tfrac{1}{4} = 4 \times 48 = 192.$$

52. Brightness of Image. This depends not upon the focal length of the object-glass but upon its size, — its area. If we estimate the diameter of the pupil of the eye at one fifth of an inch, then (neglecting the loss in transmission through the lenses) a telescope 1 inch in diameter collects into the image of a star 25 times as much light as the naked eye receives; and the great Yerkes telescope, 40 inches in diameter, gathers 40,000 times as much, or about 35,000 after allowing for the losses.

With a large telescope, therefore, objects like the stars, which appear as mere luminous points, have their brightness immensely increased, and millions otherwise invisible are brought to view. To obtain the full benefit of the aperture the entire pencil of light emerging from the eyepiece must be small enough to enter the pupil of the eye. It is clear from Fig. 12 that the ratio of the diameter of the object-glass to the diameter of the *emergent pencil* equals that of the focal lengths of the two lenses, that is,

the magnifying power. In observing the stars, therefore, higher powers must be used with larger object-glasses, in order to take full advantage of their greater light-gathering power.

For an extended luminous surface, like that of the moon or of a planet, things are very different. For magnifying powers below the limit just mentioned, the apparent area of the object is increased in exact proportion to the amount of light that enters the eye, so that its intrinsic (surface) brightness is fixed (and, on account of loss in transmission, is less than it appears to the naked eye). With higher powers the spreading out of the image further decreases its apparent surface brightness.

These effects combine to make the brighter stars visible with the telescope in the daytime, the star image being far brighter and the background of the sky seeming fainter.

53. Resolving Power; Spurious Disk. There is, however, a limit to the magnifying power which may profitably be used with a telescope of given aperture. Since light consists of waves of finite length, it can be shown that the image of a luminous point will not itself be a point, even in an optically perfect instrument, but will consist of a small central "diffraction disk" of finite diameter, brightest at the center and fading to darkness at the edge, surrounded by a series of concentric rings, each fainter than the last. Similarly, the image of a sharp line is a streak bordered by faint fringes.

Similar diffraction effects may be seen by looking through a narrow slit formed by two lead pencils held close together. As the slit is narrowed, objects appear increasingly blurred in the direction at right angles to the pencils.

The size of this disk-and-ring system can be calculated from the known wave-lengths of light and the dimensions of the lens, and the results agree precisely with observation. The *angular* diameter of the spurious disk is inversely proportional to the diameter of the objective, and independent of the focal length. According to Dawes, it is given by the equation $d = 4''.5/a$, where a is the aperture in inches.

With a 9-inch telescope, therefore, the image of any celestial object cannot be less than $0''.5$ in diameter, while with the Yerkes 40-inch telescope this is reduced to $0''.11$. Nothing is

really gained by pushing the magnifying power beyond the point at which the diffuseness of the image, arising from this cause, becomes conspicuous, that is, beyond a power of from 50 to 60 to the inch of aperture. For most purposes lower powers are more satisfactory; indeed, it is a good practical rule not to use a higher magnifying power than suffices to show the desired details clearly.

This effect of diffraction has much to do with the superiority of large instruments in showing minute details and in separating close double stars (Fig. 13). The full theoretical *resolving power* of a telescope is only practically available when the air is perfectly steady (which, unfortunately, seldom happens in most

FIG. 13. Resolving Power

The diffraction image of an artificial double star has been observed through a telescope of small aperture. The first image (from the left) shows the diffraction disk of a single star with the first two rings surrounding it, the others those of double stars with steadily increasing separation. Duplicity is evident in the third and fourth images, but clear separation of the diffraction disks occurs first in the ninth. The actual telescopic image of a star is very rarely as sharp as this. (From a photograph by J. A. Anderson)

climates). A given amount of atmospheric disturbance (§ 118) injures the performance of a large telescope much more than that of a small one; on bad nights the performance may actually be improved by reducing the aperture by means of a diaphragm in front of the objective.

54. Imperfections of Lenses; Spherical and Chromatic Aberration. A single lens, with spherical surfaces, cannot even approach this ideal performance, but suffers necessarily from various *aberrations*, of which the principal are the *spherical* and the *chromatic*. The former consists in a difference of the focal length for rays which have passed through the lens near its center and near the edge; the latter (which is more serious), in a difference in the focal length for light of different colors (wave-lengths).

The blue and violet rays are more refrangible than the yellow and red, and are brought to a focus nearer the objective, so that

the image of a star formed by such a lens is at best a round patch of light of different color at center and edge. The simple refracting telescope is therefore a very poor instrument.

By making the telescope extremely long in proportion to its diameter the distinctness of the image is considerably improved. In the middle of the seventeenth century instruments more than 200 feet in length were used by Cassini and others, and with instruments of this kind Huygens and Cassini discovered Saturn's rings and several of his satellites.

55. The Achromatic Telescope. The chromatic aberration (as was discovered in England about 1760) can be nearly corrected by making the object-glass of *two* (or more) lenses, of *different*

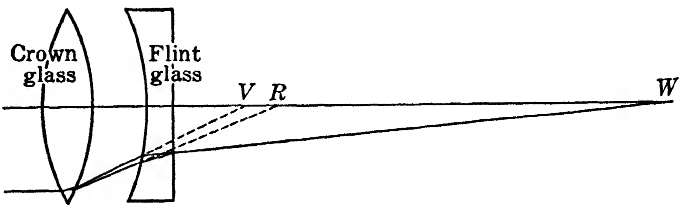


FIG. 14. Action of the Achromatic Lens

Light is strongly refracted by the crown lens, the violet more than the red. The flint lens reunites the different colors and diminishes the deviation

kinds of glass, one of the lenses being convex and the other concave. The convex lens is usually made of *crown* glass; the concave, of *flint* glass. At the same time, if the curves and the distance separating the lenses are properly chosen, the spherical aberration also can be destroyed; so that such a compound object-glass comes reasonably near to fulfilling the condition that it should gather to a mathematical point in the image all the rays that reach the object-glass from a single point in the object. The concave lens must be made of a kind of glass which has a greater *dispersive* power (separation of red and violet) in proportion to its refractive power (total deviation of the rays) than the other. It undoes a part of the deviation inward of the rays to form a focus, and almost the whole of the separation of the rays of different colors (Fig. 14).

These object-glasses admit of a considerable variety of form. In small object-glasses the lenses are often cemented together

with Canada balsam or some other transparent medium. At present some of the best makers separate the two lenses by a considerable distance, so as to admit a free circulation of air between them.

56. Secondary Spectrum. It is not possible, however, with the kinds of glass ordinarily employed, to secure a perfect correction of the color. The best achromatic lenses bring the yellowish-green rays to a focus nearer the lens than they do the red and violet. In consequence the image of a bright star is surrounded by a purple halo, which is not very noticeable in a good telescope of small size but is very conspicuous and troublesome in a large instrument, and makes it difficult to see faint stars close to a bright one.

By using the new varieties of glass which are now made at Jena, objectives of three components can be constructed which are very nearly *aplanatic*, that is, sensibly free from both chromatic and spherical aberration. Several telescopes of this sort, of apertures up to 12 inches, are in use and perform admirably; but it has not yet been possible to get disks of these glasses large enough for lenses of great size.

57. Photographic Telescopes. A visually corrected objective is practically useless for astronomical photography on ordinary plates, for the blue and violet rays, which are photographically the most effective, are not brought to any one sharp focus. When the photographic light of but a single star or planet is needed, it may be brought to one focus by introducing a correcting lens, much smaller than the objective, a few feet above the eye end. Excellent photographs of larger fields may be made, without any correcting lens, by using isochromatic plates and placing just in front of the plate a color screen (a plane-parallel glass coated with a thin film of gelatin stained with a yellow dye, which absorbs the blue and violet light); but the necessary exposures are relatively long.

When a telescope is to be used primarily for photography, the objective is designed, in the first place, so as to bring all the blue and violet rays to substantially the same focus, leaving the yellow and red uncompensated. Such a telescope is almost useless visually, but permits very much shorter exposures than the devices described above.

58. Wide-Angle Lenses. In astronomical photography it is often important to have objectives that will give good definition of the image over a field of view many degrees in diameter, as against a degree or two in the ordinary telescope. This imposes much more severe demands upon the designer and brings a number of other aberrations into importance for stars whose images are at some distance from the center of the field.

It is not possible to correct all these in an objective consisting of only two components; but by making the objective of three or four lenses, a very good correction, though not a perfect one, can be attained. The most usual form is the *doublet*, consisting of two similar pairs of lenses separated by a wide interval, as in the familiar "rapid rectilinear" lenses of hand cameras. Lenses have been constructed giving a field of good definition up to 20° in diameter. The intensity of the image of an extended surface, such as a nebula, is proportional to the square of the ratio of the aperture to the focal length. A large value of this ratio is essential if very faint objects are to be photographed, but in this case it is particularly hard to correct the aberrations completely. The calculation of such a lens system is very complicated and laborious.

59. Eyepieces. A simple convex lens performs well for a small object, like a close double star, exactly in the center of the field of view. Eyepieces are usually constructed of two or more lenses, which give a larger field of view than a single lens and define fairly well over the whole extent of the field. The image of the object formed by the object-glass lies *outside* of the *positive* eyepiece and *between* the lenses of the *negative* eyepiece. The former can be used as a hand magnifier.

60. Reticle. If the telescope is to be used for pointing upon an object, it must be provided with a reticle of some sort (Fig. 28, p. 61). The simplest is a frame with two spider lines stretched across it at right angles to each other, their intersection being the point of reference. This reticle is placed *in the focal plane* (as *ab* in Fig. 12). It is usually so arranged that it can be moved in or out a little to get it exactly into this plane, so that the images of the object and the reticle can be brought simultaneously into focus by the eyepiece. Of course, positive eyepieces only can be

used in connection with such a reticle. In order to make the lines of the reticle visible at night a faint light is reflected into the instrument by some one of various arrangements devised for the purpose.

61. The Reflecting Telescope. About 1670, when the chromatic aberration of refractors first came to be understood (in consequence of Newton's discovery of the decomposition of light), the reflecting telescope was invented. For nearly one hundred and fifty years it held its place as the chief instrument

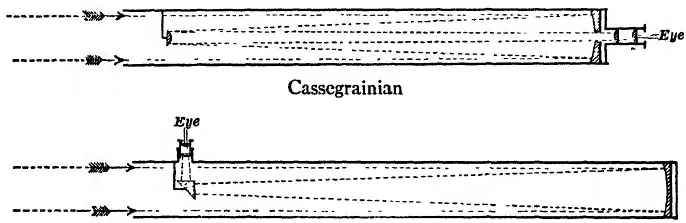


FIG. 15. Reflecting Telescopes

A concave mirror reflects the rays of light to a focus. In the Newtonian form the light is brought, for convenience, by plane reflection to the side of the tube. The main characteristic of the Cassegrainian is the reflection on a secondary convex mirror with the consequent increase of effective focal length

for star-gazing; it was almost displaced by the refractor during the nineteenth century but returned to importance in the twentieth.

The essential part of a reflector is the great mirror, which brings the light of the stars to a focus. The direct image so produced is in a very inconvenient position, and various means are adopted to bring it to a point where it can be utilized.

In the *Newtonian* form (Fig. 15) a small plane mirror standing at an angle of 45° intercepts the rays a little before they come to their focus, and throws them to the side of the tube, where the eyepiece is placed.

In the *Cassegrainian* form a small convex mirror, placed in about the same position, reflects the rays back through a hole in the center of the large mirror. With this form the observer looks directly at the stars, as with a refractor, and the image is erect.

In both forms a real image is produced, which may be photographed, but in the Cassegrainian form the effective focal length is much greater than in the Newtonian, and the image is on a correspondingly larger scale.

The great modern instruments are arranged so that they may be used in either of these forms (a "cage" carrying the small Newtonian mirror being replaced by another with the Cassegrainian mirror), and so that a plate-holder may be set directly in the *principal focus* if desired, thus avoiding the loss of light by a second reflection.

Combinations of the two forms are used, in order to avoid drilling a hole in the great mirror, to secure a greater range in the size of the image, and to meet the special requirements of certain types of observation (Fig. 16).

Formerly the mirror was made of "speculum metal" (two parts of copper to one of tin), a brittle white alloy which takes a very high polish. At present it is always made of glass, silvered on the front surface by a chemical process which deposits the metal in a brilliant film of extreme thinness. After a time this film tarnishes and loses its high reflecting power, but it may then be removed with nitric acid and the mirror resilvered, renewing its brightness without in the least altering its figure. The figuring, or shaping, of this surface is a task of the utmost nicety (as in the case of a lens), and, for a large instrument, may take months.

62. Relative Advantages of Reflectors and Refractors. Since light passes through a lens, the glass of which it is made must be *optically homogeneous*; the glass disk of a mirror, however, needs only to be *mechanically homogeneous*, so that it may not bend irregularly, — a much less severe demand. There is no present hope of casting glass disks, suitable for lenses, as large as the mirrors of the great reflectors. A reflector is far less expensive than a refractor of the same size, and its *perfect achromatism* is a great advantage in photographic and spectroscopic work.

A refractor, on the other hand, defines better under all ordinary conditions, has a larger field of good definition, and is more convenient. A lens requires only to be protected; a mirror must be resilvered every few months.

In current practice, refractors are used almost exclusively in visual observations and measures, while reflectors possess great

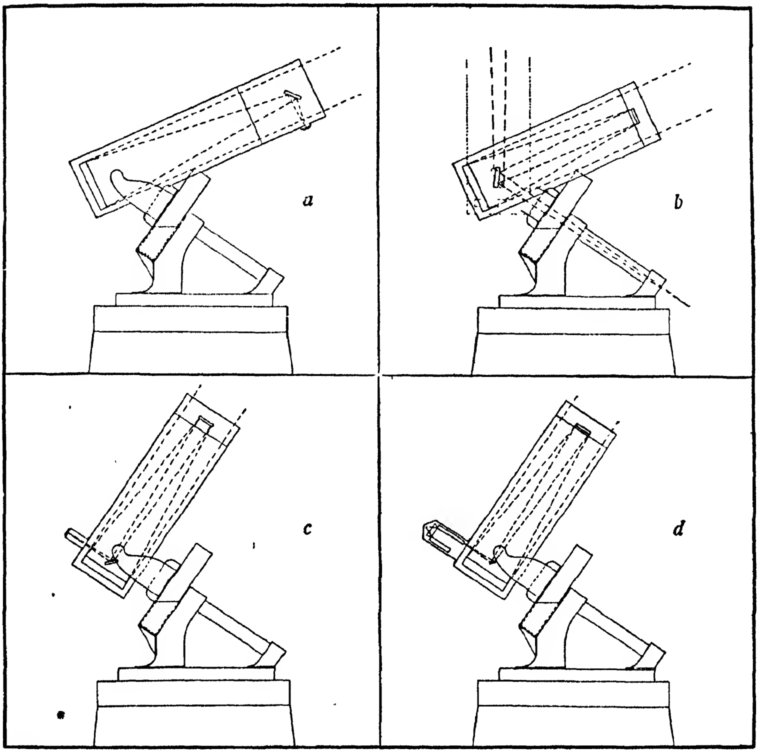


FIG. 16. Four Arrangements of the 60-Inch Reflector of the Mt. Wilson Observatory

The focal length of the 60-inch mirror is 25 feet. In the Newtonian form, *a*, the small secondary mirror is flat and simply reflects the light to the side of the tube. The focal length remains 25 feet. For use as a Cassegrain reflector the upper section of the tube is replaced by a shorter section carrying a convex mirror, which increases the equivalent focal length. By the use of a plane mirror the light is brought to the side of the tube. Different focal lengths can be obtained by the use of different convex mirrors. In the arrangement *d*, where the light is brought to a spectroscope at the side of the tube, the focal length is 80 feet; in *c*, which is used for large-scale photography, it is 100 feet; and in the coudé form, *b*, with a focal length of 150 feet, the light is brought down the polar axis to a large stationary spectroscope

advantages in spectroscopic work. In photography the great reflectors, with their great light power, excel in the observation of faint objects such as nebulæ. The doublet stands alone for work requiring a wide field.

63. Great Telescopes. The largest telescope in the world at present is the 100-inch reflector of the Mt. Wilson Observatory

(Figs. 19, 20); next comes the 72-inch reflector of the Dominion Observatory at Victoria; and, third, the 60-inch reflector also at Mt. Wilson.

The largest refractors are the 40-inch at the Yerkes Observatory (Fig. 21) and the 36-inch at the Lick Observatory. There are several other refractors 30 inches or more in aperture, and a number of reflectors of the same size or larger. The largest doublet is the Bruce telescope, of 24 inches aperture, at the Harvard station at Arequipa, Peru. All these are instruments of the highest class, both in optical parts and in mounting. Lord Rosse's reflector, constructed in 1842, with a 6-foot mirror of speculum metal, is now chiefly of historic interest.

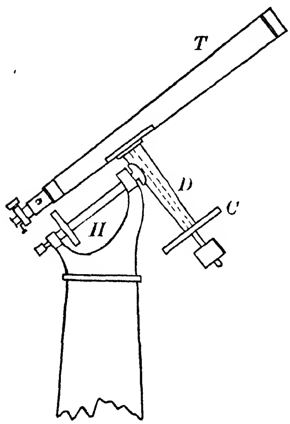


FIG. 17. The Equatorial Mounting

The telescope is set on a star by means of the declination circle C and the hour circle H, attached to the polar axis. The driving mechanism is then clamped to the polar axis, which is turned just fast enough to keep the star in the field of view

64. Mounting of a Telescope. A telescope, however excellent optically, is of little scientific use unless firmly and conveniently mounted.[1]

At present nearly all but small portable instruments are mounted as *equatorials*. Fig. 17 represents the arrangement schematically. Its essential feature is that the principal axis — the one that moves in fixed bearings attached to the pier and is called the *polar axis* — is inclined so as to point toward the celestial pole. The graduated circle H attached to it is therefore parallel to the celestial equator and is usually called the *hour circle* of the instrument. At the upper extremity of the polar axis is fastened a sleeve which carries the *declination axis D* passing through it. To one end of the declination axis is attached the telescope tube T, and at the other end the *declination circle C* and a counterpoise.

[1] It must, of course, be mounted where it can be pointed directly at the stars, *without any intervening window-glass.*

65. The advantages of the equatorial mounting are very great. In the first place, when the telescope is once pointed upon an object, it is not necessary to turn the declination axis at all in order to keep the object in view, but only to turn the *polar axis* with a perfectly uniform motion, which can be, and usually is, given by a *driving-clock* (Fig. 18). The driving-clock is driven by a heavy weight and controlled by a heavy centrifugal governor, which acts continuously, not by jerks, as in an ordinary clock. An electric motor drive, synchronized by a pendulum clock, has also been used.

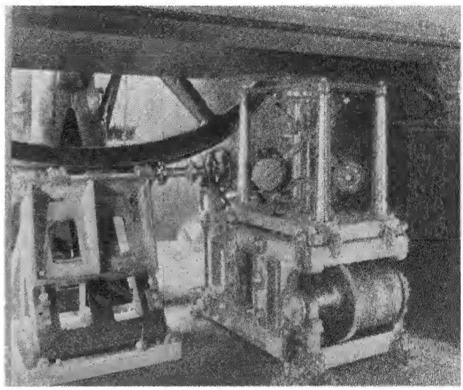


Fig. 18. The Driving-Clock of the 100-Inch Hooker Telescope

The power is transmitted through the worm gear to the worm wheel (17 feet in diameter), which may be clamped to the polar axis. The centrifugal governor is seen above, at the right

In the next place, it is very easy to find an object, even if invisible to the eye (like a faint comet or a star in the daytime), provided we know its right ascension and declination and have the sidereal time, *a sidereal clock or a chronometer being an indispensable accessory of the equatorial.* Set the declination circle to the declination of the object and then turn the polar axis until the hour circle shows the proper *hour angle*, which is simply the difference between the right ascension of the object and the sidereal time at the moment (§ 41). When the telescope has been so set, the object will be found in the field of view, *provided a low-power eyepiece is used.* On account of refraction the setting does not direct the instrument precisely to the apparent place of the object, but only very near it. Large instruments are provided with *finders* (small auxiliary telescopes with low power and wide field of view). When an object is brought to the center of the field of the finder, it is visible in the main instrument.

The equatorial does not give very accurate positions of heavenly bodies by means of the direct readings of its circles; indeed,

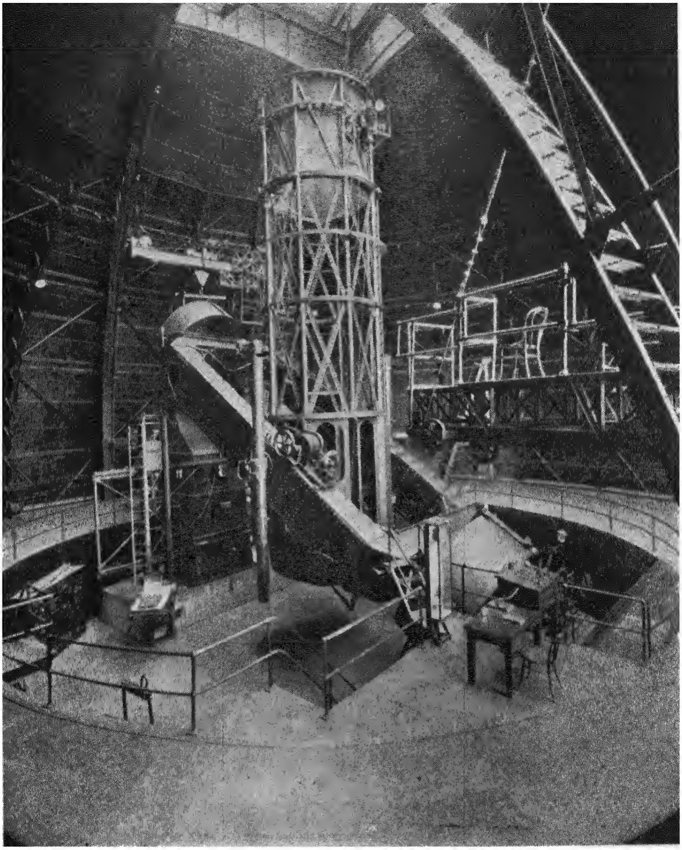


FIG. 19. The 100-Inch Hooker Telescope of the Mt. Wilson Observatory

This telescope is the largest in the world. Its mounting is of the English type: the polar axis, which is in the form of a rectangular yoke, is supported at the ends on two piers, and the telescope may be made to follow a star right across the meridian. The tube, as is usual with reflectors, is of skeleton construction. The moving parts of the telescope weigh about 100 tons. The great mirror was fashioned from a glass disk 101 inches in diameter, 13 inches thick, and weighing $4\frac{1}{2}$ tons

the most recent instruments are provided only with coarsely graduated circles, sufficient for finding the desired object (thus saving much expense). It can, however, be used to determine

with great precision *the position of any object* (such as a comet) *relative to neighboring stars of known position* (§ 100).

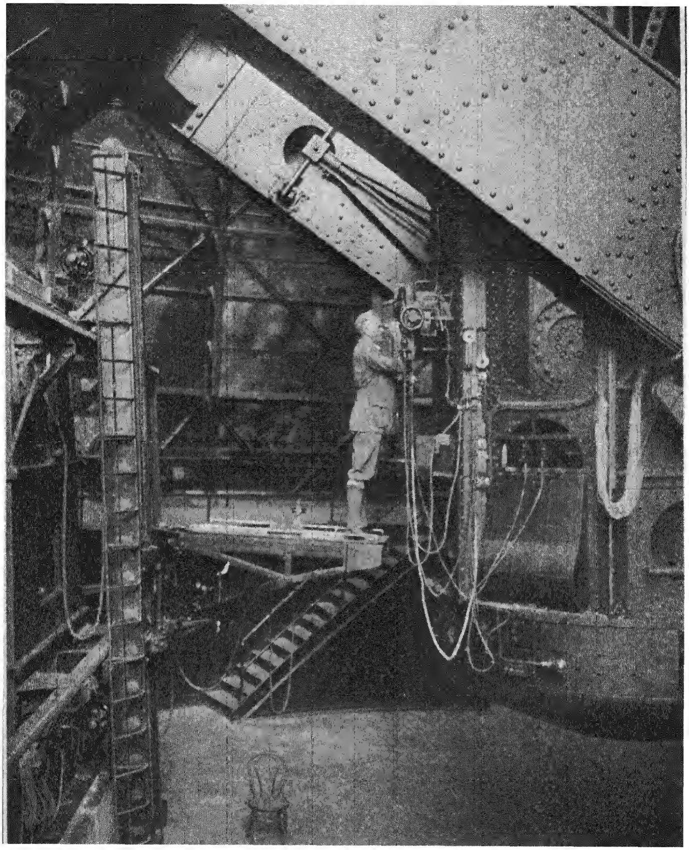


FIG. 20. Observing at the Cassegrain Focus of the 100-Inch Reflector of the Mt. Wilson Observatory

This photograph emphasizes the great size of the telescope. The observer is standing at a height of twelve feet above the floor, on a platform that is movable in any direction by electric motors. The edge of the cell which contains the great mirror may be seen at the bottom of the telescope. A part of the extensive electric wiring system may also be noticed

66. Engineering Details of the Equatorial Mounting. The great modern reflectors are exceedingly heavy (the 100-inch mirror alone weighs 4 tons), and the designing of mountings that will not sag under such loads is a difficult engineering problem. The

moving part of the 72-inch reflector weighs 45 tons. Such instruments have skeleton tubes, cross-braced like the girders of a

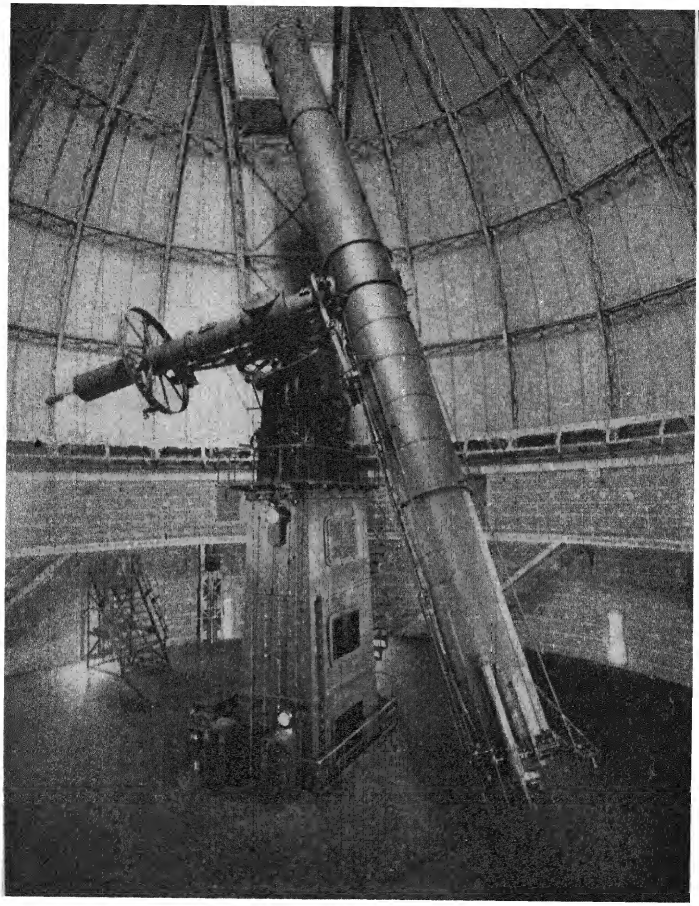


FIG. 21. The 40-Inch Refractor of the Yerkes Observatory

This refractor is equatorially mounted on a central pier which houses the driving-clock. The focal length is 62 feet. The entire floor, 75 feet in diameter, may be raised or lowered to a level convenient for observation. The dome, 90 feet in diameter, is built of steel girders, is sheathed with wood and covered with roofing-tin. It revolves upon wheels, 36 inches in diameter, which roll round the circular track fastened to the top of the wall. Two shutters, 85 feet in length, cover the observing slit, which is 13 feet wide

bridge, and various devices are used for easing the load on the main bearings. An elaborate system of electric controls enables

the observer, at the eyepiece, to turn the dome as required and to move the instrument in either coördinate, rapidly or slowly. The 100-inch telescope is so rigid that a man may climb out to the upper end without shifting a star image seriously in the field of view, and so delicately controlled that it is easy to move the image at will by one third of a second of arc. Such a mounting is a mechanical masterpiece.

As the telescope turns to follow the stars the eyepiece too must move, both horizontally and vertically. With large instruments the resulting inconvenience is minimized by means of a *rising floor*, which may be set at any desired level, or, with the great reflectors, by *observing platforms* movable by electric motors. All this paraphernalia, and the enormous rotating dome (100 feet in diameter for the 100-inch, and 90 feet for the Yerkes 40-inch) add to the already high cost of a great instrument.

The expensive parts of a telescope are the object-glass, the mounting, and the dome. The eyepieces and other accessories are relatively inexpensive. A small telescope, large enough for the purposes of the amateur, can be purchased for a few thousand dollars, or less. The 100-inch telescope, with mounting, dome, and accessories, cost $540,000, the largest sum ever spent for any single instrument of research.

67. The Cœlostat. In some cases (especially in work on the sun with instruments of long focus) it is desirable to have the main part of the instrument at rest and to reflect the light into it from a moving mirror.

In the *cœlostat* the plane of the mirror is parallel to that of the polar axis, which carries it directly and revolves only once in forty-eight hours. The reflected image in this case is thrown in a direction determined by the declination of the object. This difficulty can be overcome by using a second mirror to send the reflected beam where it is wanted. With this arrangement the image remains fixed in the focal plane. It is much used for solar work, as in connection with the 150-foot tower telescope of the Mt. Wilson Observatory, for reflecting the beam vertically downward through the objective.

Other forms of mounting, such as the *siderostat* and the *heliostat*, are used for special purposes.

Great difficulties often arise from the distortion of the mirrors by the sun's heat, which plays havoc with the definition. By making the mirrors very thick or of special glass this trouble can be greatly diminished.

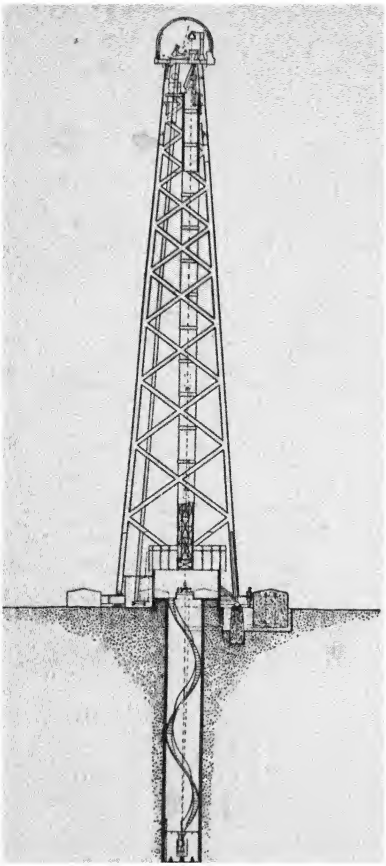


FIG. 22. Structural Plan of the 150-Foot Tower Telescope of the Mt. Wilson Solar Observatory

Sunlight is reflected from the cœlostat mirror to a second mirror, and thence vertically downward through an objective of 150 feet focal length, which forms an image of the sun about 16½ inches in diameter at the base of the tower. In the plane of the image is the slit of the 75-foot spectrograph. After passing through the slit the light descends to the collimating lens near the bottom of a well about 80 feet deep, falls on a large Michelson grating, and returns through the same lens to the plate-holder mounted close beside the slit

In the *coudé*, or elbowed equatorial, one or more mirrors make it possible for the observer to sit in comfortable shelter and look down through the polar axis in a fixed direction.

68. For astronomical photography an equatorial mounting is obviously essential. To obtain good star images the mounting should be very carefully constructed and adjusted, and should be as rigid as possible (to prevent its being shaken by the wind etc.).

Even with a good driving-clock, for exposures more than a few minutes in length the telescope cannot be trusted to follow the stars perfectly, owing to changes in the refraction and to the inevitable small outstanding errors of adjustment; and it is necessary to "guide" by hand. A star of suitable brightness, near the center of the field to be photographed, is brought to the intersection of the *cross-wires* and kept there by a watchful observer. The cross-wires may be in a

guiding telescope rigidly connected to the photographic telescope. In another arrangement they are placed in a *guiding eyepiece* just outside the plate-holder and moving with it. The guiding is then done by means of screws which move the plate-holder relatively to the rest of the telescope. In this way exposures extending even over several successive clear nights can be made, the plate-holder being shielded from the daylight and the instrument set again, with the aid of the guiding star, upon exactly the same point of the heavens as before.

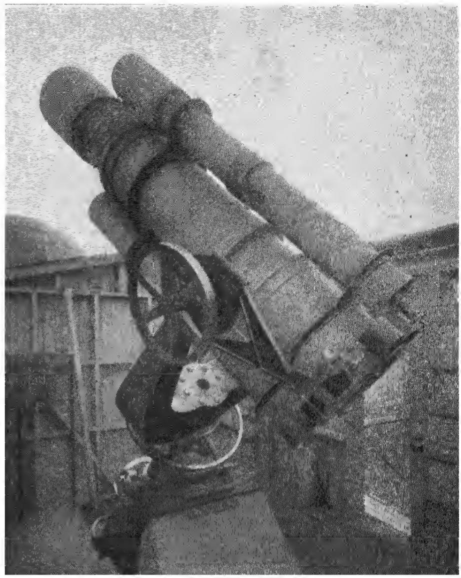


FIG. 23. The Metcalf Telescope of the Harvard College Observatory

The central tube is the 16-inch photographic doublet. The tube flares at the lower end to permit the use of a large plate. By exhausting the air in the plate-holder (which is clamped to the lower end of the tube) the plate is given a temporary curvature during the exposure, conforming to the focal surface, which is not plane. By this device well-focused star images are secured all over the plate. The large ratio of aperture to focal length makes it a very rapid camera: stars of the fifteenth magnitude are photographed in ten minutes. The upper tube is the 8-inch guiding telescope; the lower, a photographic telescope of 4-inch aperture. The driving-clock is electrical, and the yoke-mounting avoids the necessity of reversing the instrument at the meridian. (From photograph by Harvard College Observatory)

69. Time-Keepers and Time-Recorders. *The clock, chronometer, and chronograph.* The invention of the pendulum clock by Huygens in 1657 was almost as important to the advancement of astronomy as was that of the telescope by Galileo, and the improvement of the clock and chronometer through the invention of temperature compensation by Harrison and Graham in the eighteenth century is fully comparable to the improvement of the telescope by the achromatic object-glass.

So far as the principles of construction are concerned there is no difference between an astronomical clock and any other. As a matter of convenience, however, the astronomical clock is almost invariably made to beat seconds (rarely half-seconds) and has a conspicuous second-hand, while the face is marked for twenty-four hours instead of twelve. It is, of course, constructed with extreme care in all respects.

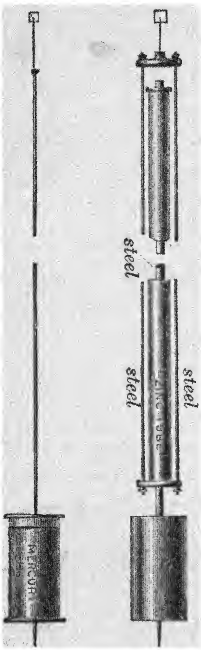


FIG. 24. Compensation Pendulums

A, Graham's pendulum; *B*, zinc-steel pendulum. In the first type the expansion of the rod downward is compensated by the expansion of the mercury upward; in the second type the tendency of the steel rods to lengthen is compensated by the upward expansion of the zinc tube, which is fastened to the steel rods

The escapement is preferably one of the numerous gravity types. The office of the escapement is to be unlocked by the pendulum at each vibration, so as to permit the wheelwork to advance one step, marking a second (or sometimes two seconds) on the clock-face; while at the same time the escapement gives the pendulum a slight impulse, just equal to the resistance it has suffered in overcoming friction during the last swing and in performing the unlocking.

The pendulum must be constructed in such a way that changes of temperature will not change its length. This is satisfactorily effected in an arrangement whereby the downward expansion of the rod is compensated by an upward expansion of the pendulum bob. The best of all materials for a pendulum is *invar*, a nickel-steel alloy whose coefficient of expansion is vanishingly small.

When the extremest accuracy of performance is required, the clock is placed in an underground chamber where the temperature varies only slightly or not at all, and is inclosed in an air-tight case, within which the air is kept at a uniform pressure, since changes in the density of the air slightly affect the swing of the pendulum. (Usually a clock loses about one quarter of a second a day for a rise of one inch in the barometer.) These clocks are

driven by the fall of a weight of only a few grams, which is automatically raised to the top of its run by an electromagnet at intervals of twenty seconds or so, and they will run for months without any attendance.

70. Clock Correction. The *correction* of a clock is the amount that *must be added* to the indication of the clock-face at any moment in order to give the true time; it is therefore plus (+) when the clock is *slow* and minus (–) when it is *fast*. The *rate* of a clock is *the daily change in its correction*, — *plus* (+) when the clock is *losing* and minus (–) when it is gaining.

A perfect clock is one that has a *constant rate*, whether that rate be large or small. It is desirable, for convenience' sake, that both error and rate should be small; but this is a mere matter of adjustment by the user of the clock, who adjusts the error by setting the hands, and the rate by raising or lowering the pendulum bob.

The final adjustment of rate is often obtained by first setting the pen dulum bob so that the clock will run slow a second or two daily, and then putting on the top of the bob little weights of a gram or two, which will raise its center of gravity and shorten its period of oscillation. They can be dropped into place or knocked off without stopping the clock or perceptibly disturbing it. A still finer adjustment can be made by altering the pressure within the air-tight clock-case.

71. The Chronometer. Since the pendulum clock is not portable, it is necessary to provide time-keepers that are so. The chronometer is merely a carefully made watch (driven by a coiled spring), with a balance-wheel compensated to run, as nearly as possible, at the same rate in different temperatures, and with a peculiar escapement which, though unsuited to ordinary usage, gives better results than any other when treated carefully. (*Never turn the hands of a chronometer backward; it may ruin the escapement.*)

The box chronometer used on shipboard is usually about five inches in diameter, and is mounted in gimbals so as to remain horizontal at all times, notwithstanding the motion of the vessel. It usually beats half-seconds.

It is not possible to secure in the chronometer balance-wheel as perfect a temperature correction as in the pendulum, and for this and other reasons the best chronometers cannot quite com-

pete with the best clocks in precision; but they are sufficiently accurate for most purposes, and of course are vastly more convenient for field operations, while at sea they are indispensable. A good ordinary watch, regulated by radio signals, will suffice for a short voyage. Chronometers are essential at observatories in countries like Japan, where small earthquakes frequently disturb pendulum clocks.

72. Eye-and-Ear Method of Observation. The old-fashioned method of time observation consists simply in noting by eye and ear the moment (in seconds and tenths of a second) when the phenomenon occurs, as, for instance, when a star passes some wire of the reticle. The tenths, of course, are merely estimated, but the skillful observer seldom errs in his estimation by a whole tenth. Skill and accuracy in this method are acquired only by long practice.

73. The Chronograph. At present such observations are usually made by the help of electricity. The clock is so arranged

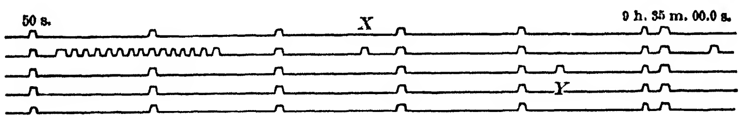


FIG. 25. A Chronograph Record

In the figure the initial minute, marked soon after the chronograph was started, happened to be $9^h\,35^m$, the zero in the case of this clock being indicated by a double beat. The signal at *X*, therefore, was made at $9^h\,35^m\,55^s.45$, and that at *Y* at $9^h\,36^m\,58^s.63$. The "rattle" just preceding *X* was the signal that a star was approaching the transit wire

that at every beat (or every other beat) of the pendulum an electric circuit is made or broken for an instant, and this causes a sudden sidewise jerk in the armature of an electromagnet, like that of a telegraph sounder. This armature carries a fountain pen, which writes upon a sheet of paper wrapped around a cylinder, six or seven inches in diameter, which is turned uniformly by a driving-clock once a minute; at the same time the pen carriage is drawn slowly along, so that the marks on the paper form a continuous helix, graduated into second or two-second spaces by the clock-beats. When taken from the cylinder, the paper presents the appearance of an ordinary page crossed by parallel lines spaced off into two-second lengths (Fig. 25).

The observer, at the moment when a star crosses the wire, presses a key which he holds in his hand, and thus interpolates a mark of his own among the clock-beats on the sheet, — for instance, at X and Y in the figure. Since the beginning of each minute is indicated on the sheet in some way by the mechanism which produces the clock-beats, it is very easy to read the time

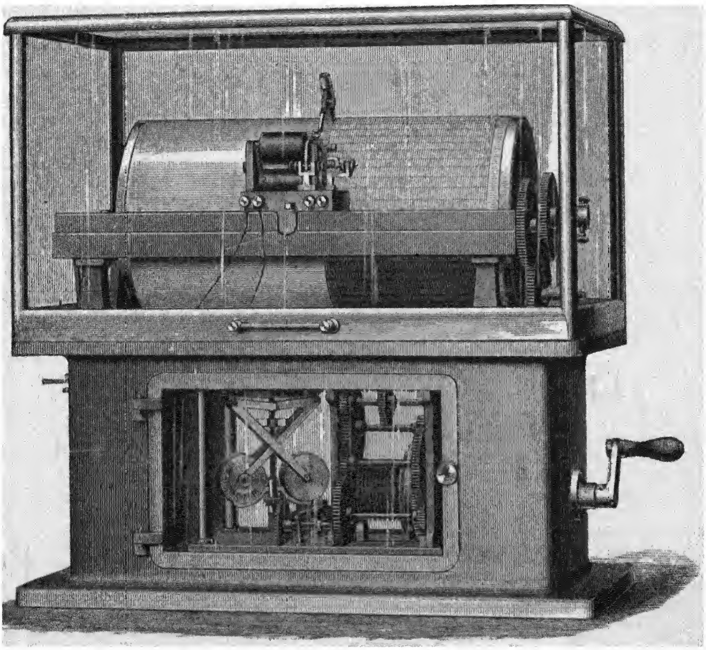


FIG. 26. A Chronograph

The clockwork of the chronograph is regulated by a centrifugal governor which acts continuously, not by beats like a clock escapement. (Built by Warner & Swasey)

of X and Y by applying a suitable scale, the beginning of the mark made by the key being the moment of observation.

74. The Transit Instrument. A large proportion of all astronomical observations for determining the position of a heavenly body are made when the body is crossing the meridian or is very near it. At that time the effects of refraction and parallax (to be discussed later) are a minimum; and as they act only vertically, they do not affect the *time* when a body crosses the meridian, or, consequently, its observed right ascension.

The transit instrument is used in connection with a sidereal clock or a chronometer, and often with a chronograph, to observe the time of a star's *transit*, or passage across the meridian. If the correction of the sidereal clock at the moment is known and allowed for, the corrected time of the observation will be the right ascension of the star (§ 41). Vice versa, if the right ascension is known, *the clock correction* will be the difference between the right ascension of the object and the time observed.

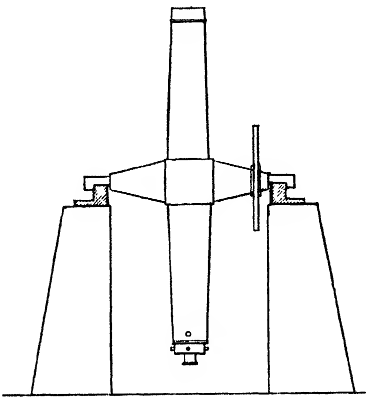


FIG. 27. The Transit Instrument (Schematic)

In the ideal instrument the axis is horizontal and lies east and west. The middle wire of the reticle (near the eyepiece) is in the meridian at whatever altitude the telescope may be pointed

The instrument (Fig. 27) consists essentially of a telescope carrying a reticle at the eye end and mounted on a stiff axis, perpendicular to the line of sight, that turns in V-shaped bearings called "Y's," which can have their position adjusted so as to make the axis exactly perpendicular to the meridian. A delicate spirit-level, which can be placed upon the pivots of the axis to measure any slight deviation from horizontality, is an essential accessory; and it is practically necessary to have a small graduated circle attached to the instrument, in order to set it at the proper altitude for the star that is to be observed, according to the declination.

It is desirable, also, that the instrument should have a reversing apparatus by which the axis may be easily lifted and safely reversed in the Y's without jar or shock.

The reticle (Fig. 28) usually contains from five to fifteen vertical wires crossed by two horizontal ones. In order to make the wires visible at night the field must be illuminated by some suitable device.

75. The instrument must be thoroughly rigid, without any loose joints or shakiness, *especially in the mounting of the object-glass and reticle.* Moreover, the two pivots should be of the same

diameter, accurately round, without taper, and precisely in line with each other; in other words, they must be *portions of one and the same geometrical cylinder.* The fulfilling of this condition taxes the highest skill of the mechanician. The Y's must be worked with corresponding accuracy.

When exactly adjusted, the middle wire of such an instrument *affords a visible image of a part of the invisible meridian,* wherever the instrument may be turned on its axis; and *the sidereal time when a star crosses that wire is therefore the star's right ascension.*

76. Adjustments of the Transit. These are four in number:

(1) The reticle must be exactly in the *focal plane* of the object-glass, and the middle wire must be *accurately vertical.*

When the wires have been adjusted, the reticle slide should be tightly clamped and never disturbed again. The *eyepiece* can be moved to secure distinct vision for different eyes, reticle and star coming into focus together.

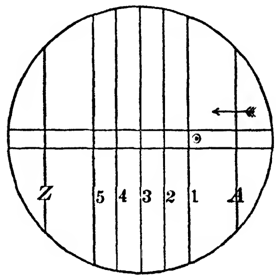


FIG. 28. Reticle of a Transit Instrument

The observation consists in noting the instant at which a star crosses each of the transit wires of the reticle. When the chronograph is used, transits may be taken over wires which are quite close together (1 to 5). Eye-and-ear observations may be taken on wires A, 3, Z, the wider spacing of which permits the observer to make his record before the star reaches the next wire

(2) The *line of sight* (that is, the line which joins the optical center of the object-glass to the middle wire) must be *exactly perpendicular to the axis of rotation.* It then coincides with the *line of collimation.* This may be tested by pointing on a distant mark and then reversing the instrument. The middle wire must still bisect the mark after the reversal; if it does not, the reticle must be adjusted by the screws provided for the purpose.

(3) The axis must be *level.* This adjustment is made mechanically by the help of the striding level, which may be set across the pivots. One of the Y's has a screw by which it can be slightly raised or lowered. When correct it is permanently clamped.

(4) The *azimuth* of the axis must be exactly 90°; that is, the axis must point exactly east and west. This adjustment is made by means of observations of a number of stars, by shifting one

of the Y's horizontally until the interval between successive transits of a circumpolar star across the instrumental meridian above and below the pole is exactly twelve sidereal hours.

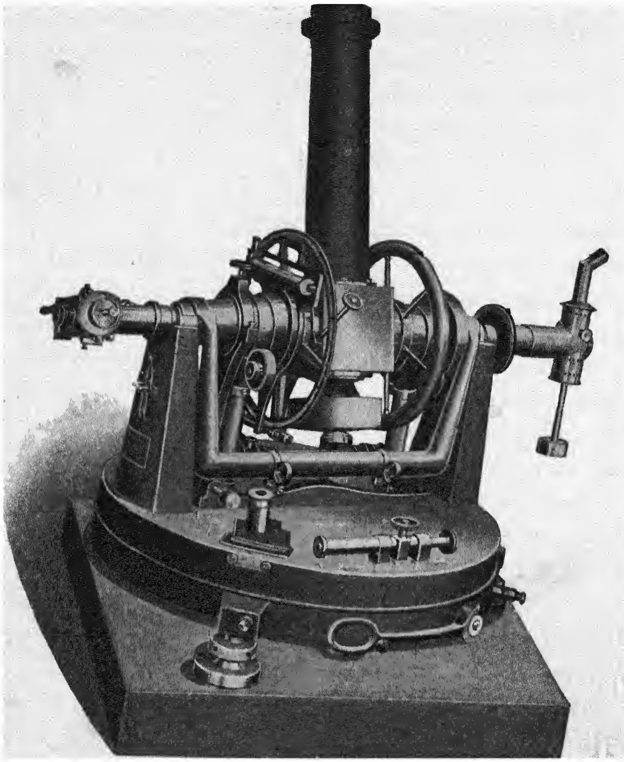


FIG. 29. A Broken Transit

A modified form of the transit instrument, now much used, is often called the *broken transit.* A reflector (usually a right-angled prism) in the central cube of the instrument directs the rays horizontally through one end of the axis where the eyepiece is placed, so that whatever may be the elevation of the star the observer looks straight forward horizontally, without needing to change his position. (Built by Warner & Swasey)

77. Elimination of Remaining Errors. The final test of *all* the adjustments, and of the accurate going of the clock, is obtained by observing a number of Almanac stars of widely different declination, reversing the instrument in its pivots midway in the process. If they all indicate *identically* the same clock correction, the instrument is in adjustment; if not, and if the differences

are not very great, it is nevertheless possible to deduce from the observations themselves the true clock correction and the adjustment errors of the instrument.

The astronomer can never assume that *adjustments are perfect*; even if they were once perfect, they would not stay so, on account of changes of temperature and other causes. Nor are observa tions ever absolutely accurate. The problem is, from observations more or less *inaccurate* but *honest*, with instruments more or less *maladjusted* but *firm*, to find the result that would have been obtained by a perfect observation with a perfect and perfectly adjusted instrument. It can be done more nearly and more easily than one might suppose (see Campbell's *Practical Astronomy*), but the discussion of the subject belongs to practical astronomy.

78. Personal Equation. The Transit Micrometer. Even the best observers habitually note the passage of a star across the fixed wires of the reticle slightly too late or too early, by an amount which is different for each observer. This *personal equation* (though usually less than $0^s.1$ for chronographic observations) is an extremely troublesome error, because it varies with the observer's physical condition and also with the nature and brightness of the object. Faint stars are almost always observed too late, in comparison with bright ones; this gives rise to the so-called *magnitude equation.*

The effect of personal equation has been very greatly diminished by the introduction of the *transit micrometer*. In this instrument the reticle of fixed wires is replaced by a movable wire, carried by a micrometer screw which can be turned by hand, or by mechanism controlled by the observer, at any desired rate. The observer devotes his whole attention to keeping the moving star accurately bisected by this wire. An electric signal is automatically given whenever the screw reaches certain points in every revolution (that is, when the moving wire and the star bisected by it reach each of a definite series of positions in the field), thus furnishing an equivalent for the transits of the star over an equal number of fixed wires. The relative personal equations of observers, after a little practice with this apparatus, become almost vanishingly small and are almost independent of the brightness of the stars observed.

79. The Meridian Circle. To find the declination of an object we must find where it crosses the meridian. The instrument used for this purpose is the *meridian circle* (Fig. 30). This is a transit

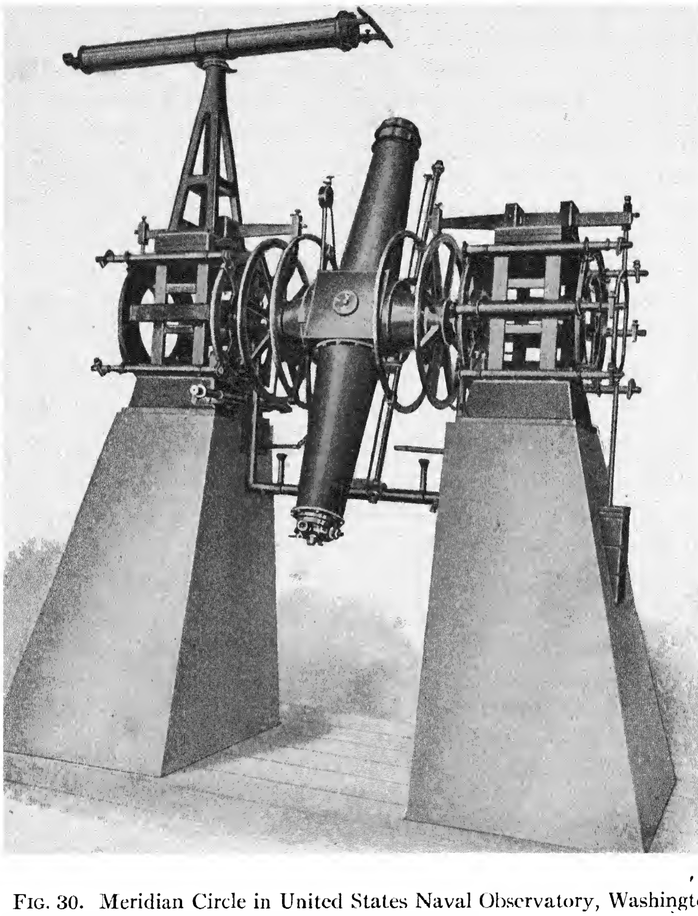


FIG. 30. Meridian Circle in United States Naval Observatory, Washington

The declination of the star is deduced from the readings of the circles attached to the axis. The circles are read by means of a number of microscopes. (Built by Warner & Swasey)

instrument of large size and most careful construction, *with a large graduated circle attached to the axis and turning with it.* When a star enters the field of view, the instrument is moved by the "slow-motion" screw until the star is bisected by the fixed horizontal wire of the reticle; and the star is kept bisected until it

reaches the middle vertical wire marking the meridian. The utmost resources of mechanical art are expended in graduating the circle with precision. The divisions are now usually made either two minutes or five minutes of arc, and the further subdivision is effected by so-called reading microscopes, four of which, at least, are always used in the case of a large instrument (see Campbell's *Practical Astronomy*). By means of these microscopes the reading of the circle is made to a tenth of a second of arc.

On a circle 2 feet in diameter, 1″ of arc is only about 1/17,000 of an inch; an error of that amount is now very seldom made by the best constructors in placing a graduation line. Even these minute *division errors* are carefully determined and allowed for by astronomers.

80. Zero Points. To reduce a circle reading to altitude or declination we must determine some *zero point* upon the circle, — the *nadir* point or the *horizontal* point if we wish to measure altitudes or zenith distances, the *polar* point or the *equator* point if we wish to measure polar distances or declinations. The polar point is determined by taking the circle reading for some star near the pole when it crosses the meridian above the pole, and then doing the same thing again twelve hours later when it crosses it below. The mean of the two readings (each corrected for refraction) will be the reading which the circle gives when the telescope is pointed exactly to the pole, — technically, the *polar point*. The *equator point* is, of course, 90° from the polar point.

The *nadir point* is the reading of the circle when the telescope is pointed vertically downward. It is determined by the reading of the circle when the instrument is set so that the horizontal wire of the reticle coincides with its own image formed by reflection from a basin of mercury placed on the pier below the instrument. To make this reflected image visible it is necessary to illuminate the reticle by light thrown toward the object-glass from behind the wires. The *zenith point* is just 180° from the nadir point thus determined.

81. Extra-Meridian Observations. Many objects are not visible when they cross the meridian: a comet, for instance, or a planet may be in such a part of the heavens that it transits only by day-

light. To observe such objects we may employ a so-called *altazimuth* (Fig. 31), — a telescope provided with both horizontal and vertical circles. By means of this the altitude and azimuth of an object may be measured; and if the time is recorded, from these the right ascension and declination can be deduced. The instrument may also be used for the determination of latitude, longitude (clock correction), and the direction of the meridian. The theodolite used in precise surveying, and the familiar engineer's transit, are smaller instruments of the same type.

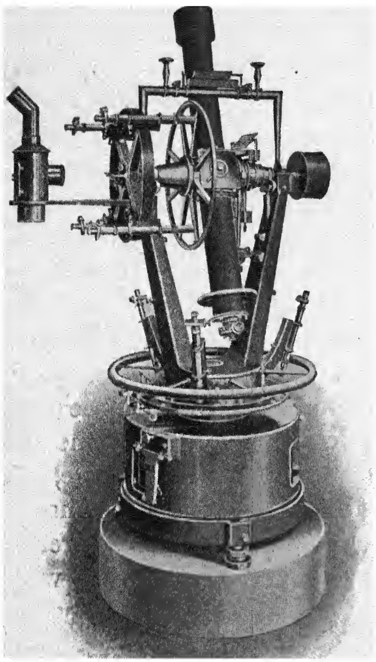


FIG. 31. A 5-Inch Altazimuth

This instrument may be directed at an object in any part of the heavens. Altitudes and azimuths are read from the circles by means of microscopes. (Built by Warner & Swasey)

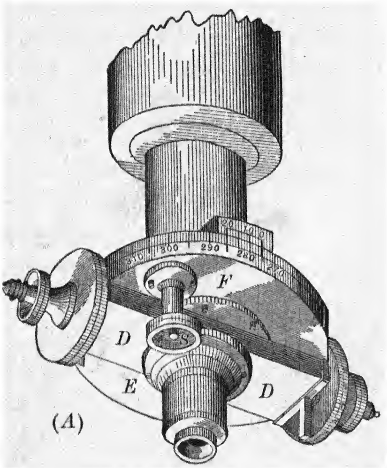
The *astrolabe à prisme* is an instrument with which very accurate simultaneous determinations of latitude and time may be made by observing the times when a number of stars reach the same fixed altitude.

Observations for the positions of bodies not on the meridian are often made with the equatorial telescope, with which the *difference* between the right ascension and declination of the observed body and those of some star in its neighborhood is determined by means of a *micrometer*, or often (at present) by photography.

82. The Micrometer. The *filar position micrometer* is a comparatively small instrument attached at the eye end of the telescope (Fig. 32). In its simplest form it contains two parallel wires, — one fixed, the other movable by means of a micrometer screw which affords the means of setting it at any desired distance from the fixed wire and of measuring this distance accurately.

The box containing the wires can itself be rotated around the optical axis of the telescope so that the wires can be set in any desired position angle. By rotating the box, first, so that the wire passes through two stars, and, second, so that a star follows the wire exactly when the driving-clock is stopped, the direction of one star from the other can be measured. By turning the box until the wires are at right angles to the line joining the two stars the distance between them can be measured (§ 764).

The micrometer can also be used for measuring differences of right ascension and declination. It is widely employed in measuring the diameters of planets and satellites, and in determining the position of comets, the relative positions of close double stars, and the positions of faint objects near bright ones. For the last purpose it has no rival. For the precise measurement of larger distances up to two or three degrees the heliometer was formerly used. That instrument is now only of historical interest.



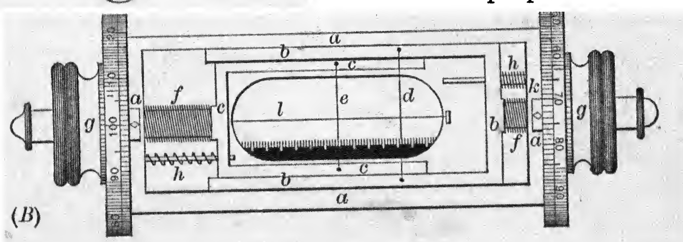


Fig. 32. The Filar Position Micrometer

Under the plate which carries the fixed wires lies a fork *b* moved by a carefully made screw with a graduated head *g*. This fork carries one or more wires parallel to the first set, so that the distance between the wires *e* and *d* can be varied and read off by means of the screw-head graduation

83. Measurement of Photographs. For larger fields, or when many stars are to be observed in one region, photography now affords the most valuable method of measurement. With an

exposure of only a few minutes, images of dozens or hundreds of stars can be permanently recorded on a plate, and afterwards measured at leisure. Exposures of an hour or two show stars too faint to be seen visually with the same instrument. If care is taken with the guiding (§ 68), so that the star images are sharp and round, the measurement of the plate will give the relative positions of the stars more accurately than they could be obtained by direct micrometric measures with the same telescope. If three or more stars whose places are already known from meridian observations are included among those measured, the right ascensions and declinations of all the others can easily be calculated.

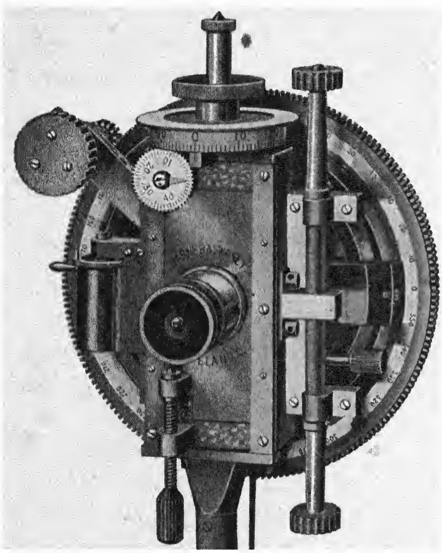


FIG. 33. A Complete Position Micrometer
This instrument is fitted with electric illumination. (Built by Warner & Swasey)

It is customary to measure the *rectangular coördinates* of the star images, that is, their distances from a pair of real or imaginary lines drawn on the plate at right angles to one another. With the aid of suitably constructed *measuring machines* this can be done quickly and accurately. In some of these the plate is moved under a fixed microscope by a micrometer screw; in others the microscope is similarly moved. There are many other types. Often a *réseau*, consisting of a network of lines ruled with the utmost attainable precision and forming squares five millimeters on a side, is printed photographically on the plate before development, thus affording a system of coördinates permanently attached to the plate (Fig. 154). The exact position of a star image inside one of the réseau squares can then be measured by a device similar in principle to the reading microscope.

Under the best conditions, as in the photographs taken with the great Yerkes refractor, a single plate will fix the position of a star relative to its neighbors with a probable error of only 0″.02, — a degree of precision not yet equaled by any other method of observation.

The images of bright stars or planets are often overexposed and cannot be measured with accuracy. Again, even the smallest photographic star images are about ten times the diameter of the diffraction disks (§ 53) for the aperture employed. Close double stars and fine planetary detail must therefore be studied visually.

84. The Sextant. All the instruments so far mentioned, except the chronometer, require some firmly fixed support and are therefore absolutely useless at sea. The *sextant* is the only one upon which the mariner can rely. By means of this he can measure the angular distance between two points (for instance, between the sun and the visible horizon), not by pointing first to one and afterwards to the other, but by sighting them both *simultaneously* and in *apparent coincidence*. A skillful observer can make the measurement accurately even when he has no stable footing.

The graduated arc of the instrument (Fig. 34) is usually, as its name implies, about a sixth of a complete circle, with a radius of about six inches. It is graduated in *half degrees* (which are, however, *numbered as whole degrees*), and it can measure any angle not much exceeding 120°. The *index-arm* is pivoted at the center of the arc and carries a *vernier*, which slides along the limb and can be fixed at any point by a clamp, with an attached *tangent screw* T. The reading of this vernier gives the angle measured by the instrument; the best instruments read to 10″.

Just over the center of the arc the *index-mirror* M, about 2 inches by $1\frac{1}{2}$ inches in size, is fastened to the index-arm, moving with it and keeping always perpendicular to the plane of the limb. At H the *horizon-glass*, about an inch wide and about twice the height of the index-glass, is secured to the frame of the instrument in such a position that when the vernier reads zero the index-mirror and horizon-glass will be parallel to each other. Only half the horizon-glass is silvered, the upper half being left transparent. E is a small telescope screwed to the frame and directed toward the horizon-glass.

If the vernier stands near but not exactly at zero, an observer looking into the telescope will see, together in the field of view, two separate images of the object toward which the telescope is directed; and if he slides the vernier, he will see that one of the images remains fixed while the other moves. The fixed image is

formed by the rays which reach the object-glass *directly* through the unsilvered half of the horizon-glass; the movable image, on the other hand, is produced by rays which have suffered *two reflections*, having been reflected from the index-mirror to the horizon-glass and then reflected a second time from the lower,

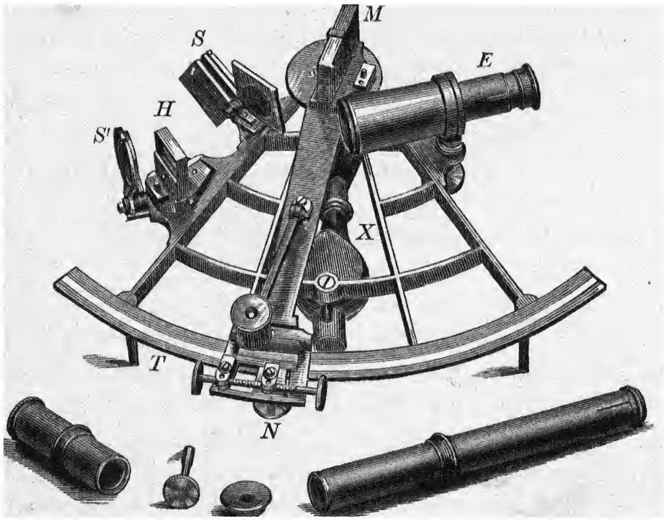


FIG. 34. The Sextant

The index-arm carries the index-glass M and a vernier which slides along the arc. Through the telescope E the observer looks directly through the unsilvered half of the horizon-glass H, while receiving light also which is reflected first from the index-glass and then from the silvered half of the horizon-glass

silvered half of the horizon-glass. When the two mirrors are parallel, the two images coincide, provided the object is at a considerable distance.

If the vernier does not stand at or near zero, an observer looking at an object directly through the horizon-glass will see not only that object but also, in the same telescopic field of view, whatever other object is so situated as to send its rays to the telescope by reflection from the mirrors. The *reading of the vernier will give the angle at the instrument between the two objects whose images thus coincide*, the angles between the planes of the two mirrors being, as is easily proved, just half the angle between the two objects, and the half degrees on the limb being numbered as whole ones.

If the vernier does not read strictly zero when the mirrors are parallel, all the sextant readings will be too great (or too small) by a fixed amount. This *index correction* is, however, very easy to determine and allow for.

85. Observation with the Sextant. The principal use of the instrument is in measuring the altitude of the sun. At sea the observer usually proceeds as follows: Holding the sextant in his right hand, with its plane vertical, he points the telescope at the horizon vertically beneath the sun. Then he

FIG. 35. "Shooting the Sun" with the Sextant

This photograph, taken on a destroyer, well shows the proper way to hold a sextant. The right hand grasps the handle. The index finger of the left hand supports the arc, while the thumb and middle finger are used on the clamp and slow motion. (From an official photograph by the United States Navy)

slides the vernier along the arc with his left hand until he brings the reflected image of the sun down to the horizon. Tightening the clamp and using the tangent screw, he makes the lower edge, or limb, of the sun just graze the horizon as he swings the sun's image back and forth by a slight motion of the instrument, to make sure that he is measuring the vertical distance between sun and horizon. As soon as the contact is satisfactory he marks the time and afterwards reads the angle. The reading of the vernier, after due corrections (see Chapter III), gives the sun's true altitude at the moment.

The image of a star is usually brought to the horizon more expeditiously by pointing the telescope directly at the star, then moving the index-arm slowly forward as the telescope is lowered, thus keeping the star in the field

of view until the horizon is reached. On land, recourse is had to an "artificial horizon." This is a shallow basin of some liquid that is a good reflector. The angle between the sun and its image reflected in the liquid is measured. The reading of the instrument corrected for index error then gives *twice* the sun's apparent altitude. Mercury gives the brightest image but is so mobile that it must be protected from the wind by a glass case, which is costly, since the surfaces must be worked accurately plane and parallel. Heavy lubricating oil is much cheaper and, if reasonably protected from the wind, requires no cover.

It is also possible (though less accurate) to use a telescope that has a spirit-level reflected into the field of view in such a manner that the center of the bubble indicates the true horizon. Instruments of this sort ("bubble-sextants") have been successfully used in airplanes (where the vibration renders the ordinary type of artificial horizon useless), but the accelerations due to the roll of the ship make them worthless at sea.

An artificial horizon that would work at sea in rough weather, when the stars can be seen and the horizon cannot, would be of great practical value, but the problem has not yet been successfully solved.

The skillful use of the sextant requires considerable dexterity, and on account of the low power of the telescope the angles measured are less precise than those determined by large fixed instruments; but the portability of the instrument and its applicability at sea render it invaluable. It was invented in practical form by Godfrey, of Philadelphia, in 1730, though Newton, as Halley announced, had really hit upon the same idea long before.

86. With the instruments described above, all the fundamental observations required in the investigations of spherical and theoretical astronomy can be carried out. The sextant and chronometer are, however, the only ones available in nautical astronomy.

Astrophysical studies require numerous physical instruments of entirely different character, — spectroscopes, photometers, heat-measuring instruments, and various kinds of photographic apparatus. These will be considered later, as occasion arises.

EXERCISES

1. If a firefly were to alight on the object-glass of a telescope, what would be the appearance to an observer looking through the instrument? Would he think he saw a comet?

2. When a person is looking through a telescope, if you hold your finger in front of the object-glass, will he see it?

3. If half the object-glass of a telescope pointed at the moon is covered, how will it affect the appearance of the moon as seen by the observer?

4. If a certain eyepiece gives a magnifying power of 60 when used with a telescope of 5 feet focal length, what power will it give on a telescope of 30 feet focal length?

5. What is the angular distance (theoretically) between the centers of two star disks which are just barely separated by a telescope of 24 inches aperture (§ 53)?

6. Why is it important that the two pivots of a transit instrument should be of exactly the same diameter?

7. If the middle wire in the reticle of a transit instrument is to the west of its proper position, what error in the observed time of transit will result? Will this error be the same for stars of all declinations? What will be the effect of reversing the instrument?

8. If the eastern Y is too far north, what will be the effect upon the transit of a star south of the zenith? north of the zenith? and how will these change with the declination?

9. What errors will result if the western Y is too high?

10. If the wires of a micrometer (Fig. 32) are set so that, used with a telescope of 10 feet focal length, a star moving along the right-ascension wire will occupy 15 seconds in passing from d to e, how long will it take when the micrometer is transferred to a telescope of 50 feet focus?

11. If the pitch of a micrometer screw is 1/75 of an inch, what is the angular value of one revolution of the screw when the micrometer is attached to a telescope of 30 feet focal length?

12. Does changing the eyepiece of a telescope for the purpose of altering the magnifying power affect the value of the revolution of the micrometer screw?

13. When the planet Mars is 20 seconds in apparent diameter, how large will its image be on a photograph taken with the Yerkes refractor of 19.36 meters focal length? *Ans.* 1.88 mm., or 1/14 inch.

REFERENCES

Louis Bell, *The Telescope* (McGraw-Hill Book Co., New York), a book "written for the many observers who use telescopes for study or pleasure and desire more information about their construction and properties."

The principal instruments employed in observations of position are described in detail, and their errors and adjustments discussed, in the works on practical and nautical astronomy. Most of these treat also of spherical astronomy.

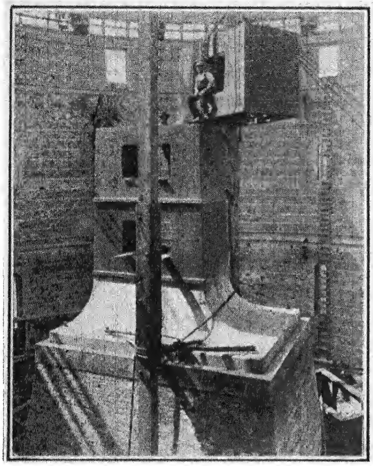
W. W. Campbell, *The Elements of Practical Astronomy* (The Macmillan Company, New York) — an excellent introduction.

William Chauvenet, *A Manual of Spherical and Practical Astronomy* (J. B. Lippincott, Philadelphia), an old but standard work, detailed and reliable; more advanced than the preceding, covering the field of practical astronomy as known in its day.

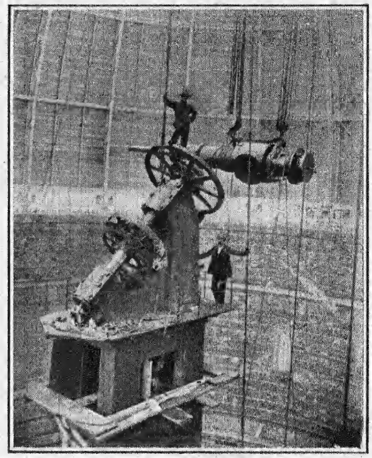
Simon Newcomb, *A Compendium of Spherical Astronomy* (The Macmillan Company, New York) — an authoritative work dealing exclusively with spherical astronomy.

The student interested in the theory of the diffraction image may be referred to treatises on optics (for example, R. W. Wood's *Physical Optics*).

P. G. Nutting, *Outlines of Applied Optics* (P. Blakiston's Sons & Co., Philadelphia) may be consulted for the theory of lenses and their aberrations.



Erecting the Iron Column of the 40-Inch Telescope. (By courtesy of the Yerkes Observatory)



Preparing to attach the Declination Axis to the Polar Axis. (By courtesy of the Yerkes Observatory)

CHAPTER III

PROBLEMS OF PRACTICAL ASTRONOMY

FUNDAMENTAL AND DIFFERENTIAL METHODS • LATITUDE, LONGITUDE, TIME, RIGHT ASCENSION, DECLINATION, AND AZIMUTH • THE POSITION OF A HEAVENLY BODY • NAVIGATION • REDUCTION AND CORRECTION OF OBSERVATIONS • ERRORS OF OBSERVATION • COMPUTATIONS

87. There are certain problems of practical astronomy which are encountered at the very threshold of all investigations respecting the heavenly bodies, the earth included. The student must know how to determine his *position on the surface of the earth*, that is, his latitude and longitude; how to ascertain the *exact time at which an observation is made*; and how to observe the *precise position of a heavenly body* and fix its right ascension and declination.

The first astronomers had no body of collected data at their disposal; the observer had to rely, for the complete solution of his problem, entirely on his own observations, and devise his method accordingly. Such methods are called *fundamental*, because they are employed in laying the foundations of the science. They are still as important as ever, since each generation strives for an increase of accuracy on the work of its predecessors. Such methods are used to obtain the positions and motions of a sufficient number of fundamental objects, — the sun, moon, and planets, and a selected list of suitable stars. The possession of these data makes it possible to employ much less laborious methods in the great bulk of astronomical observation. In particular, the fundamental determination, with great precision, of the places of many hundreds of stars has made it possible, by measuring *differences* of right ascension and declination, to determine the places of many thousands of other stars. We shall follow the same order in our presentation of the methods of practical astronomy, — first the *fundamental* methods, then the *differential* methods.

FUNDAMENTAL METHODS

88. Latitude and Declination. The latitude, as already defined, is the *arc of the meridian*, ZQ (Fig. 36), *between the zenith and the equator*, or its equivalent, the *angle of elevation of the celestial pole.* The zenith or the horizon can be located by the direction of the force of gravity. This is usually accomplished with the aid of the spirit-level or by reflection, as in the determination of the nadir point of the meridian circle (§ 80). There is no such convenient accessory available for the location of the point Q, where the equator crosses the meridian, or of the celestial pole P. Their places must be found by observation of the heavenly bodies.

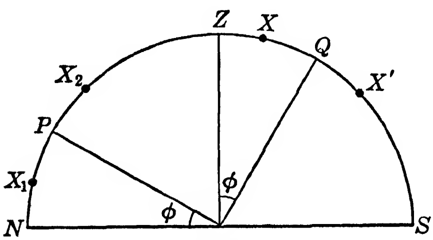


FIG. 36. Meridian Altitudes and the Determination of the Polar Point

Obviously the position of Q can be found from an observation of the star X on the meridian, if we know this star's declination. Such a method calls for information not supplied by our own observations, and is not, therefore, a fundamental method.

The matter stands otherwise with the determination of the *polar point.* With a suitable instrument — the meridian circle or, with much less precision, the sextant or theodolite — we may measure the altitude X_2N of some star near the pole when it is crossing the meridian above the pole, and, twelve sidereal hours later, the altitude X_1N when it is again on the meridian but below the pole. In the first case its altitude is the greatest possible; in the second, the least. The *mean of the two altitudes* (each corrected for atmospheric refraction) is the *altitude of the pole* or the *latitude* of the observer.

Half the difference between the two corrected altitudes is the *north polar distance* of the star, or $90° - \delta$. The observation, therefore, furnishes an equally fundamental determination of the declination of the star.

The altitude, as is readily apparent, does not necessarily enter into the problem. The mean of the circle readings when the in-

strument is pointed to the star at X_2 and X_1 is the circle reading of the pole. The north polar distance, and hence the declination, of any other star is simply the difference of the circle readings on the star and on the pole.

In practice, however, it is found unsafe to rely on the stability of the instrument. The polar point of the circle may change from day to day and even from hour to hour. It is necessary to have some fixed, observable point of reference which may be set on at any time. The *nadir point* (§ 80) is the most convenient and satisfactory. Thus the direction of the force of gravity is actually utilized, after all.

The method fails for stations very near the equator, because there the pole is so near the horizon that the necessary observations cannot be made.

89. Time and Right Ascension. In practice the problem of determining time always consists in ascertaining the *correction of the timepiece*, that is, the *amount by which the clock or chronometer is fast or slow as compared with the time it should indicate.* The latter, or true time, must be found from observation.

The sidereal clock should indicate $0^h\ 0^m\ 0^s$ when the vernal equinox transits the meridian; but we cannot directly observe the equinox. When any star crosses the meridian, the sidereal-clock reading should equal the right ascension of the star, and the time elapsing between the transits of any two stars should equal their difference of right ascension. Until we know where the equinox is we cannot find the former, but we can observe the latter. Observations on successive days will show how much the clock is gaining or losing, but not how fast or slow it is. Correcting for the rate of the clock, we may therefore make a map or catalogue of all the observed stars with their correct declinations and correct differences of right ascensions from any arbitrarily chosen star, such as Sirius. We have now to put the equinox on this map. This is done by observing the sun in the same fashion as the stars, and thus determining its track (the ecliptic) in the heavens relative to the stars. If this is plotted on the map, its intersection with the equator will be the equinox. This could be done approximately by plotting on a globe, and much more accurately by computation.

The positions of the equinox and a large number of stars are now known so accurately that the fundamental work of today takes the form of determining small corrections to the older values and hence obtaining increasingly accurate positions.

90. The Gnomon. It is appropriate to mention here the remarkable results which the ancients obtained with the simplest of all astronomical instruments, the gnomon.

The gnomon is merely a vertical shaft or column of known height erected on a perfectly horizontal plane. The shadow, at noon, grows shorter each day as the sun travels northward. The shortest noon shadow, AC, of the year, and the longest, AD, are measured. The height AB being given, we can easily find, in the right-angled triangle, the angle ABC, which equals SBZ, the sun's zenith distance when farthest north, and the angle ABD, which is the sun's corresponding zenith distance when farthest south. Now, since the sun travels equal distances north and south of the equator, the mean of the two zenith distances is the angular distance between the equator and the zenith, that is, the *declination of the zenith*, which is the latitude of the place (Fig. 36).

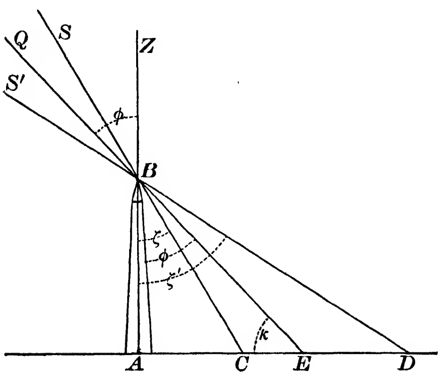


FIG 37. An Ancient Astronomical Instrument — the Gnomon

By measurements of the length and direction of the shadow of a vertical pillar the ancients determined their latitude, the obliquity of the ecliptic, the length of the year, and the time of day

Taking half the difference between the two zenith distances, the ancients obtained a good approximation to the *obliquity* of the ecliptic; from the direction of the shadow at noon they determined azimuth; the interval between observations of the shortest shadows gave them the length of the year; while the changing direction of the shadow furnished them with a measure of time, — a great deal of first-hand information to be derived from a stake driven into the ground. Measures with this instrument do not admit of much accuracy, however, since the penumbra surrounding the shadow makes it impossible to measure the length precisely.

It may be noted that the ancients, instead of designating the position of a place by its latitude, used its *climate*, the climate (from κλίμα) being the slope of the plane of the celestial equator, — the angle AEB, which is the *colatitude.*

Many of the Egyptian obelisks are known to have been used for astronomical observations, and perhaps were erected mainly for that purpose.

DIFFERENTIAL METHODS

Latitude, Time, Longitude, and Azimuth

91. Latitude. Suppose we travel southward one degree of latitude; our zenith moves one degree southward among the stars, and our horizon tips down in the south and up in the north, rotating about the east-west line. The altitude of every star on the meridian to the south of the zenith increases by one degree, and the altitude of every star to the north decreases by the same amount. Since the separation of the two horizons is greatest at the meridian, an altitude measured here will distinguish much better between two such horizons (and determine the latitude more accurately) than the altitude of an object far from the meridian. When a star is on the meridian, its altitude depends only on its declination and on the latitude of the observer; when it is away from the meridian, its altitude depends also on the hour angle.

The general underlying principles of the many methods of determining the latitude from a measured altitude may be outlined as follows: The altitude is to be measured with a suitable instrument; the declination of the object — sun, moon, planet, or star — may be found in the *Nautical Almanac* or some other catalogue; and if we note the time at which the altitude is measured, we can find the hour angle of the object (§ 41). The latitude may then be calculated by a suitable formula.

92. Latitude by Meridian Altitude. In Fig. 36 the semi-circle $SQPN$ is the meridian, Z the zenith, P the pole, and Q the point where the equator crosses the meridian. QZ is the latitude (ϕ) of the observer — to be determined. If, when the star is on the meridian, we observe its zenith distance, Z_m (arc ZX in the figure), its declination δ (QX) being known, then evidently QZ equals QX plus XZ; that is, *the latitude of the place equals the declination of the star plus its zenith distance.* If the star is at X', south of the equator, the same equation still holds *algebraically*, because the declination (QX') is then a negative quantity. Therefore in all such cases (when the star and the pole are on opposite sides of the zenith) $\phi = \delta + Z_m = 90 + \delta - h_m$. When the star crosses the meridian between the zenith and the pole, as at X_2, the formula is

$\phi = \delta - Z_m$; and, finally, for stars (X_1) below the pole it is necessary to use the formula $\phi = 180 - \delta - Z_m$. *These formulæ apply only to stars on the meridian.*

If the meridian circle is used in making the observations, stars can always be selected that pass near the zenith, where the refraction is small, which

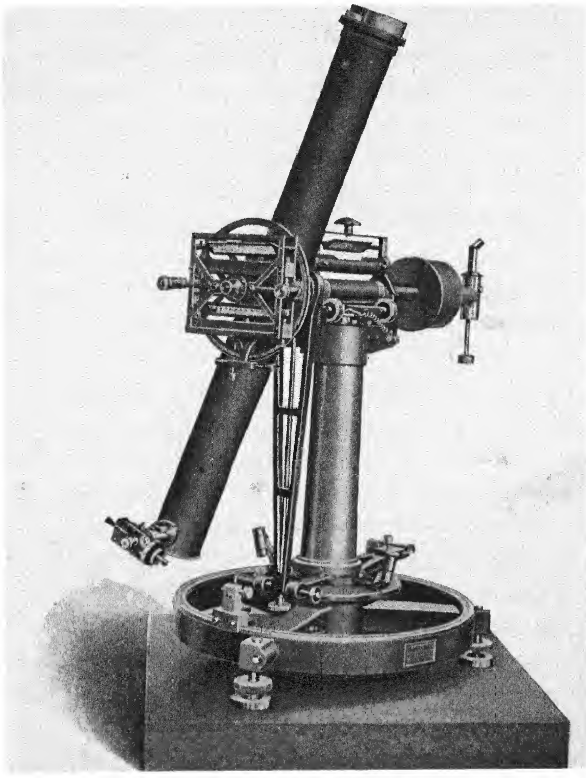


FIG. 38. A Zenith Telescope
Built by Warner & Swasey

is in itself a great advantage. Moreover, they may be selected in such a way that some will be as far north of the zenith as others are south, and this will practically eliminate even the slight refraction errors that remain.

93. Latitude by the Zenith Telescope. The essential characteristic of the method is the measurement with a *micrometer* (rather than by the graduated circle) of *the difference between the nearly*

equal zenith distances of two stars which pass the meridian within a few minutes of each other, one north and the other south of the zenith, and not very far from it. Such pairs of stars can now always be found listed in our star-catalogues and almanacs. A sensitive level plays an important rôle in the measurement.

A special instrument, known as the *zenith telescope* (Fig. 38), is generally employed, though a simple transit instrument, provided with reversing apparatus, a delicate level attached to the telescope, and a declination micrometer, is now often used.

The telescope is set at the proper altitude for the star that first comes to the meridian, and the "latitude level," as it is called, which is attached to the telescope, is set horizontal. As the star passes through the field of view its distance north or south of the central horizontal wire is measured by the micrometer. The instrument is then reversed so that the telescope points toward the north (if it was south before), and, if necessary, the telescope so readjusted that the level is again horizontal. Great care must be taken, however, *not to disturb the angle between the level and the telescope itself.* Evidently the telescope is then elevated at exactly the same angle as before, but on the opposite side of the zenith. As the second star passes through the field we measure with the micrometer its distance north or south of the central wire. The comparison of the two measures gives the difference of the two zenith distances with great accuracy and *without the necessity of depending upon any graduated circle.*

In field operations, like those of geodesy, this is an enormous advantage, as regards both the portability of the instrument and the attainable precision of results.

By section 92 we have

for star *south* of zenith, $\phi = \delta_s + Z_s$;

for star *north* of zenith, $\phi = \delta_n - Z_n$.

Adding the two equations and dividing by 2, we have

$$\phi = \frac{\delta_s + \delta_n}{2} + \frac{Z_s - Z_n}{2}.$$

The *Nautical Almanac* gives the declinations of the two stars $(\delta_s + \delta_n)$; and the difference of the zenith distances $(Z_s - Z_n)$ is determined by the micrometer measures.

Refraction is almost eliminated, because the two stars of each pair are at very nearly the same zenith distance.

Evidently the accuracy depends ultimately upon the exactness with which the level measures the slight but inevitable difference between the inclinations of the instrument when pointed on the two stars.

One night's work with a portable zenith telescope such as is used by the United States Geodetic Survey will give the latitude with a probable error (§ 123) not exceeding $\pm 0''.10$. The larger instruments at fixed observatories do twice as well.

There are numerous other methods for obtaining the latitude. Some of them are discussed later (§§ 105, 106).

94. Time by the Transit Instrument. The method most employed by astronomers is observation with the transit instrument (§ 74). We observe the time shown by the sidereal clock at which a star of *known right ascension* crosses each wire of the reticle. The mean is taken at the instant of crossing the instrumental meridian, and when the instrument is in perfect adjustment the difference between the star's right ascension and the observed clock time will be the clock correction, or, as a formula, $\Delta T = \alpha - T$, ΔT being the usual symbol for the clock correction, and T the observed time.

The *Nautical Almanac* supplies a list of several hundred stars whose right ascension and declination are accurately given for every tenth day of the year, so that the observer at night has no difficulty in finding a suitable star at almost any time. In the daytime he is, of course, limited to the brighter stars.

By observing a number of stars, reversing the instrument upon its Y's during the program, and using a transit micrometer to eliminate personal equation, the clock correction may be determined within about a thirtieth of a second of time. The sun cannot be observed as accurately as the stars. The correction of the solar clock can best be found by comparing the readings of the sidereal and solar clocks, correcting the former to local sidereal time by applying the error found from star observations, and then converting first into local mean time and then into standard time (§ 45).

Other methods of determining the clock correction will be found in section 107 (p. 91).

In some cases a person is so situated that it is necessary to determine the time roughly, without instruments; this can be done with an error less than about half a minute by establishing a noon-mark, which is nothing but a line drawn exactly north and south, with a plumb-line or some vertical edge, such as the edge of a door-frame or window-sash, at its southern extremity. The shadow will always fall upon the meridian line at *local apparent noon.*

95. Longitude. Having now the means of finding the true local time at any place, we can take up the problem of the longitude, the most important of all the economic problems of astronomy. The great observatories at Greenwich and Paris were established expressly for the purpose of furnishing the observations which could be utilized for its accurate determination at sea.

The longitude is simply *the difference between the local time and the Greenwich time at the same instant.* Since the observer can determine the former by the methods already given, the crux of the problem is to find, without leaving his place, the Greenwich local time corresponding to his own.

96. Telegraphic comparison may be made between his own clock and that of some station whose longitude from Greenwich is known. The difference between the two clocks will be the difference of longitude between the two stations after the proper corrections for *clock errors, personal equation, and time occupied by the transmission of the electric signals* have been applied.

The process usually employed is as follows: The observers, after ascertaining that they both have clear weather, proceed early in the evening to determine the local time at each station by an extensive series of star observations with the transit instrument. Then, at an hour agreed upon, the observer at the eastern station takes a telegraph key and makes a series of arbitrary signals, which are recorded, by means of relays, upon the chronographs at *both* stations. After fifteen or twenty such signals have been sent, the western observer takes his key and sends a series of thirty or forty signals; then the eastern observer sends another set like the first. The night's work is closed by another series of transit observations by each observer.

Upon each chronograph there are now the records of the clock times of sixty or more signals which would be simultaneous at the two stations if the transmission of the electric signals were instantaneous.

In practice the amount by which the western clock appears to be slow compared with the eastern clock will not be the same at the two stations. This discrepancy is evidently the sum of the times of transmission of the

signals in the two directions. If these are equal (as is safe to assume if the electrical conditions are exactly the same in both cases), the true difference of the clocks will be the mean of the values found from the two sets of signals. The transmission time over a land line is only a few hundredths of a second, but for a long ocean cable it may be much greater.

By applying to the difference of the two clocks the difference of the corrections required to reduce the reading of each to *true local time*, as determined by the two observers, we should obtain the difference in longitude, *provided that the personal equations of the observers were the same.* The effect of this source of error may be minimized by the exchange of observers after several nights' work and then observing for several nights more.

For observations with the transit micrometer (§ 78) the relative personal equations of experienced observers are very small (averaging about $0^s.01$), and the exchange of observers may be dispensed with except in work of unusual precision, when the relays and other electrical apparatus should be exchanged along with the observers. In field work chronometers are used instead of clocks.

97. Use of Radio Signals. The best of all methods of exchanging signals in longitude work seems to be by *wireless telegraphy.*

Regular time-signals at specified hours of Greenwich time are now sent out every day from a number of stations in Europe and America, and may be utilized by any observer within range on land or at sea. He can thus find the error of his chronometer daily within one fifth of a second. The signals, if fairly strong, can be recorded on a chronograph by the use of suitable amplifiers combined with a relay; but in most cases they are received by ear. The time-signals from the Naval Observatory are sent out from Arlington daily at noon and at 10 P.M. Eastern standard time. The signals (short dashes every second) begin five minutes before the hour. The twenty-ninth second of each minute is omitted, also those from 55 to 59 inclusive. At the end of the last minute a pause from the fiftieth to the fifty-ninth second is followed by a long signal beginning at the instant of noon (or of 10 P.M.).

Vernier signals are sent out from Eiffel Tower and some other stations when greater accuracy is required. The interval between successive signals is slightly less than one second, and the coincidences with the beats of the observer's own timepiece determine the clock correction within a small fraction of a second, just as the coincidence of a vernier division enables one to read a setting to a small fraction of a scale division.

A very precise determination of the difference of longitude between Paris and Washington was made by wireless in 1913–1914. Signals sent across the Atlantic in both directions indicated that the transmission time over the distance of 3830 statute miles was only $0^s.022$, giving a velocity of 175,000 miles per second, which agrees, within the (large) experimental error, with that of light.

98. Azimuth. An important problem, though one not so often encountered as that of latitude and longitude determinations, is that of determining the azimuth, or "true bearing," of a line upon the earth's surface.

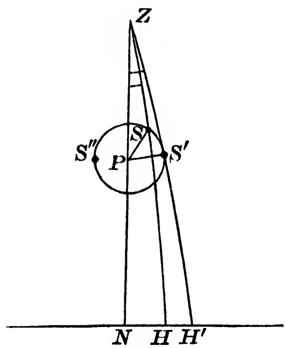


FIG. 39. The Azimuth of Polaris

When its hour angle is ZPS, the azimuth is the angle PZS, or the arc NH

With a theodolite having an accurately graduated horizontal circle the observer points alternately upon the polestar and upon a distant signal erected for the purpose, — usually an "artificial star" consisting of a small hole in a plate of metal, with a lantern behind it. At each pointing on the star he notes the time by a sidereal chronometer. The theodolite must be carefully adjusted for collimation (or reversed for the elimination of the collimation error), and special pains must be taken to have the axis of the telescope perfectly level. The next morning by daylight the observer measures the angle or angles between the night signal and the objects whose azimuths are required.

If the polestar were exactly at the pole, the mere difference between the two readings of the circle, obtained when the telescope is pointed on the star and on the signal, would give directly the azimuth of the signal. As this is not the case, the azimuth of the star must be computed for the moment of each observation, which demands only the solution of the "astronomical triangle" (Fig. 39, — P being, as usual, the pole, Z the zenith, NH' the northern horizon, and S the star) (compare § 42).

The polestar (or another fainter star close to the celestial pole) is used because a slight error in the assumed latitude of the place

or in the time of the observation will produce hardly any effect upon the result, especially if the star be observed near its greatest elongation east or west of the pole. One night's observations with a large theodolite should determine the azimuth with a probable error (§ 123) not exceeding $\pm 0''.5$.

There happens to be a convenient way of knowing when Polaris is on the meridian and hence due north. The two bright stars, ζ Ursae Majoris and δ Cassiopeiae, on opposite sides of the pole (Fig. 3), are very close to the hour-circle of Polaris. They cross the meridian about fourteen minutes ahead of Polaris. The meridian may be located quite closely by watching for the moment when Polaris and one of these stars are on the same vertical circle and then following Polaris about fourteen minutes longer (the exact interval is given in the *Nautical Almanac*). The observation may be made with two plumb-lines, keeping the northern one fixed and moving the southern one. It may be made more accurately with a theodolite, moving the telescope up and down until the two stars are on the vertical wire, then keeping Polaris bisected for the requisite number of minutes. An instrument will require only very slight adjustment after it has been oriented by this method.

The sun, or any other heavenly body whose position is given in the *Almanac*, can also be used as a reference point in the same way when near the horizon, provided sufficient care is taken to secure an *accurate observation of the time* at the instant when the pointing is made; but the results are usually rough compared with those obtained from the polestar.

The Position of a Heavenly Body

The "position" of a heavenly body is defined by its right ascension and declination. These may be determined:

99. (1) *By the meridian circle,* provided the body is bright enough to be seen by the instrument and crosses the meridian at night. In the differential use of the instrument we observe some neighboring standard star or stars of accurately known position just before or after the object whose place is to be determined, and thus obtain the *difference* between the right ascension and declination of the object observed and of the stars of known position. In this case slight errors in the adjustment of the instrument affect the final result very little. In collecting material for

a star-catalogue the observer, by limiting his attention to a very narrow *zone* of declination, is able to observe nearly all the brighter stars as they come to the meridian. In order to secure accuracy it is desirable that the observations should be repeated many times. Even in the best work, however, after correction for all known sources of error, there remain minute errors of obscure origin, varying with the right ascension, declination, and brightness of the star, which can be detected only by comparison with the results of other observers. The detection and determination of these "systematic errors" demand a high order of skill and judgment.

(2) *By the equatorial.* When a body (a comet, for instance) is too faint to be observed by the telescope of the meridian circle, which is seldom very powerful, or when it does not come to the meridian during the night, we must accomplish our observation with some instrument that can pursue the object to any part of the heavens. An equatorially mounted telescope is most suitable for the purpose.

100. With this instrument the position of a body is determined by measuring the *difference of right ascension and declination* between it and some neighboring star whose place has been accurately determined by the meridian circle and is given in a star-catalogue.

In measuring this difference of right ascension and declination a filar micrometer (§ 82) is usually employed, which is fitted with a number of fixed wires, set accurately north and south in the field of view, and one or more wires at right angles which can be moved by the micrometer screw. The difference of *right ascension* between the star and the object to be determined is measured by clamping the telescope firmly and then simply observing and recording upon the chronograph the transits of the two objects across the wires that run north and south; the difference of *declination* is measured by bisecting each object by one of the micrometer wires as it crosses the middle of the field of view. The difference of the two micrometer readings gives the difference of declination.

The observed differences must be corrected for refraction and for the motion of the body during the time of observation.

101. (3) *By photography.* Theoretically only three known stars need be measured (§ 83) on the plate in order to find the right ascensions and declinations of all objects that can be observed on it. In practice, measures of all the known stars on the plate are used, to get as accurate values as possible of the constants used in the reduction.

This photographic method is very accurate and very rapid and economical, especially when great numbers of stars are to be observed, and is extensively used. Schlesinger finds that the cost of determining the positions of eight or ten thousand stars by photography is less than that for six hundred stars by the meridian circle. One notable advantage is that the corrections for refraction, aberration, etc., as well as those for the instrumental adjustments, take a very simple form and can all be made at once. But the photographic method still depends on the meridian circle for the places of its reference stars.

NAVIGATION

The only instruments suited to the determination of the place of an observer at sea, on the unstable deck of a ship, are the sextant and the chronometer. The sextant is used to measure the altitude of a heavenly body; the chronometer furnishes the Greenwich time of the observation.

102. The Ship's Chronometer. While the ship is in port the chronometer is carefully compared with the time-signals, to determine the correction and daily rate which are to be applied to the chronometer reading to give the true Greenwich time at any desired moment during the voyage.

Chronometers are but imperfect instruments, and theoretically the ship should have at least *three*, for if only two chronometers are carried, and they disagree, there is nothing to indicate which is the delinquent. Practically, however, a single chronometer can be sufficiently trusted for voyages of moderate length, and only warships and liners carry three. Indeed, with the reception of daily radio signals any good watch would suffice.

103. Solution of the Astronomical Triangle. In order to calculate one of the sides or angles of the *ZPX* triangle it is necessary

(we recall) to know three other elements — sides or angles. If we measure the *altitude* of an object of known *declination*, and know also its *hour angle* (and this requires a knowledge of the *longitude*), we may calculate the *latitude*; if we know the *latitude*, as well as the *altitude* and *declination*, of the body observed, we can calculate the *longitude*. We must know the latitude *or* the longitude in order to calculate the other. A single measured altitude will not give both the latitude *and* the longitude of a ship.

104. Dead Reckoning; Traverse Tables. By the older methods of navigation separate altitudes for latitude and longitude were measured several hours apart and each coördinate worked up with an assumed value of the other. These assumed values are obtained by *dead reckoning*. The change in latitude, for example, since the noon observation, may be calculated from the course and distance run, and used for the afternoon longitude observation. The actual calculation is avoided by the use of "traverse tables," which give the solution, in tabular form, of a right-angled plane triangle (Fig. 40). All three sides are measured in nautical miles, which are equal, very nearly, to minutes of arc of latitude. The change in latitude (DL), therefore, is given directly by the table. The *departure* (Dep.), however, which is the number of miles of *easting* or *westing* the ship makes, on the *course* steered, is not directly equal to the change of longitude, on account of the convergence of the meridians toward the poles; but the conversion is readily made by means of the traverse table used in a different way.

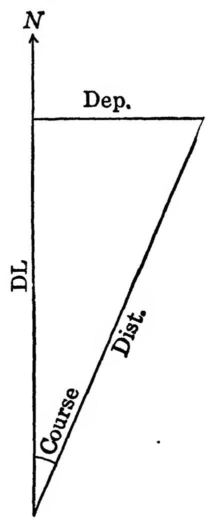


FIG. 40. Plane Sailing

105. Latitude by Meridian Altitude. The latitude is usually obtained by observing with the sextant the *sun's maximum altitude*, which occurs, of course, at *local apparent noon*.

If the navigator is somewhat uncertain of his position and does not know very closely the moment of local noon, he takes care

to begin his observations some minutes earlier. He brings the image of the sun down until it just touches the horizon.

Looking again, he sees the sun lifted slightly above the horizon (the altitude is still increasing) and pushes the index-arm forward to bring its image down again. Now the sun's lower limb hangs on the horizon and finally dips. He calls out to "make it eight bells," and reads the sextant. A number of corrections (to be explained later) have to be applied to the maximum[1] altitude to obtain the *true meridian altitude.*

We have given (§ 92) the astronomer's formulæ for finding the latitude from a noon altitude. Their application in different latitudes requires careful attention to signs. The navigator may be in the northern or in the southern hemisphere; he may face toward the south or toward the north when he measures the noon altitude of the sun; he may observe the sun or a star below the pole. He has developed a single rule to cover all cases, — a rule which sounds rather complicated but which is simple in its application and is, as the saying goes, well-nigh "fool-proof": "Mark the zenith distance *north* if the zenith is north of the body, *south* if the zenith is south of the body. Mark the declination of the body north or south as given in the *Almanac*. If the zenith distance and declination are of the same name, their sum is the latitude and is of the same name; if they are of opposite names, their difference is the latitude, which has the name of the greater." For sub-polar observations, *180° — declination* replaces the declination and has the same name as the declination.

It is well, in any case, to represent the situation graphically. Draw a semi-circle for the meridian and mark the north and south points and the zenith; mark the position of the observed body by its distance from the horizon or from the zenith; then put in Q by the distance and direction of the equator from the body. We can then see at once how far it is from Z to Q, and which pole is visible.

[1] Also, on account of the sun's motion in declination and the northward or southward motion of the ship itself, the sun's maximum altitude is usually attained not precisely on the meridian but a short time earlier or later. This necessitates a slight correction. It is customarily considered better practice for the observer to set his watch to the local apparent time of the prospective noon longitude and take the sun at the moment when his watch indicates noon.

The method of noon altitude depends but indirectly upon the observer's knowledge of the time, which is used only in interpolating the declination from the *Almanac*. The declination of the sun changes most rapidly near the equinoxes, and then only at the rate of about 1′ per hour. An error of six minutes in the assumed time will produce, therefore, an error of only 10″ in the calculated latitude.

106. Latitude by Reduction to the Meridian and by a Single Altitude. If the observer knows his time with fair accuracy, he can obtain his latitude from altitudes measured *near* the meridian, applying to each the small difference, easily calculated from the hour angle, between it and the meridian altitude. By this method the observer is not restricted to a single observation at each meridian passage of the sun or of the selected star, but may utilize a considerable interval before and after this moment and gain the greater accuracy inherent in an average of several observations, or take advantage of a break in the clouds.

Finally, the altitude of an object some distance from the meridian may be measured, and the latitude calculated by a solution of the *ZPX* triangle. Obviously the method requires a more accurate knowledge of the time the farther the object is from the meridian, and is practically useless, when the sun is the body observed, beyond an hour angle of three hours. An observation of the slow-moving Polaris at any hour angle will yield a good value of the latitude.

107. Longitude by an Altitude of Sun or Star near the Prime Vertical. The observation consists in measuring the altitude and noting accurately the corresponding chronometer time. The declination is taken from the *Almanac*, and the latitude is assumed by dead reckoning. The hour angle t may then be computed. This hour angle, if the sun is observed, is the *local apparent time* at the moment of observation, and may be converted, by means of the equation of time, into *local civil time*. The difference between this time and that shown by the chronometer, at the moment of observation, is the longitude, provided the chronometer indicates true Greenwich civil time. As previously explained, the chronometer time will usually require correction for error and rate.

If a star, a planet, or the moon is observed, the local sidereal time may be derived from the computed hour angle and the right ascension (§ 41), and then converted into local civil time.

In order to insure accuracy it is desirable that the sun or other object to be observed should be on the prime vertical, or as near it as practicable. It *should not be near the meridian*, for at that time the sun is rising or falling very slowly, and the slightest error in the measured altitude is magnified many times in the error of the computed hour angle. Furthermore, if the sun is exactly east or west at the time of observation, an error of even several minutes of arc in the assumed latitude produces no sensible effect upon the result.

Before the days of reliable chronometers, navigators were obliged to depend, for their Greenwich time, upon the observation of the place of the moon (by "lunar distances" from stars of known position) or of such phenomena as eclipses which were predicted in the *Almanac.*

108. Circle of Position; Sumner's Method. The great advance in modern methods of navigation lies in the realization, first practically expressed by Captain Sumner, of Boston, in 1843, that a single observation of the sun's altitude places the ship on a *line* and not at a *point.* There are innumerable other places where the altitude or zenith distance of the sun would be the same. Such places all lie on a circle, the so-called *circle of position,* whose radius (reckoned in degrees of a great circle) is the observed *zenith distance* of the sun. The center of this circle of position on the earth's surface is the point directly under the sun — the "sub-solar point." The *longitude* of this point is the *Greenwich apparent time*[1] at the moment of observation as determined by the chronometer, and its *latitude* is the sun's *declination.*

In practice the ship's place is known, by dead reckoning, within twenty miles or so (much nearer unless there has been a spell of cloudy weather), and the portion of the circle of position on which there is any chance of its actually lying is so small that it may be *replaced by a straight line.* This "Sumner line" may be drawn on a chart in various ways.

[1] Or, in general, the hour angle at Greenwich of the body observed. Except for this one statement the description of the method is general for any observed body.

(1) Having observed an altitude at a known Greenwich time, assume a latitude north of the dead-reckoning position and compute the longitude from the observation. Repeat with the assumed latitude to the southward. Plot the two positions thus obtained, and the line joining them will be the Sumner line.

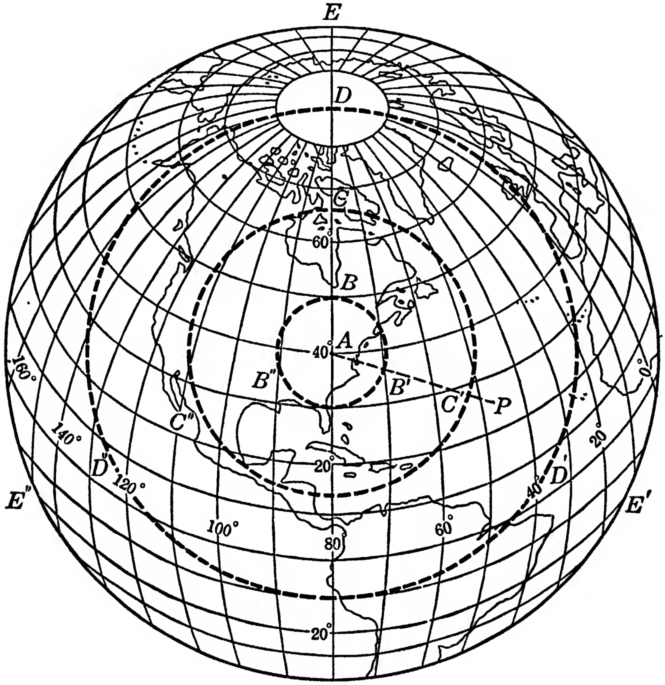


Fig. 41. Circles of Position

The observed body is vertically over A. Its declination is 40° North. The hour angle of the body at Greenwich is $5^h\ 20^m$. The dotted circles are circles of position corresponding to measured zenith distances. P represents the assumed dead-reckoning position of the ship. The observed zenith distance gives the improved position C', through which the Sumner line is to be drawn at right angles to the line $C'A$. (From the *American Practical Navigator*)

(2) A quicker method, due to Admiral St. Hilaire of the French navy, depends on the fact that *the direction of the Sumner line is at right angles to the bearing of the sun.* Starting with any convenient assumed position, we compute the sun's zenith distance and azimuth as seen from this point at the Greenwich time of observation. If this computed zenith distance agrees with the

observed zenith distance, the assumed position lies on the Sumner line. Otherwise *its distance from this line* (in nautical miles) *is equal to the difference between the observed and computed altitudes* (in *minutes of arc*). If the observed zenith distance is *smaller* than the computed, we are nearer the point beneath the sun than we thought we were. The difference, in this case, is to be measured off *toward* the sun, and the Sumner line is to be drawn through the improved point at right angles to the direction of the sun.

The computations required by this method may be much shortened by using special tables and by choosing the assumed position on an even degree of latitude and at such a longitude that the sun's hour angle is also an exact number of degrees, when hardly any calculation is necessary.

Any other bright heavenly body may be used instead of the sun. During twilight, when the brighter stars and the sea horizon are both visible, observations upon two objects in different azimuths can usually be secured almost simultaneously, which give two intersecting Sumner lines and hence furnish a "fix," that is, the actual position of the ship. Sometimes the sun and moon can be observed in the same fashion by day.

The circles of position intersect at *two* points, of course; but it is never doubtful at which one the ship is situated, because they are usually thousands of miles apart.

Successive observations of the sun may be utilized, provided they are separated by an interval sufficiently long to allow a considerable change in the azimuth of the sun. Allowance for the motion of the ship between times is easily made by moving the earlier Sumner line parallel to itself on the chart in the direction and over the distance the ship has traveled. The intersection with the later line is the position of the ship at the time of the later observation. It is good modern practice always to "bring up" each Sumner line in this manner.

A ship near the coast may now get a good "fix" from a single Sumner line and the radio bearing of a shore station.

It is frequently important to have the Sumner line run in a certain direction. Thus, if the ship is passing a dangerous shoal, the navigator will try to get a Sumner line parallel to his course,

which will give him his distance from the danger; and he will therefore observe the sun when it bears directly toward or away from the danger.

It should be noticed that in Sumner's method, as in all others, *the correctness of the position depends upon the accuracy of the Greenwich mean time given by the chronometer.* On this account the chronometer must be treated with great care.

THE REDUCTION OF OBSERVATIONS

Observations as actually made always require corrections before they can be used in deducing results. Those that depend on the errors or maladjustment of the instrument, which have already been referred to as belonging to the technical field of practical astronomy, will not be considered here, but only such as are due to other causes, external to the instrument (and the observer).

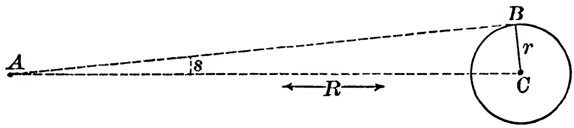


FIG. 42. Relation of Linear Diameter to Apparent Diameter and Distance

Thus far we have carefully avoided reference to the distance of the heavenly bodies; the discussion has been rather closely confined to their geometrical positions on the celestial sphere; but the element of distance enters into many of the methods of practical astronomy that have been described, and cannot be neglected.

109. Relation between the Distance and Apparent Size of an Object. The apparent size of an object depends upon its linear size and its distance from the observer; the larger it really is, and the nearer it is, the larger it will look.

Imagine a sphere having a (linear) radius BC equal to r. As seen from the point A (Fig. 42), its *apparent* (that is, angular) semidiameter will be BAC or s, its distance being AC or R.

From trigonometry, since B is a right angle, $\sin s = r/R$, whence also $r = R \sin s$, and $R = r/\sin s$.

When we are dealing with a small angle and are not insistent upon high accuracy, we may secure a close approximation to the values of the *sine*, *tangent*, and *cosine* by using their expressions in the form of series and discarding all but the first terms, thus,

$$\sin s = s; \quad \tan s = s; \quad \cos s = 1.$$[1]

Even for an angle of 5°, sin s and s differ only by one unit in the fourth decimal place. The difference between the sine and the tangent is nearly as small.

For small angles,[1] therefore, $\sin s = \tan s = s''/206{,}265$.

It is quite allowable, then, in the case of the sun, moon, or planets, to set $\sin s = s$, and we have

$$s = \frac{r}{R}.$$

or, in ordinary angular units,

$$s^\circ = 57.30\, r/R, \text{ or } s' = 3437.7\, r/R, \text{ or } s'' = 206{,}265\, r/R;$$

also $$R = 206{,}265\, r/s'' \text{ and } r = Rs''/206{,}265,$$

where s° means s in *degrees*, s' in *minutes* of arc, s'' in *seconds* of arc. In either form of the equation we see that the apparent diameter *varies directly as the linear diameter and inversely as the distance.*

In the case of the moon $R =$ about 239,000 miles; and r, 1081 miles. Hence $s = \frac{1081}{239{,}000} = \frac{1}{221}$ of a radian, which is about 933″, — a little more than one fourth of a degree.

The sun, of very nearly the same apparent size as the moon, is both larger and farther away.

The navigator, near shore, may use this principle to get his position by measuring the vertical angle subtended by a lighthouse. If, for example, he finds this apparent height to be 2°, he concludes that his distance from the light is $\frac{57.3}{2}$ times its linear height.

110. Semidiameter. In the case of the sun or moon the edge, or *limb*, of the object is usually observed, and to get the true position

[1] The angle s is here expressed in *radians*, the radian being the angle that is measured by an arc equal in length to the radius. As the length of the circumference is $2\pi r$, the radian equals 57°.30 (that is, $360^\circ \div 2\pi$), or 3437′.7 (that is, $21{,}600' \div 2\pi$), or 206,265″ (that is, $1{,}296{,}000'' \div 2\pi$). *Hence, to reduce to seconds of arc an angle expressed in radians, we must multiply its value in radians by 206,265,* — a relation of which we shall make frequent use.

of its center the angular semidiameter must be added or subtracted. For all objects except the moon this may be taken directly from the ephemerides, but the moon's apparent diameter increases slightly with its altitude, being about 1/60, or about 30″, greater when in the zenith than at the horizon, because at the zenith it is about 4000 miles, or 1/60 its whole distance from the center of the earth, nearer than at the horizon.

This augmentation is tabulated in works on navigation, and must be taken into account in accurate work. It has, of course, nothing whatever to do with the optical illusion, already referred to (§ 11), which makes the moon seem larger when near the horizon.

111. Parallax. In general the word "parallax" means the difference between the direction of a heavenly body as seen by the observer and as seen from some standard point of reference.

The *annual*, or *heliocentric*, parallax of a *star* is the difference of the star's direction as seen from the *earth* and from the *sun*. With this we have nothing to do for the present.

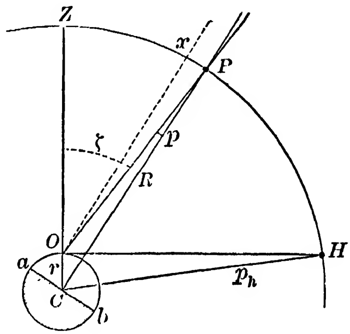


FIG. 43. The Geocentric Parallax

The *diurnal*, or *geocentric*, parallax of the sun, moon, or a planet is the difference of its direction as seen from the *center of the earth* and from the *observer's station* on the earth's surface; or, what comes to the same thing, it is the angle at the body made by two lines drawn from it, — one to the observer, the other to the center of the earth. In Fig. 43 the parallax of the body P is the angle OPC, which equals xOP and is the difference between ZOP and ZCP. Obviously this parallax is zero for a body directly overhead at Z, and a maximum for a body rising at H. Moreover (and this is to be especially noted), this parallax of a body at the horizon — the *horizontal parallax* — is simply *the angular semidiameter of the earth as seen from the body*. When we say that the moon's horizontal parallax is 57′, it is equivalent to saying that, seen from the moon, the earth has an apparent diameter of 114′.

112. Law of the Parallax. From the triangle OCP we have

$$PC : OC = \sin COP : \sin CPO,$$

or

$$R : r = \sin \zeta : \sin p,$$

since COP is the supplement of ζ. This gives

$$\sin p = \frac{r}{R} \sin \zeta, \tag{1}$$

or (since p is always a small angle), expressed in seconds of arc,

$$p'' = 206{,}265'' \frac{r}{R} \sin \zeta. \tag{2}$$

When a body is at the horizon, its zenith distance is 90° and $\sin \zeta = 1$. Hence the horizontal parallax, p_h'', of the body is given by the formula

$$\sin p_h = r/R, \quad \text{or} \quad p_h'' = 206{,}265\, r/R; \tag{3}$$

and

$$p'' = p_h'' \sin \zeta, \tag{4}$$

or, in words, *the parallax at any altitude equals the horizontal parallax multiplied by the sine of the apparent zenith distance.*

From equation (3) we have also, for finding R, the distance of the body,

$$R = \frac{r}{\sin p_h}, \quad \text{or} \quad R = \frac{206{,}265\, r}{p_h''}, \tag{5}$$

a relation of great importance as determining the distance of a heavenly body when its parallax is known.

113. Equatorial Parallax. Owing to the ellipticity, or oblateness, of the earth, the horizontal parallax of a body varies slightly at different places, being a maximum at the equator, where the distance of an observer from the earth's center is greatest. It is agreed to take as the standard the *equatorial horizontal parallax,* that is, the earth's *equatorial* semidiameter in seconds as seen from the body.

If the earth were exactly spherical, the parallax would act in an exactly vertical plane and would simply diminish the altitude of the body without in the least affecting its azimuth. Really, however, it acts along great circles drawn from the *geocentric* zenith to the *geocentric* nadir (§ 12), and these circles are not exactly perpendicular to the horizon. For this reason the azimuth of the *moon,* which has a parallax of about a degree, is sensibly affected.

The calculation of the parallax corrections to observations of the moon's right ascension and declination is also modified and greatly complicated (see Campbell's *Practical Astronomy*).

In the calculation of the parallax of all other bodies it is sufficient to regard the earth as spherical.

The parallax (4) is added to the measured altitude to reduce the latter to the earth's center. This correction is necessarily applied in order to prepare the altitude for use in formulæ involving almanac positions, which are always geocentric.

114. Refraction. The waves of light all travel with the same speed in empty space, but in transparent media, and particularly in air, their velocity diminishes by an amount which is proportional to the density of the air. If a wave passes through a region where the density is not uniform, the parts of it which move through the denser air may be supposed to lag behind those which traverse more rarefied air, and the *rays of light* (which are perpendicular to the wave-fronts) *will always be curved toward the region of greater density* (Fig. 44).

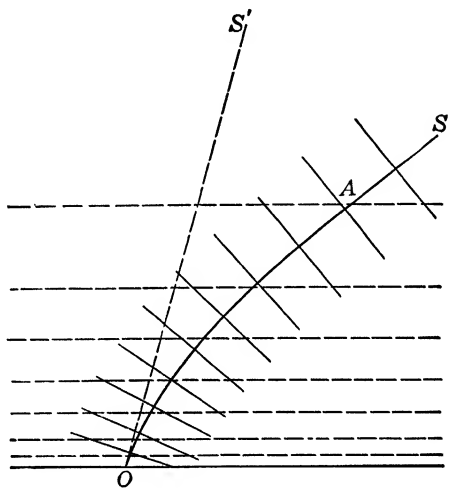


FIG. 44. Refraction of Light in the Earth's Atmosphere

Light from a star enters the atmosphere from the direction *SA*. Its velocity becomes less and less as it penetrates denser and denser layers. The direction of the wave-front changes more and more rapidly. The observer at *O* sees the star in the direction *OS'*. (The effect is much exaggerated here)

Since the density of the atmosphere increases downward, the rays from all heavenly bodies will be bent downward. We see them in the direction in which the rays enter our eyes, and *their apparent altitudes will exceed the true altitudes.*

Refraction, like parallax, will evidently vanish for rays which come vertically downward, and be a maximum for those which are nearly horizontal (Fig. 45). But the *law* of refraction is very

different from that of parallax. Its amount depends upon the density of the air (which is determined by the barometric pressure and temperature), as well as upon the altitude of the object, but is independent of the distance. The theory of refraction is too complicated to be discussed here.

The computation of the correction, when precision is required, is made by means of elaborate tables provided for the purpose and given in works on practical astronomy, the data being the observed altitude of the object, the temperature, and the height of the barometer. Increase of atmospheric pressure somewhat increases the refraction, and increase of temperature diminishes it.

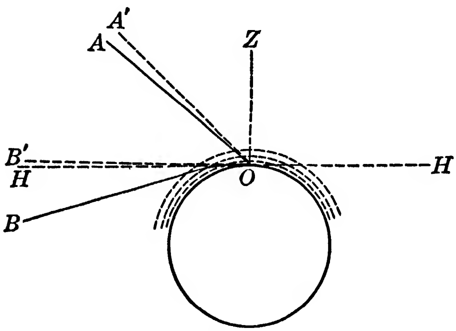


FIG. 45. Atmospheric Refraction Increases the Altitude

At the horizon, where refraction increases the altitude most of all, it raises the sun through its own diameter

In Table 6 (Appendix) the refraction, under the circumstances stated in the heading there, is given by inspection with fair accuracy, and by applying the approximate corrections for readings of the barometer and thermometer, indicated in the note below it, we obtain results which will seldom be more than 2″ in error.

It is hardly necessary to add that this refraction correction, required by most astronomical observations of position, is very troublesome and usually involves more or less uncertainty and error from the continually changing and unknown condition of the atmosphere along the path followed by the rays of light.

For methods by which the amount of refraction *is determined by observation* the reader is referred to works on practical astronomy.

Near the horizon the refraction does not increase indefinitely but reaches a finite value (about 35′ under average conditions) which is somewhat greater than the diameter of either the sun or the moon. At the moment, therefore, when the sun's lower limb appears to be just rising or setting, the whole disk is really below the plane of the horizon; and the time of sunrise in the latitude of New York is thus accelerated by three or four minutes, accord-

ing to the inclination of the sun's diurnal circle to the horizon, which varies with the time of the year. Sunset is delayed by the same amount, and thus at both ends the day is lengthened at the expense of the night.

Refraction at the horizon, for average conditions, and also semidiameter are taken into account in the computation of the times of sunrise and sunset. The problem is the same as that of finding the time by a single altitude of the sun. The zenith distance of the sun's center at the moment when its upper edge is rising (sunrise) equals, on the average, 90° 51′, that is, 90° plus 16′ (the mean semidiameter of the sun) plus 35′ (the mean refraction at the horizon). The *Nautical Almanac* contains tables of the times of rising and setting of the sun and moon. In the latter case the parallax is important, and direct calculation is rather complicated.

115. Mirage. When the surface of the land or the sea, and the air in contact with it, is considerably warmer than the air a few yards above, the density of the air may (locally) increase *upwards*. Rays of light passing through this region will be concave *upwards*. Under these conditions distant objects may appear greatly distorted, inverted, or suspended in the sky above the apparent horizon, producing the effects called *mirage* or looming, and the dip of the horizon may be quite abnormal. The image of the setting sun, especially when seen from a mountain, is often greatly distorted by similar anomalies of refraction.

116. Atmospheric Dispersion. The index of refraction of air, like that of almost all transparent bodies, is greater for green light than for red, and still greater for violet light. Hence a star at a low altitude, when observed with a high power, appears elongated vertically into a *spectrum*, with red at the bottom and green at the top (the blue being usually, and the violet always, lost by absorption in passing through so great a thickness of air). For the same reason the disk of a planet, or of the sun, seems to be bordered by a narrow red edge below and a green one above. This can be seen, even with the naked eye, at sunset, if the horizon is distant and sharp and the air perfectly clear. Just as the last glimpse of the sun disappears, its color changes from reddish yellow to green. Care must be taken, in observing this "green flash," that the eye is not fatigued by looking at the sun before it has almost completely disappeared.

The light of a red star is, on the average, less refracted in our atmosphere than that of a white star, so that white stars are raised a little more by refraction than are neighboring red ones.

117. Twinkling, or Scintillation, of the Stars. This is a purely atmospheric phenomenon, usually conspicuous near the horizon and small at the zenith. It differs greatly on different nights, according to the steadiness of the air, and is probably due to two coöperating causes, both depending on the fact that the air is generally full of streaks and wavelets of unequal density, carried by the wind.

In the first place, light transmitted through such a medium is concentrated in some places and turned away from others by simple refraction, so that, if the light of a star were strong enough, a white surface illuminated by it would look like the sandy bottom of a shallow, rippling pool of water illuminated by sunlight, with light and dark mottlings which move with the irregularities of the surface. So, as we look at a star, and the mottlings due to the irregularities of the air move by us, we see the star alternately bright and faint; in other words, it twinkles.

If the light of Sirius (the brightest of the fixed stars) is admitted through an open window into a perfectly dark room, it produces upon a white screen an illumination strong enough to be easily seen (if the observer's eye has been thoroughly rested by remaining ten minutes or more in complete darkness), and presents exactly the appearance described in the preceding paragraph (which was written by Professor Young many years before the phenomenon had been observed).

The other cause of twinkling is optical interference. Pencils of light coming from a star (optically a mere luminous point) reach the observer's eye by slightly different paths and are just in a condition to interfere. The result is the temporary destruction of rays of certain wave-lengths, and the reënforcement of others. Accordingly the light of the star appears to vary both in brightness and in color. This can very easily be seen by looking at a bright star with a field-glass and giving the glass a rapid rotatory motion, so that the star appears drawn out into a circle of light. This will appear beaded with brilliant colors.

The planets do not twinkle, because they are not luminous points but have disks of sensible diameter. While each point of

the disk twinkles like a star, the different points do not keep step, so to speak, in their twinkling, and the general sum of light remains nearly uniform. When very near the horizon, however, the irregular refraction is sometimes sufficiently violent to make them dance and change color, — especially in the case of Mercury, whose disk is very small.

118. Telescopic Effects; "Bad Seeing." These disturbances of refraction play havoc with telescopic definition. When a star is twinkling at all strongly, it appears in the telescope to dance madly about, and often, when the tremors are violent, to burst into an ill-defined mass of light many seconds of arc in diameter. The larger the telescope, the more pronounced are these effects. Such "bad seeing" is, next to actual cloudiness, the most serious of all hindrances to astronomical observation; and in most places really good nights, when the theoretical defining power of a great telescope can be approximately attained, are lamentably rare.

The most important of all factors in choosing the location of a great observatory is now recognized to be the *character of the seeing*. Mountains, high plateaus, and oceanic islands all have their advantages. Much depends on the nature of the work to be undertaken; the seeing at a given place may be good by day and poor by night, or vice versa.

The Lick Observatory on Mt. Hamilton, 40 miles from San Francisco and 4000 feet high, the Mt. Wilson Observatory, at an altitude of 6000 feet above Pasadena, and the Lowell Observatory at Flagstaff, at an altitude of 7200 feet, are noteworthy examples.

119. Twilight. Although this has nothing to do with the correction of observations, it is an atmospheric phenomenon and may most conveniently be treated here. It is caused by the *reflection* of sunlight from the upper portion of the earth's atmosphere. After the sun has set, its rays, passing over the observer's head, still continue to shine through the air above him, and twilight continues as long as any portion of the illuminated air remains in sight from where he stands. It is considered to end when stars of the sixth magnitude become visible near the zenith, which does not occur until the sun is about 18° below the horizon; but this varies considerably for different places, according to the purity of the air.

The length of time required by the sun, after setting, to reach this depth varies with the season and with the observer's latitude. In latitude 40° it is from one and one-half to two hours. In latitudes above 50°, when the days are longest twilight never disappears even at midnight. In the tropics, even on the mountains of Peru where the air is exceptionally clear, it lasts only about an hour and a quarter.

From the fact that twilight lasts until the sun is 18° below the horizon, the height of the twilight-producing atmosphere can easily be computed, and comes out about 50 miles. This, however, is not the real limit of the atmosphere. The incandescence of meteors shows that at an elevation of 100 miles there is still air enough to resist their motion (§ 528).

Soon after the sun has set, the *twilight bow* appears, rising in the east, — a dark blue segment bounded by a faintly reddish arc. It is the shadow of the earth upon the air, and as it rises the arc becomes rapidly diffuse and indistinct, and is lost long before it reaches the zenith.

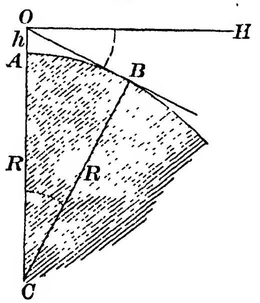


FIG. 46. Dip of the Horizon

120. Dip of the Horizon. In observations of the altitude of a heavenly body at sea, where the sextant measurement is made from the *visible* horizon, or *sea-line*, it is necessary to take into account the depression of the visible horizon below the true astronomical horizon by a small angle called the *dip*. The amount of this dip depends upon the observer's altitude above the sea-level.

In Fig. 46, C is the center of the earth, AB a portion of its level surface, and O the eye of the observer at an elevation h above A. The line drawn perpendicular to OC is truly horizontal (if the earth is regarded as spherical), while the tangent OB is the line drawn from O to B, the visible horizon. The angle HOB is the dip and is obviously equal to OCB.

Owing to refraction, however, the actual line of sight *curves downward*; this will obviously make the dip *less*, and the distance of the horizon *greater*, than would otherwise be the case. Making allowance for the average amount of this effect of refraction, the very convenient approximate formula may be derived,

$$\text{Dip (minutes)} = \sqrt{h \text{ (feet)}};$$

that is, *the dip in minutes of arc equals the square root of the observer's elevation in feet.*

For *the distance of the sea horizon* (*OB*, Fig. 46) the approximate formula is

$$\text{Distance (miles)} = \sqrt{\tfrac{3}{2}\,h\ \text{(feet)}}.$$

This, however, takes no account of refraction, and the actual distance is always greater.

121. Precession and Nutation. It is also in place to refer briefly to the corrections to a star's position which are made necessary by the motion of the coördinate systems. The equinoxes, the equator, and the ecliptic are all in constant motion (their motions are known as precessions and nutations, and are explained in § 165, p. 141); and so the right ascension, declination, etc. of every star is constantly changing. Formulæ for allowing for this (which are quite complicated) are given in the *Nautical Almanac* and in star-catalogues.

DISCUSSION OF OBSERVATIONS: ERRORS; COMPUTATIONS

122. Errors of Observation. No actual observations can be made with absolute precision, the result of any measurement being more or less influenced by a multitude of circumstances. The study of these, and of the resulting *errors of observation*, forms an important part of all methods of precision.

Observational errors may be divided into two classes: (1) *systematic errors*, arising from causes which repeat themselves whenever the observation is repeated under similar conditions, and (2) *accidental errors*, arising from causes which do not so repeat themselves. The latter make the results of successive measurements of the same quantity differ slightly from one another, while the former affect all alike and can only be detected by repeating the observations under different conditions or by a different method. It is clear, therefore, that systematic errors are much more difficult to detect and troublesome to get rid of.

Systematic errors may arise from many causes: from peculiarities of the observer, such as personal equation (§ 78); from the unavoidable small imperfections of construction of an instrument, such as those of the transit (§ 77); or from external causes, such as refraction. In all work which aims at high precision it is necessary above all things to avoid systematic errors, either by

finding the law of their occurrence and calculating and allowing for their amount (as in the case of refraction), or by arranging the program of observation so that the effect of the error is eliminated from the final mean result (as when observers change places in longitude work), or, when possible, by devising such methods of observation that the systematic errors become very small (as with the transit micrometer). Any influence whatever which can affect the results of observation in a *predictable* manner should be got rid of in one of these ways.

Accidental errors probably arise from a combination of many minor influences, which change, from one observation to the next, in a manner incapable of prediction. It does not follow, however, that we can know nothing about such errors. For example, positive and negative errors (that is, deviations on opposite sides of the true value) are equally likely to occur; large errors are less frequent than small ones; and there is a definite relation between the numbers of errors of different magnitudes.

123. Probable Error. If we know the accidental errors of a series of observations, we can find a quantity such that half these errors are numerically less than this, and the other half greater. This is called the *probable error* and is obviously an indication of the degree of precision of the observations. The mean error, or standard deviation, which is 1.48 times the probable error, is often used for the same purpose.

It follows from certain simple and probable assumptions (by analysis far too difficult to reproduce here) that, out of 1000 observations, of *probable error* r, we ought on the average to find 500 with errors numerically less than r, 323 with errors between r and $2r$, 134 between $2r$ and $3r$, 36 between $3r$ and $4r$, and only 7 with errors numerically exceeding $4r$. Experience shows that even for fairly small groups of observations this law of error very closely represents the facts.

It should be carefully remembered that the law of error tells us nothing about the way in which the errors are attached to the individual observations. We know, for instance, that one observation out of 140, on the average, will have an error exceeding four times the probable error, but there is no way of foretelling when the bad observations will appear.

124. The Method of Least Squares. On account of the inevitable errors of observation it is impossible to obtain absolutely accurate values of any physical constant. To minimize the effects of such errors it is usual to make a large number of observations — much greater than would theoretically suffice if the measures were perfect — for the determination of the unknown quantities which we are seeking. In such a case the values of the unknowns are determined in such a way that the outstanding differences, or residuals, between the actual observations and the values calculated from these unknowns shall be as small as possible. More specifically, *the sum of the squares of the residuals must be a minimum.*

When the observations are successive, and presumably are equally accurate measures of the same quantity, this principle leads to the *arithmetical mean* of the individual observations as the best result which can be got from them. The probable error of this mean may be obtained by dividing that of one observation by the square root of the number of observations. It follows that the relative values, or weights, of observations of *different* probable error are *inversely proportional to the squares of their probable errors.*

The proof of this, and the discussion of the more complicated cases in which several unknown quantities have to be found from the same set of observations, may be found in treatises on the Method of Least Squares.

Probable errors are always written with the double sign $\pm$; for example, if a quantity was found to be $10''$, with a probable error of $2''$, it would be written $10'' \pm 2''$. It must always be remembered that the probable error of an observed quantity is a reliable measure of its precision *only* if the observations on which it is based are not affected by undetected systematic errors, and that there is an even chance, at best, that the observed quantity is wrong by more than the probable error.

125. Computations in Astronomy. It is already apparent, from the scope of the subject thus far covered, that the student of elementary astronomy should have some familiarity with the rudiments of trigonometry and with the use of logarithms. When calculations are to be made involving very large or very small distances, masses, or times, it will usually be found most convenient to express these in terms of centimeters, grams, and seconds,

using the notation in powers [1] of 10. Thus, the mean distance from the earth to the sun = 149,450,000 km. = 1.4945×10^{13} cm. If a distance is known to be 100 meters to the nearest meter, we may write it simply as 100 meters; but if it is known to have this value to within one millimeter, it should be written 100.000 meters. In general, observed quantities should be so recorded as to give all the significant figures that are reliably determined by the methods of observation, and one or, at most, two uncertain figures. It is evident that the sum or difference of two such quantities cannot be more certain than the more poorly determined one, and that the percentage error of the product or quotient of such quantities will be at least as great as that of the weaker of the two. This should be borne in mind in all calculations, and the number of decimal places should be chosen so that all but the last one or two figures of the final result are significant. The inexperienced student is likely to waste time using 7-place logarithms on 3-place data, that is, data for which the fourth place is entirely uncertain.

Astronomical observation, with its attendant computations, calls for facility in calculation. Much labor may be saved by knowing how and when to use a slide-rule and calculating machines. The professional astronomer must, in addition, familiarize himself with many methods of computation, including the theory of least squares, and must be able, in the face of new difficulties, to devise new methods. Mastery of celestial mechanics can be acquired only with the aid of an extensive mathematical equipment.

EXERCISES

In cases in which corrections for refraction are required, they are to be taken from Table 6 (Appendix), taking into account the temperature and barometric pressure, if these are given among the data. If preferred, the student may also use Comstock's formula (Appendix, Table 6). The results for example 1 have their corrections computed by the regular refraction tables, and the approximate results obtained by the student may differ from them by a considerable fraction of a second.

[1] According to the English numeration a billion is a million million, a trillion is a million billion, etc. In the French method, also used in the other Continental countries and in the United States, a billion is a thousand million, a trillion is a thousand billion, etc.

1. Given the following meridian-circle observations on β Ursae Minoris at its upper and lower culminations respectively, namely:

Altitude $55° 48' 6''.0$, temperature 30° F., barometer 30.1 inches;

Altitude $24° 58' 56''.4$, temperature 25° F., barometer 30.1 inches.

The nadir reading (§ 88) was $270° 1' 6''.8$ in both cases. Required the latitude of the place and the declination of the star.

Ans. Lat. $40° 20' 57''.8$.
Dec. $+ 74° 34' 40''.1$.

2. Given the meridian altitude of the sun's lower limb, $62° 24' 45''$, the height of the observer's eye above the sea-level being 16 feet (§ 120). The sun's declination was $+ 20° 55' 10''$, and its semidiameter $15' 47''$. Its parallax at the observed altitude was $5''$, and the mean refraction from Table 6 (Appendix) may be used. Required the latitude of the ship.

Ans. $48° 19' 3''$ N., if the sun is in the south.

3. The meridian altitude of the sun, above the south horizon, is observed, on a ship at sea, to be 30° 15′ (after being duly corrected); the sun's declination at the time is 19° 25′ *south*. What is the ship's latitude?

4. At sea, the sun being on the meridian and south of the zenith, the altitude of its lower limb is observed to be 84° 21′. The sun's declination is $+ 18° 39'$. Find the latitude.

5. Find the latitude from the meridian altitude of the moon's lower limb, 49° 37′, the moon being south of the zenith, its declination $+ 3° 13'$, its semidiameter $15'.0$, and its horizontal parallax $55'.2$. The height of eye is 40 feet.

6. The *midnight* sun is observed on the meridian at a corrected altitude of 4° 11′. Its declination is $+ 22° 8'$. What is the latitude?

7. Show by means of a Sumner line that a considerable error in the dead-reckoning latitude will make no difference in the longitude calculated from an observation of the sun when on the prime vertical.

8. On January 10, 1919, the moon's altitude was observed from an airplane with a bubble-sextant to be 38° 55′ at $8^h 37^m 33^s$ G. M. T., and the sun's altitude to be 12° 59′ at $8^h 43^m 58^s$, — each observation being the mean of seven readings corrected for instrumental and refraction errors. The known position of the aircraft was 37° 4′ North, 76° 24′ West. The computed altitude of the sun at the time it was observed was 12° 55′, and its azimuth S 39 W, while for the moon the computed altitude (including the effect of parallax) was 39° 4′ and its azimuth S 81 E. Draw the Sumner lines and find the error of the position of the "ship" given by the observations.

Ans. 9 nautical miles too far west and 2 miles too far north.

(These are actual observations made during experimental tests of navigating devices. See *Publications of the Astronomical Society of the Pacific*, June, 1919).

9. What is the approximate dip of the horizon from a hill 900 feet high?

10. How high must a mountain be in order that the dip of the horizon from its summit may be 2°?

11. What is the distance of the horizon in miles as seen from the summit of this mountain?

12. An observer at sea-level on the equator, and another in an airplane 10,000 feet above him, watch the same sunset. How much longer will the sun be visible to the aviator?

13. Assuming the horizontal parallax of the sun as $8''.8$, what is the horizontal parallax of Mars when nearest us, at a distance of 0.378 astronomical units? (The astronomical unit is the distance from the earth to the sun.)

14. What is the greatest apparent diameter of the earth as seen from Mars?

15. What is the horizontal parallax of Jupiter when at a distance of 6 astronomical units?

16. Does atmospheric refraction increase or decrease the apparent area of the sun's disk when it is near the horizon?

17. If the water of the sea is warmer than the air about it, will the dip of the horizon be increased or diminished?

18. What is the lowest latitude at which twilight can last all night? Can it do so at New York? at London? at Edinburgh?

REFERENCES

Detailed formulæ for calculating the influence of parallax and refraction upon the coördinates of a heavenly body may be found in the works of Campbell, Newcomb, and Chauvenet (p. 73). The last two contain also a discussion of the theory of refraction and of the theory of errors.

WILLIAM BOWIE, *Determination of Time, Longitude, Latitude, and Azimuth*, United States Coast and Geodetic Survey, Special Publication, No. 14, an admirable summary of the methods employed in the practice of the Survey.

NATHANIEL BOWDITCH, *American Practical Navigator* (revised) (Washington Printing Office), a standard compendium of the subject, clearly written, kept up to date, and exceedingly interesting. It gives a full discussion of the determination of the place of a ship at sea, and of all other problems of nautical astronomy and navigation.

CHAPTER IV

THE EARTH AS AN ASTRONOMICAL BODY

ITS FORM, ROTATION, AND DIMENSIONS · THE EARTH'S MASS AND DENSITY · CONSTITUTION AND AGE OF THE EARTH

126. In a science which deals with the heavenly bodies it might at first seem to the student that the earth has no place; but certain facts relating to it are similar to those we have to investigate in the case of other planets and are ascertained by astronomical methods, and a knowledge of them is essential as a basis of all astronomical observations. Moreover, astronomical methods reveal important facts about the constitution and history of the earth which are not ascertainable otherwise. In fact, astronomy, like charity, begins at home, and it is impossible to go far in the study of the bodies which are strictly celestial until some accurate knowledge has been acquired of the dimensions and motions of the earth itself.

127. The astronomical facts relating to the earth are broadly these:

(1) The earth is a great ball approximately 7920 miles in diameter, and 24,880 miles in circumference.

(2) It rotates on its axis once in twenty-four sidereal hours.

(3) It is not exactly spherical, but is flattened at the poles, the polar diameter being nearly 27 miles, or about one three-hundredth part, less than the equatorial.

(4) Its mean density is between 5.5 and 5.6 as great as that of water, and its mass is represented in tons by 6 with twenty-one ciphers following (six thousand millions of millions of millions of tons), or 6×10^{21}.

(5) It is flying through space in its orbit around the sun with a velocity of about $18\frac{1}{2}$ miles a second, or nearly 100,000 feet a second, — about twenty times as fast as the swiftest modern projectiles.

I. ROTUNDITY AND SIZE OF THE EARTH

128. The Earth's Approximate Form. It is not necessary to dwell on the ordinary familiar proofs of the earth's globularity. One, first quoted by Galileo as absolutely conclusive, is that the outline of the earth's shadow seen upon the moon during a lunar eclipse is such as only a sphere could cast.

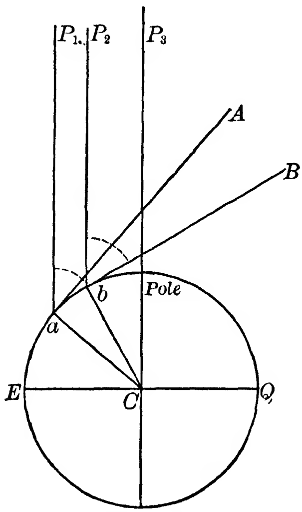


FIG. 47. Measuring the Earth's Diameter

The astronomical measurement of the difference of latitude

We may add, as to the smoothness and roundness of the earth, that if represented by an 18-inch globe, the difference between its greatest and least diameters would be only about 1/16 of an inch, the highest mountains would project only about 1/75 of an inch, and the average elevation of continents and depths of the ocean would be hardly greater on that scale than the thickness of a film of varnish. Relatively, the earth is much smoother and rounder than most of the balls in a bowling-alley.

129. Approximate Measurement of the Diameter of the Earth regarded as a Sphere. There are various ways of determining the diameter of the earth. The simplest and best is by measuring *the length of a degree.* It consists essentially in *astronomical* measurements which determine the distance between two selected stations (several hundred miles apart) in *degrees* of the earth's circumference, combined with *geodetic* measurements giving their exact distance in miles or kilometers.

The astronomical determination is most easily made if the two stations are on the same terrestrial meridian. Then (Fig. 47) the distance *ab* in degrees is simply the difference of latitude between *a* and *b*. The latitudes are best determined by zenith-telescope observations (§ 93), but any accurate method may be used.

The *linear distance* (in feet or meters) is measured by a geodetic process called *triangulation.* It is not practicable to measure it with sufficient accuracy directly, as by simple chaining. Between

the two terminal stations (A and H, Fig. 48) others are selected, such that the lines joining them form a complete chain of triangles, each station being visible from at least two others. The angles at each station are carefully measured, and the length of one of the sides, called the *base*, is also measured with all possible precision.

It can be done, and is done, with an error not exceeding half an inch in 10 miles. (BU is the base in the figure.) Having the length of the base and all the angles, it is then possible to calculate every other line in the chain of triangles and to deduce the exact *north-and-south distance* (Ha) between H and A. An error of more than 3 feet in a hundred miles would be unpardonable.

130. The ancients understood the principle of the operation perfectly. Their best-known attempt at a measurement of the sort was made by Eratosthenes of Alexandria about 250 B.C., his two stations being Alexandria and Syene in Upper Egypt. At Syene he observed that at noon of the longest day in summer there was no shadow in the bottom of a well, the sun being then vertically overhead. On the other hand, the gnomon at Alexandria, on the same day, by the length of the shadow, gave him 1/50 of a circumference (7° 15′) as the distance of the sun from the zenith at that place. This 1/50 of a circumference is therefore the difference of latitude between Alexandria and Syene, and the circumference of the earth must be fifty times the *linear distance* between those two stations.

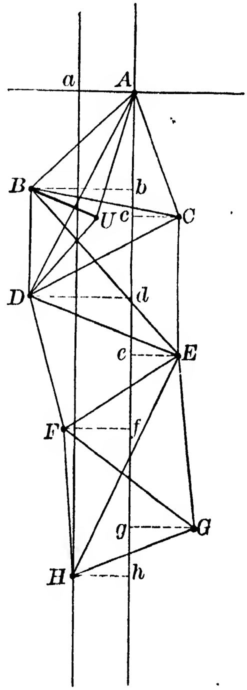


FIG. 48. Measuring the Earth's Diameter

Triangulation to find the linear distance between the two stations

The weak place in his work is the absence of details concerning the measurement of the linear distance between the two places. He states it as 5000 *stadia*. According to Dreyer the distance was probably measured in *paces* by specially trained men, and the stadium was 517 feet. This would make the earth's circumference 24,500 miles, — remarkably near the truth.

The first really valuable measure of an arc of the meridian was that made by Picard in northern France in 1671, — the measure which served Newton so well in his verification of the idea of gravitation. Since then many arcs of meridian have been accurately measured (§ 139).

II. THE ROTATION OF THE EARTH

131. Evidence of the Earth's Rotation. At the time of Copernicus the only argument he could bring in favor of the earth's rotation was that this hypothesis seemed much more probable than the older one that the great heavens themselves revolved. All the phenomena *then known* would be sensibly the same on either supposition. The apparent diurnal motion of the heavenly bodies can be completely accounted for, within the limits of observation *then* possible, either by supposing that all the stars are actually attached to an immense celestial sphere which turns around daily or that the earth itself rotates upon an axis; and for a long time the latter hypothesis seemed to most people less probable than the older and more obvious one.

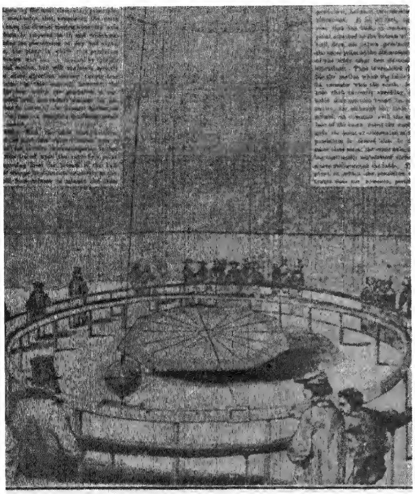


FIG. 49. Foucault's Pendulum Experiment

This illustration is copied from a newspaper of 1851 and shows the appearance of the apparatus and its surroundings. The lines ruled on the table in the center made it easier to notice the change in direction of the swinging pendulum

A little later, after the invention of the telescope, analogy could be adduced; for with the telescope we can see that the sun, the moon, and many of the planets are rotating globes, and it is reasonable to suppose that the earth is also.

Within the last century it has become possible to adduce experimental physical proofs of the earth's rotation, which are independent of observation of the heavenly bodies.

132. Foucault's Pendulum Experiment. Among these experimental proofs the one most often employed is the pendulum experiment, devised and first executed by Foucault in 1851. From the dome of the Panthéon in Paris he hung a heavy iron ball about a foot in diameter by a wire more than 200 feet long. A circular

rail some 12 feet across, with a little ridge of sand built upon it, was placed in such a way that a pin attached to the swinging ball would just scrape the sand and leave a mark at each vibration. To put the ball in motion it was drawn aside by a cotton cord and left for hours to come absolutely to rest; then the cord was *burned* and the pendulum started without jar to swing in a true plane.

But this plane at once began apparently to *deviate slowly toward the right*, in the direction of the hands of a watch, and the pin on the pendulum ball cut the sand ridge in a new place at each swing, shifting at a rate which would carry it completely around in about thirty-two hours if the pendulum did not first come to rest. In fact, the floor of the Panthéon was really and visibly turning under the plane of the vibrating pendulum.

A Foucault pendulum is of daily interest to visitors to the building of the National Academy of Sciences at Washington.

133. Explanation of the Foucault Experiment. The approximate theory of the experiment is very simple. A swinging pendulum, suspended so as to be *equally free to swing in any plane* (unlike the common clock pendulum in this), if set up at the pole of the earth, would appear to shift completely around in twenty-four hours.

It is easy to see that at the south pole the rotation will appear to be reversed. At the earth's equator there will be no such tendency to shift, while in any other latitude the effect will be intermediate and the time for the pendulum to complete the revolution of its plane will be longer than at the pole.

It can be proved that the hourly deviation of a Foucault pendulum equals 15° multiplied by the *sine* of the latitude. In the latitude of New York it is not quite 10° an hour.

The northern edge of the floor of a room in the northern hemisphere is nearer the axis of the earth than is its southern edge, and therefore is carried more slowly eastward by the earth's rotation. Hence the *floor* must *skew around* continually, like a postage stamp gummed upon a whirling globe, anywhere except at the globe's equator. The pendulum is constrained by the force of gravity to follow the changes in the direction of the vertical, but is otherwise free. Its plane of vibration, therefore, will appear to deviate in the opposite direction from the real skewing motion of the ground, and at the same rate. In the northern hemisphere it apparently moves in the same direction as the hands of a watch; in the southern hemisphere, in the opposite direction.

134. There are several other mechanical proofs of the earth's rotation.

(1) The plane in which a projectile moves also remains fixed while the earth rotates beneath it. The projectile therefore appears to deviate to the *right* in the northern hemisphere and to the *left* in the southern. Allowance for this deviation has to be made in precise artillery work.

(2) Freely falling bodies deviate slightly to the eastward.

(3) The rotation of the earth supplies the directive force for the *gyro-compass*, an instrument of great practical importance which will be explained later (§ 168).

(4) The best laboratory observations of the earth's rotation are secured with an apparatus devised by A. H. Compton (described in *Popular Astronomy*, Vol. XXIII (1915), p. 199). A circular tube, like a bicycle tire, filled with water, is suddenly turned over, through 180°, about an axis in its own plane; and through a small window in the tube the water, after such rotation, is seen to be slowly flowing around the tube. (This effect is due to the conservation of angular momentum.) From observations of the motion of the water, when the tube is turned over about an axis in various directions, the length of the day, the geographical latitude of the laboratory, and the azimuth of a given line can be determined independently of astronomical observations.

135. Climatic Effects of the Earth's Rotation. The great circulatory movements in the oceans and in the atmosphere — the ocean currents, the trade and anti-trade winds, and the vorticose revolution of cyclones — are modified and controlled by the earth's rotation. In the northern hemisphere the wind in a cyclone (or "low") moves spirally toward the center, whirling *counterclockwise*, while in the southern the spiral motion is *clockwise*. The motion is explained in either case by the fact that currents of air, setting out for the center of disturbance, do not meet squarely in the center, but deviate like projectiles.

This uniformity of motion is of great importance both for weather forecasting and for the avoidance of hurricanes at sea.

It may at first seem that the rotation of the earth once a day is not a very rapid motion; but a point on the equator travels over 1000 miles an hour, or about 1500 feet a second.

136. Secular Changes of the Earth's Rotation. Within the last few years it has become generally accepted that the rate of rota-

tion of the earth is not constant, but rather is very gradually decreasing, with a consequent lengthening of the sidereal day amounting to about 1/1000 of a second per century.

With the lengthening of the day the sun and moon *appear* to be moving faster *per day* than they used to. The cumulative effects of even this very minute change are clearly indicated by the comparison of modern with ancient eclipses. This has been suspected for a century, and has recently been shown conclusively by Fotheringham of Oxford.

The principal cause of this change is to be found in the friction of the tides (§ 355).

The sun, moon, and planets, during the last century, have shown a tendency to run ahead of their calculated positions in some years, and behind in others, in very much the way that would happen if the earth, considered as a clock, were sometimes as much as twenty seconds fast for a decade or two, and at other times slow, to the same maximum amount. E. W. Brown (1926) concludes that these changes of rotation are real and arise from alterations in the earth's diameter, due to internal causes, and amounting at most to a few feet.

137. Variation of Latitude; Motion of the Poles of the Earth. Any alteration in the arrangement of the matter of the earth must bring about a bodily shifting of the earth with respect to its axis of rotation. There results an apparent wandering of the terrestrial poles, which may be detected by a corresponding variation of the latitude of stations far from the pole.

The first satisfactory evidence of this fact was obtained at Berlin by Küstner in 1888, and at other German stations, and it has since been abundantly confirmed. Chandler found the same variations clearly exhibited in almost every extended body of reliable observations made since 1750. In 1900 a continuous series of observations for the study of the variations of latitude was started at six stations, distributed around the earth on the parallel of 39° 8′ north latitude, in Japan, Turkestan, Sardinia, Maryland, Ohio, and California. From the whole mass of evidence it is apparent that the movement of the terrestrial pole (Fig. 50) is composed mainly of two motions: one an *annual* revolution in a narrow ellipse about 30 feet long (as measured

on the surface at either pole) but varying in form and position; the other, a revolution in a circle about 26 feet in diameter with *a period of about 433 days.* Both motions are *counterclockwise.* The annual component is probably the result of meteorological causes which follow the seasons, such as the deposit and melting of snow and ice. The fourteen-month component arises from the fact that the earth's axis of rotation differs slightly from its axis

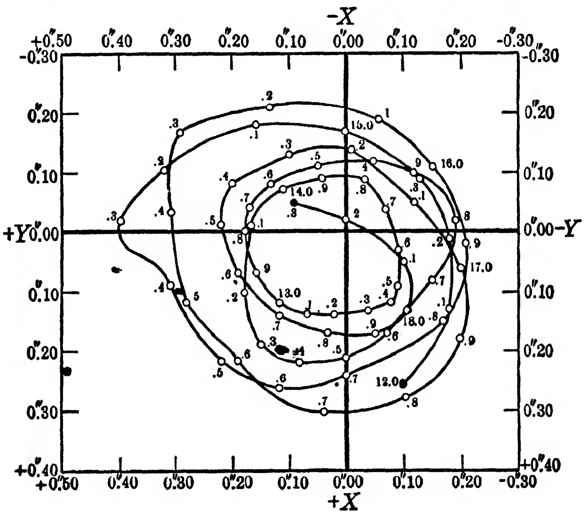


FIG. 50. The Variation of Latitude

We see that the north pole of the earth moved, from 1912 to 1918, in a series of loops in a counterclockwise direction. Every six or seven years the 12-month and 14-month terms come into step, and the pole may wander as far as 0''.4 from its mean position. Halfway between these times the two terms counteract each other, and their resultant is less than half as great. The path is quite irregular and cannot be predicted as accurately as it can be observed

of figure (§ 154). The fact must be clearly grasped that variation of latitude is due to bodily oscillations of the earth. While the *terrestrial poles* appear to wander about, the *direction in space* of the axis of rotation remains fixed. The position of the *celestial poles* is unaffected.

There appears to be no satisfactory evidence of *great* changes of this kind, such as have been invoked to explain changes in climate during geological times.

III. THE EARTH'S FORM

As the earth rotates, every particle of it (except along its axis) is subjected to a centrifugal force directed perpendicularly away from the axis.

The vertical component (Fig. 51) of the centrifugal force neutralizes the force of gravity to a slight extent, which varies from point to point. The horizontal component, being directed toward the equator, acts to make the plumb-line hang away from the radius toward a point on the axis beyond the center. As things actually are, the earth is adjusted to bring the surface everywhere approximately perpendicular to the resultant direction of gravity, and takes, as a result of its rotation, the approximate form of an oblate spheroid (that is, an ellipsoid generated by revolving an ellipse about its minor axis).

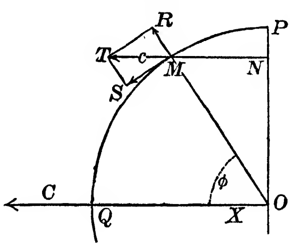


FIG. 51. Centrifugal Force caused by Earth's Rotation

138. Dimensions of the Earth and its Oblateness. The form of such a body is usually defined by its *oblateness f*, which is the fraction obtained by dividing the difference between the polar and equatorial semidiameters by the equatorial, that is,

$$f = \frac{a - b}{a}.$$

There are three ways of determining the form of the earth: First, by *geodetic measurement of distances upon its surface, in connection with the astronomical determination of the points of observation*. This gives not only the form but also the linear dimensions in miles or kilometers (§ 139).

Second, by observing the varying *force of gravity* at points in different latitudes; these observations are made by means of *a pendulum apparatus* of some kind, and determine *only the form*, not the *size*, of the earth (§ 151).

Third, by means of *purely astronomical phenomena*, known as *precession* and *nutation* (discussed in § 165 *et seq.*), and by *certain irregularities in the motion of the moon* (§ 342). These methods, like the pendulum method, give *only the form* of the earth.

The most reliable values of the oblateness of the earth, determined by the three principal methods, are as follows:

METHOD	OBLATENESS	AUTHORITY
Geodetic	$1/297.0 \pm 0.5$	Hayford, 1909
Gravimetric	$1/297.4 \pm 1.0$	Bowie
Precession	$1/296.0 \pm 0.2$	de Sitter

All three determinations are in excellent agreement, and the best available evidence, therefore, points to a value of the oblateness of about 1/297. The dimensions of the earth,[1] according to Hayford (1909), are

Equatorial radius	(a)	6378.388 kilometers =	3963.34 miles
Polar radius	(b)	6356.909 kilometers =	3949.99 miles
Difference	$(a - b)$	21.479 kilometers =	13.35 miles

139. Geodetic Method, by which the Dimensions of the Earth, as well as its Form, are Determined. This method in its most convenient shape consists essentially in the measurement of *two* (or more) *arcs of meridian in widely different latitudes.* These measurements are effected by the same combination of astronomical and geodetic operations already described for the measurement of a single arc (§ 129). More than twenty have thus far been measured in various parts of the earth, the most extensive reaching from Hammerfest to the mouth of the Danube (28° long); from the Shetland Islands to Algeria (28°); from Texas to Minnesota (23°); from the Himalayas to the southern point of India (25°); and from Cape Colony nearly to Lake Tanganyika (21°). Shorter arcs have been measured in Peru, Japan, Spitzbergen, and many other countries.

From these measures it appears in a general way that the *higher* the latitude the *greater* the length of each astronomical degree. In other words, the earth's surface is *flatter near the poles*, as illustrated by Fig. 52. It is necessary to travel about 3000 feet farther in Sweden than in India to increase the latitude by one degree, as measured by the elevation of the celestial

[1] These are the dimensions of the regular geometrical spheroid which, according to the data of measurement, most nearly fits the surface of the earth. The "Hayford spheroid" is used as the basis of the calculations of the *American Ephemeris and Nautical Almanac.*

pole. The length of a degree of latitude varies from 68.7 miles at the equator to 69.4 at the pole.

It will be understood, of course, that the length of a degree at the pole is obtained by extrapolation from the measures made in lower latitudes.

140. The deduction of the exact form of the earth from such measurements is an abstruse problem. Owing to local deviations in the direction of gravity, due to unevenness of surface and variation of density in the rocks near the station, the different arcs do not give strictly accordant results, and the best that can be done is to find the result which most nearly satisfies all the observations.

It is probably safe, judging by the probable errors of the observations, to put the practical limits of uncertainty of the value of the equatorial radius a, according to Hayford's results, as less than ± 150 meters.

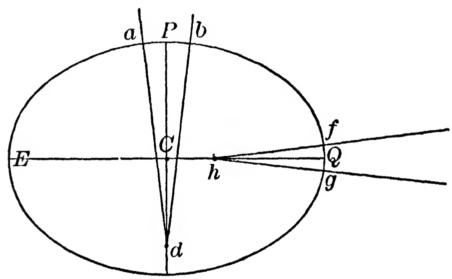


FIG. 52. Length of Degrees in Different Latitudes

The radius of curvature is longest at the pole. Therefore the degree of astronomical latitude is longest there

141. Arcs of longitude are also available for determining the earth's form and size. A degree of latitude is longer near the pole, and shorter near the equator, than on a sphere which has the same equator. Hence a point in a given astronomical latitude is farther from the earth's axis than it would be on the sphere, and a degree of longitude is longer. From this difference the oblateness can be computed.

In fact, *arcs in any direction between stations of which both the latitude and the longitude are known* can be utilized for the purpose, and thus all the extensive geodetic surveys that have been made by different countries contribute to our knowledge of the earth's dimensions.

142. Station Errors. If the latitudes of all the stations in a triangulation, as determined by astronomical observations, are compared with their differences of latitude, as deduced from the geodetic operations, we find discrepancies by no means insensible.

They are far beyond all possible errors of observation and are due to *irregularities in the direction of gravity*, which depend upon the variations in density and form of the crust of the earth in the neighborhood of the station. Such irregularities in the direction of gravity *displace the astronomical zenith* of the station. They are called the *station errors* and can be determined only by a comparison of astronomical positions by means of geodetic operations. Station errors affect both the longitudes and the latitudes of the stations and the astronomical azimuths of the lines that join them. Station errors of from 4″ to 6″ are not uncommon, and they occasionally rise to 30″ or 40″.

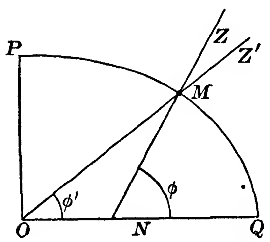


FIG. 53. Astronomical and Geocentric Latitude

The angle MNQ is the *astronomical* latitude of the point M (it is also the geographical latitude, provided the station error at that point is insensible), and MOQ is the *geocentric* latitude. The angle ZMZ', the difference of the two latitudes, is called *the angle of the vertical* and is about 11′ in latitude 45°

143. Astronomical, Geographical, and Geocentric Latitudes. (1) The *astronomical* latitude of the station is that actually determined by astronomical observations, — simply the observed altitude of the pole.

(2) The *geographical* latitude is the astronomical latitude *corrected for station error*. It is defined as the angle formed with the plane of the equator by a line drawn from the place *perpendicular to the surface of the standard spheroid* at that station. Its determination involves the adjustment and evening off of the discrepancies between the geodetic and astronomical results over extensive regions. The geographical latitudes (sometimes called *topographical*) are those used in constructing an accurate map.

(3) *Geocentric latitude.* While the astronomical latitude is the angle between the *plane* of the equator and the *direction of gravity* at any point, the *geocentric* latitude, as the name implies, is the angle *at the center of the earth*, between the plane of the equator *and a line drawn from the observer to that center*; this line evidently does not coincide with the direction of gravity, since the earth is not spherical.

Geocentric degrees are longest near the equator and shortest near the poles (Fig. 53).

Geocentric latitude is employed in certain astronomical calculations, especially such as relate to the moon and to eclipses, in which it becomes necessary to "reduce observations to the center of the earth."

144. Surface and Volume of the Earth. The earth is so nearly spherical that we can compute its surface and volume (or bulk) with sufficient accuracy by the formulæ for a perfect sphere, provided we put the earth's *mean* semidiameter for radius in the formulæ.

This mean semidiameter of an oblate spheroid is not $\frac{a+b}{2}$ but $\frac{2a+b}{3}$, because if we draw through the earth's center three axes of symmetry at right angles to each other, only one will be the axis of rotation, and both the others will be equatorial diameters.

The *mean* radius r of the earth thus computed is 6371.23 kilometers, or 3958.89 miles; its surface ($4\,\pi r^2$) is 5.101×10^8 square kilometers, or 196,950,000 square miles, and its volume ($\frac{4}{3}\,\pi r^3$), 1.083×10^{27} cubic centimeters, or 259,000 million cubic miles, in round numbers.

IV. THE EARTH'S MASS AND DENSITY

145. Mass, Volume, Density, Weight. It will be well, at this point, to remind ourselves, in a very elementary manner, of the proper meanings of such words as "mass," "density," etc. Very briefly, then: the *mass* of a body is the "quantity of matter" in it, and may be expressed in grams or pounds; its *volume* is the amount of space occupied by it, and may be expressed in cubic centimeters or cubic feet. *Density* is the mass contained in unit volume. The *weight* of a body is the force with which the earth attracts it. The childish conundrum "Which weighs more, a pound of feathers or a pound of lead?" illustrates the danger of confusing mass with density.

146. Gravitation. Science cannot yet explain "why" bodies tend to fall toward the earth, but Newton discovered that this phenomenon is only a special case of the much more general *law of gravitation: any two particles of matter attract each other with a*

force proportional to the product of their masses and inversely proportional to the square of the distance between them, or

$$F = G\frac{M_1 M_2}{d^2},$$

where M_1 and M_2 are the masses of the two bodies, d is the distance between them, and G is the force between unit masses at unit distance, commonly called the "constant of gravitation." When the bodies are not points, the question arises how the distance is to be measured. Newton demonstrated that for spherical bodies, in which the density is the same for all points at equal distances from the center, the distance is to be measured from the center of the sphere. How Newton reached these conclusions will be described in Chapter X. At present we are concerned with the application of his results to the determination of the earth's mass and form.

The *weight* of any body at the earth's surface (neglecting the centrifugal force due to the earth's rotation) is given by $W = G\frac{mE}{R^2}$, where m is the mass of the body, E is the mass of the earth, and R the distance to the center of the earth. At a given station, W is strictly proportional to m, and we may write $W = mg$, where $g = G\frac{E}{R^2}$ and represents the weight of unit mass or the acceleration of gravity. At a different station, however, the value of g may be different, because of a difference in the value of R (and also on account of a difference in the centrifugal force arising from the earth's rotation).

147. Measurement of Mass. To find the mass of any small body we ordinarily put it into one pan of a balance and counterpoise its weight by a standard weight. If the attraction of the earth on the two is the same, the weights are equal and so are the masses. With larger quantities, such as the rock excavated from the Panama Canal, another method must be resorted to: the volume is determined by a survey, the mean density is found from the density of an average sample of the material, and the mass is the product of the two.

Neither of these two methods can be applied to the earth, — the first, for obvious reasons; the second, because the only ob-

tainable samples come from depths hardly more than 1/4000 of the distance to the center and are not typical of the whole earth. It is necessary, therefore, to use a wholly different method and to determine the mass of the earth by comparing the attraction which the earth exerts on a body, m, with the attraction exerted upon m by some other body of known mass at known distance.

148. The simplest method theoretically (and one capable of very considerable precision) is by the use of the common balance, first carried out by von Jolly at Munich in 1881.

An accurately constructed balance, capable of carrying considerable loads, is set up as illustrated diagrammatically in Fig. 54, with two sets of scale-pans, the lower hung from the upper by long wires. If two equal spherical masses, m_1 and m_2, are put in the two upper or the two lower pans, they will exactly balance, the earth's attraction on the two being equal. If m_2 is in the upper pan and m_1 in the lower, m_1 will be heavier than the other, since it is nearer the earth's center and more strongly attracted; but the balance can be restored by a small counterpoise c. If, after this is done, a large sphere of lead, of mass M, is brought underneath the lower pan, the equilibrium will again be disturbed because of the mutual gravitational attraction of the masses M and m_1, and an additional small mass n must be placed in the upper pan to restore the balance. Since the upper pan is so far from M that the attraction between the two is negligible, the attraction of M upon m_1 is equal to that of the earth upon n. But, by the law of gravitation, the first of these forces has the magnitude $G\frac{Mm_1}{d^2}$, where d is the distance between the centers of the spheres M and m_1; while the second is $G\frac{En}{R^2}$, where E is the mass of the

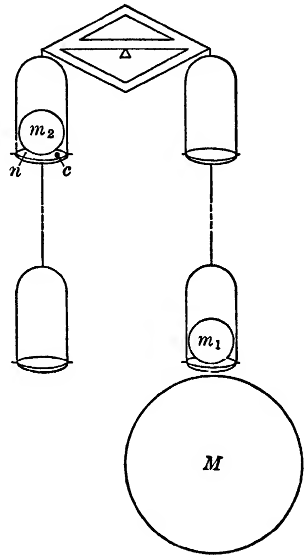


FIG. 54. The Earth's Mass measured by the von Jolly Balance

earth and R the distance between n and its center, which is the radius of the earth. We have, therefore,

$$G\frac{Mm_1}{d^2}=G\frac{En}{R^2},$$

whence, solving for E, we find

$$E=\frac{Mm_1R^2}{nd^2}.$$

All the quantities which appear in the second member of this equation are known, and the earth's mass is therefore determined.

In von Jolly's experiments the observed values of these quantities were $m_1 = 5.00$ kilograms, $M = 5775.2$ kilograms, $n = 0.589$ milligram, $d = 56.86$ centimeters, and, for the latitude of Munich, $R = 6366$ kilometers. Hence we find $E = 6.15 \times 10^{27}$ grams. (Verifying this result will be a good exercise for the student.)

Comparison with the results of other observers shows that these values for the earth's mass and density are rather more than 2 per cent too great. An error of only one sixtieth of a milligram in the determination of the small quantity n will explain the discrepancy.

More accurate results are obtainable by the torsion balance, in which the gravitational force between the known masses acts horizontally to displace a pair of spheres connected by a light rod, which is suspended at the middle by a long wire. By using a very fine wire or a quartz fiber the force necessary to produce a measurable twist may be made exceedingly minute and the measurement correspondingly accurate.

The first determination of the earth's mass (by Maskelyne in 1774) was made by comparing the attraction of the earth and that of a mountain, as measured by the deflection of the vertical (§ 152). In precision this method falls far below the laboratory methods.

The mean result of the best methods is, according to Woodward,

$$\text{Earth's mass} = (5.974 \pm 0.005) \times 10^{27} \text{ grams}.$$

The volume of the earth is 1.083×10^{27} cc., whence, by simple division,

$$\text{Mean density} = 5.515 \pm 0.004 \text{ g./cc.},$$

or $5\frac{1}{2}$ times that of water.

MEASUREMENT OF GRAVITY

149. The Law of Falling Bodies. In accordance with the principles of mechanics (§ 298) the acceleration a of a body moving under the influence of any force f (that is, the *rate at which it gets up speed* in the direction of the force) is given by the equation $a = f/m$, where m is the mass of the body. If the force is due to the gravitational attraction of a mass M at the distance R, $f = GmM/R^2$ (§ 146), whence $a = GM/R^2$. The rate at which the body falls depends, therefore, only on the mass of the *attracting* body, not on that of the one attracted.

If a body falls from rest, its downward velocity will be a at the end of one second, and at at the end of t seconds. Its average downward speed during this interval will be half as great, that is, $\frac{1}{2}at$. Multiplying this by the elapsed time, the distance fallen comes out $\frac{1}{2}at^2$. The acceleration at the earth's surface is denoted by g. The distance fallen in one second is approximately 16 feet, or 490 centimeters.

If the particle does not fall from rest but is projected horizontally, its horizontal velocity will not be affected at all by gravity, nor will the amount by which it drops below the starting level be affected by its horizontal velocity (Fig. 55 a).

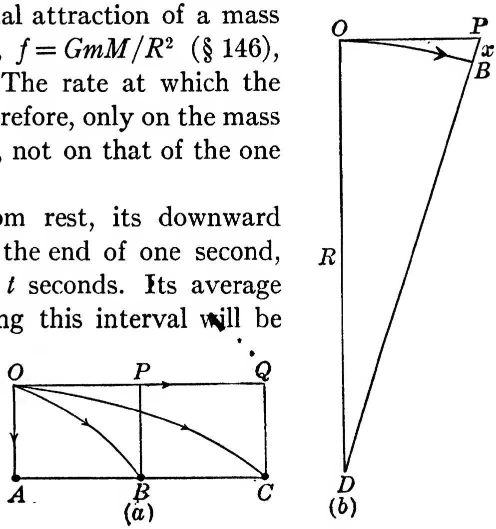


FIG. 55. Falling Bodies

From the point O (see a) a particle A is allowed to fall from rest, while two other particles B and C are simultaneously projected horizontally with velocities proportional to OP and OQ. At the end of t seconds A, B, and C will be on exactly the same level and directly underneath the points which they would have reached had there been no gravitational attraction, the distances OA, PB, and QC all being equal to $\frac{1}{2}at^2$. The distance that a particle falls under the action of a central force (see b) is likewise $\frac{1}{2}at^2$

150. Centrifugal Force. We can now calculate the centrifugal force due to the earth's rotation. Suppose that a particle moves in a circle of radius R with uniform velocity v. If no force acted

upon it, it would move in the straight line OP instead of in the arc OB (Fig. 55 *b*). Just as in Fig. 55 *a*, the distance PB must be $\frac{1}{2}at^2$ (if t is the elapsed time). Now $OD = R$, $OP = vt$, and $\overline{PD}^2 = \overline{OD}^2 + \overline{OP}^2$ (since the angle at O is a right angle). Setting $PB = x$, we have $PD = R + x$, and therefore $R^2 + v^2t^2 = R^2 + 2Rx + x^2$. For a short time interval x is very small and x^2 negligible. In this case $x = v^2t^2/2R = \frac{1}{2}at^2$, whence $\boldsymbol{a = v^2/R}$. If T is the period during which the moving point traverses the circumference of the circle, then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$.

For a point at the earth's equator $R = 6.3784 \times 10^8$ cm. and $T = 86{,}164$ sec. Hence $a = 3.3917$ cm./sec.^{2}, which is 1/289 of the acceleration due to the earth's attraction.

In latitude ϕ the whole centrifugal force is $a \cos \phi$; the vertical component, $a \cos^2\phi$; and the horizontal, $a \cos\phi \sin\phi$, as may be readily seen from Fig. 51. The vertical component is a maximum at the equator and decreases to zero at the pole; the horizontal component and the angle of the vertical are both a maximum at latitude 45° (neglecting the ellipticity of the earth).

151. The force of gravity, at a point on the earth's surface, is the *resultant* of the attraction of the earth and the centrifugal force. It is this resultant force which determines the *weight* of a body at rest or its velocity and direction of fall.

The time of vibration of a pendulum of invariable length depends upon the force of gravity. If allowance is made for the centrifugal force, the corrected pendulum observations provide a measure of the relative distances to the center of the earth from different stations.

Pendulum observations can be made at many places (as in the polar regions) where it would be impracticable to carry out a geodetic survey, and the whole land surface of the earth has been fairly well covered by a network of gravity stations, while only relatively small areas have been covered by triangulation. As it has proved practicable to make pendulum observations in a submarine, a way is open to complete the survey of the entire earth.

It is found, from the pendulum surveys, that the force of gravity at the pole exceeds that at the equator by about 1 part in 189. The centrifugal force accounts for 1 part in 289, leaving

about 1 part in 555 to be explained by the difference in the polar and equatorial radii. This difference amounts to more than thirteen miles. Since the earth is not spherical, the simple inverse-square formula is not applicable, and a more complicated one must be used (§ 341).

152. Deflection of the Vertical; the Geoid. We are now prepared for a better understanding of station errors. Near the foot of a high mountain or a high plateau the plumb-bob is attracted toward the excess of mass, and the direction of the zenith is deflected away from it.

One of the most striking examples of such deflection has been observed on the island of Porto Rico, which has very deep water to the north and south and is really the top of a great mountain range fully 20,000 feet high. The plumb-lines of stations on the north and south coasts, 33 miles apart, are drawn together 56″. The surface of the ocean, therefore, slopes up toward the island, near which it is several feet higher than at a distance, when compared with a uniform spheroid.

The sea-level surface, to which measures of height are referred, is distorted all over the world. This surface, which is the one found by observation, is known as the geoid. It is found, however, that the geoid deviates but slightly (at most about 100 meters) from a smooth spheroid. The dimensions of the earth (§ 138) are the dimensions of this standard spheroid, and geographical latitudes (§ 143) are referred to it.

153. Isostasy. The most notable example of local attraction is to be found in northern India, where the enormous mass of the Himalayas and Tibet deflects the nadir strongly northward. A few hundred miles south of the mountains, however, the deflection has fallen off much more rapidly than the inverse-square law predicts. This has been explained by the hypothesis that the earth's crust underneath the mountains is of lower density than elsewhere; the total mass per square kilometer to a moderate depth below sea-level is everywhere very nearly the same. When the observer is far enough away to make his distance from all points of such a column very nearly the same, the attraction of the column is almost exactly the same as it would be if the surface were at sea-level and the density uniform. Close to the foot of a

mountain range, however, the observer is much nearer the projecting upper end of the column than he is to its lower end, and he observes a strong lateral deviation. The intensity of gravity is there *less* than normal, for the attraction of the excess of mass at the top of the column acts almost horizontally, and the influence of the deficiency of mass at the bottom almost vertically. The deep ocean basins are similarly compensated by an excess density in the crust below.

From the manner in which deflections of the vertical and gravity anomalies vary with the distance from mountains and other topographical features it has been found that the depth of compensation is about 100 kilometers. This means, then, that the whole quantity of matter in a column, perhaps 100 miles square, to this depth, is very nearly the same, no matter whether it consists of 100 kilometers of rock beneath a plain lying near sea-level, of 104 kilometers of rock beneath a plateau 4 kilometers high, or of 95 kilometers of rock with the overlying 5 kilometers of sea-water.

This condition, and the geological process by which it must have been brought about, are both known by the term "isostasy."

154. Constitution of the Earth's Interior. Most of our knowledge of the nature of the main mass of the earth's interior is derived from astronomical sources. Three things are definitely known concerning it:

(1) *The earth's interior is of high density.* This follows at once from the fact that the mean density of the earth is 5.52, while that of its superficial layers is about 2.71. There are two obvious explanations for the high central density:

(*a*) The effects of pressure. At a depth of 100 miles the weight of the overlying rocks amounts to about 300 tons per square inch. Though the force of gravity diminishes toward the center, the pressure in the deeper regions must be enormous. All known substances are increased in density by pressure.

(*b*) If the interior of the earth was ever liquid, or even viscous, the denser materials had a tendency to sink toward the center.

The speeds with which vibrations caused by earthquakes are transmitted through the earth's interior to distant seismographs depend upon the density and elasticity of the material along the

paths which they follow. Adams and Williamson have recently shown that the observed velocities cannot be harmonized on the assumption that the high density at the center is due solely to the diminution of volume by pressure.

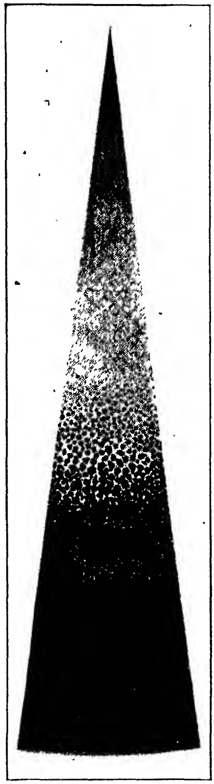


FIG. 56. What a Wedge-Shaped Sample of the Earth Might Look Like

From an article by H. S. Washington, by courtesy of the *Scientific American*

It is necessary, therefore, to fall back on the second hypothesis, that the center is composed of denser materials. The occurrence of metallic iron in meteorites (§ 531) makes it probable that this dense substance is iron or an alloy of iron and nickel. This was first suggested by Wiechert in 1897. The latest evidence indicates that this metallic core is about 4000 miles in diameter, and of a density ten to twelve times that of water (which nickel-iron might attain under the great pressure). Adams and Williamson conclude that a layer of mixed iron and rock, about 1000 miles thick, surrounds the core (Fig. 56); Jeffreys (1926), that the transition to the rocky crust is abrupt. Most of this crust is composed of heavy basic rocks, and its mean density is about 4. The outer granitic layer, of density 2.7, appears to be only about 40 miles thick.

(2) *The interior is solid and more rigid than steel.* This is proved by three independent lines of evidence:

(*a*) By the transmission of vibrations caused by earthquakes through the earth's interior to distant points. Investigations of this lead to the conclusion that the earth, taken as a whole, is considerably more rigid than steel. The inner core, however, does not transmit transverse vibrations, and is probably of low rigidity (Oldham, 1906). According to Jeffreys, it may actually be fluid. The rocky crust is very rigid.

(*b*) By the variation of latitude. Any body which, like the earth, is approximately symmetrical about a certain "axis of figure" will, if set rotating about this axis, continue to rotate uniformly unless acted upon by external forces; but if the initial axis of rotation is slightly inclined to the axis of figure, it will slowly change its position in the body in such a way that the "instantaneous pole" moves along a small circle whose center is the "pole of figure." The more nearly spherical the body, the longer the period of this motion. For a rigid body of the mass and figure of the earth it would be 305 days. The principal term of the variation of latitude is of this nature, but its period is 433 days (§ 137). Newcomb and Hough have shown that if the earth yields slightly to the centrifugal force produced by its rotation, the period of the pole's motion will be lengthened, and will have the observed value if it is about as rigid as steel.

(*c*) By certain tidal phenomena (to be discussed in Chapter X), which indicate a rigidity of the same order of magnitude.

This great rigidity with respect to the effects of rapidly changing forces is not necessarily inconsistent with the idea that the deeper layers of the earth's crust may slowly yield in the course of ages to long-continued stresses acting always in the same direction. The fact that the isostatic compensation beneath geologically young mountain ranges is nearly complete indicates that such slow readjustments actually take place.

(3) *The temperature of the interior is very high.* In all deep mines and borings the temperature increases downward, the average rate being not far from 1° centigrade for every hundred feet. How far down this rate of increase extends we do not know, but at a depth of less than fifty miles the rocks must be very hot.

There must be a steady flow of heat, by conduction, from the interior to the surface, where it is radiated into space. It is now known that only a small part of the heat conducted out of the earth at the present time is due to cooling of the interior; the rest is supplied by radioactivity. Jeffreys estimates the latter at 83 per cent. If the earth contained as much uranium and thorium throughout its whole mass as is found in the superficial layers of the earth's crust, more heat would be produced than is lost, and

the interior of the earth would be getting hotter. The rate of increase of temperature downward indicates, however, that radioactivity is largely confined to a thin superficial layer, and that it becomes insignificant at a depth of the order of 50 kilometers. Jeffreys concludes that the cooling of the earth, since solidification, amounts, at a depth of some 300 kilometers, to between 200° and 300°; at a depth of 700 kilometers the cooling is as yet insignificant; the physical state of the matter at great depths can scarcely have changed since the solidification of the earth.

155. The Age of the Earth. In recent years this problem, originally in the field of geology, has passed mainly into those of physics and astronomy, and more definite statements may be made concerning it than were formerly possible.

Much the best and most powerful line of attack is through the study of *radioactivity*. To state briefly a long and fascinating story, the heavy elements uranium and thorium disintegrate spontaneously but gradually, their atoms changing into atoms of quite different sorts (many of them, including radium, short-lived), but ultimately becoming atoms of lead. The lead produced from uranium has an atomic weight of 206; and that from thorium, 208. Both may thus be distinguished, by careful analysis, from ordinary lead, which is of atomic weight 207. When lead of this sort is found in a uranium mineral, it is reasonably certain that it has been formed by a radioactive change since the mineral crystallized from the melted rock. One per cent of the uranium is transformed in 66,000,000 years. In this way the ages of minerals in lower Pre-Cambrian rocks (the oldest geologically) from different parts of the world are found to be about 1,200,000,000 years. The earth's crust as a whole must be older than this.

On the other hand, a maximum age of the crust can be found from the relative proportion of uranium, thorium, and lead in its general composition (which is fairly well known from numerous rock analyses). It is thus found that all the existing lead would have been produced from the uranium and thorium in about 8,000,000,000 years. This does not, of course, date the creation of matter, but only the time within which the present crust was formed upon the planet. It appears likely, therefore, that the

estimate that the age of the earth is 4,000,000,000 or 5,000,000,000 years can hardly be more than twice too great or too small. The time during which life has existed on the earth is probably about 1,000,000,000 years. These numbers are, of course, subject to modification if other factors in the problem, at present neglected or unknown, have in the future to be considered. They represent, however, the best values on the basis of our present scientific knowledge.

REFERENCES

United States Coast and Geodetic Survey, Special Publications, Nos. 10, 12, 40, 69, 99, 100, and 110 give a thorough discussion of the measurement of gravity and of the deflection of the vertical, and of the theory of isostasy.

HAROLD JEFFREYS, *The Earth: its Origin, History, and Physical Constitution* (Cambridge University Press), is difficult reading (in parts) but authoritative and well written; it contains the best account of the problems of geophysics.

CHAPTER V

THE ORBITAL MOTION OF THE EARTH

THE APPARENT MOTION OF THE SUN, AND THE ORBITAL MOTION OF THE EARTH • ABERRATION OF LIGHT • PRECESSION AND NUTATION • THE EQUATION OF TIME • THE SEASONS AND THE CALENDAR

156. The Sun's Apparent Annual Motion among the Stars. This must have been among the earliest recognized of astronomical phenomena, and it is obviously one of the most important.

As seen in the northern hemisphere, the sun, starting in the spring at the vernal equinox, mounts higher in the sky each day at noon for three months, until the summer solstice, and then descends toward the south, reaching in the autumn the same noonday elevation that it had in the spring. It keeps on its southward course to the winter solstice in December, and then returns to its original height at the end of a year, marking and causing the seasons by its course.

Nor is this all. The sun's motion is not merely north and south, but it also advances continually *eastward* among the stars. In the spring the stars rising on the eastern horizon at sunset are not those found there at that hour in summer or winter.

In March the most conspicuous of the eastern constellations at sunset are Leo and Boötes. A little later Virgo appears; in the summer, Ophiuchus and Libra; still later, Scorpio; while in midwinter Orion and Taurus are ascending as the sun goes down.

So far as the obvious appearances are concerned, it is quite indifferent whether we suppose the earth to revolve around the sun or vice versa. That it is the earth which moves, however, is demonstrated by three phenomena too delicate for observation without the telescope, but accessible to modern methods. The most conspicuous of them is *the aberration of light* (§ 162); the others are *the regular annual shift of the lines in the spectra of stars* (§ 732) and *the annual parallax of the stars* (§ 710).

157. The earth's orbit is the path in space pursued by the earth in its revolution around the sun. The *ecliptic is not the orbit* and must not be confounded with it. The ecliptic is the great circle traced on the infinite celestial sphere by the plane in which the orbit lies; the orbit itself is a closed curve, of finite diameter, in space. The fact that the ecliptic is a great circle gives us no information about the orbit, except that it lies *wholly in one plane, which passes through the sun*; it tells us nothing as to the orbit's real *form* or *size*. But by reducing the daily observations of the sun's right ascension and declination made with a meridian circle to celestial longitude and latitude,[1] and combining these data with observations of the *sun's apparent diameter*, we can ascertain the *form* of the earth's orbit and the *law of its motion* in this orbit. The *size* of the orbit cannot be fixed until we find some means of determining the scale of miles.

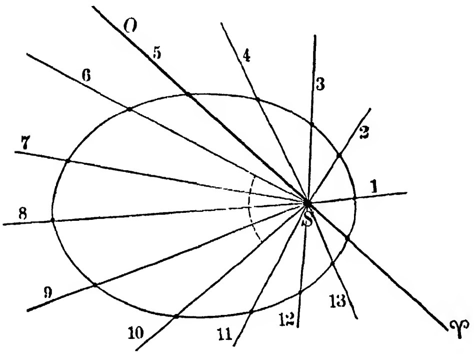


FIG. 57. Determination of the Form of the Earth's Orbit

Lines are drawn, radiating from S (the sun), to represent the directions of the earth from the sun, and are then cut off at lengths inversely proportional to the apparent diameters of the sun

158. To find the Form of the Orbit. Take a point S (Fig. 57) for the sun, and draw from it a line $S♈$ directed toward the vernal equinox, from which longitudes are measured. Lay off from S lines indefinite in length, making angles with $S♈$ equal to the earth's *longitude* as seen from the sun ($180° +$ the sun's longitude *as seen from the earth*) on each of the days when observations were made. We shall thus get a sort of spider, showing the *direction* of the earth as seen from the sun on each of those days.

Next, as to the *distances*. While the apparent diameter of the sun does not determine its *absolute* distance from the earth unless

[1] The latitude would always be exactly zero, except for some slight perturbations (§ 326) of the earth.

we know the diameter in miles, yet the *changes in the apparent diameter* do inform us as to the relative distance at different times, — the distance being *inversely proportional* to the sun's apparent diameter (§ 109). If we divide 206,265 by the number of seconds in the sun's measured diameter at any date, we shall obtain (very approximately) the earth's distance from the sun, measured in *solar diameters* as units. If we lay off these distances on the arms of our spider, the curve joining the points thus obtained *will be a true map of the earth's orbit,* though without any scale of miles.

When the operation is performed, we find that the orbit is an ellipse of small eccentricity (about 1/60), with the sun not in the center but *at one of the two foci.*

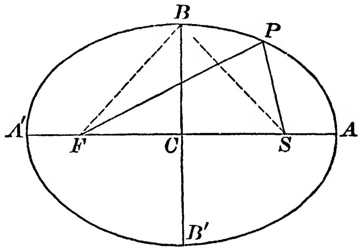


Fig. 58. The Ellipse

$SP + FP = SB + FB = AA'$, the major axis. AC is the *semi-major axis*, denoted by a. BC is the *semi-minor axis*, denoted by b. The *eccentricity* $e = SC/AC = \frac{\sqrt{a^2 - b^2}}{a}$

159. Definitions relating to the Orbital Ellipse. *The ellipse is a curve such that the sum of the two distances from any point on its circumference to two points within, called the foci, is always constant and equal to the major axis of the ellipse* (Fig. 58).

Perihelion and *aphelion* are, respectively, the points where the earth is nearest to and remotest from the sun, the line joining them being the major axis of the orbit. The *line of apsides* is the major axis indefinitely produced in both directions. A line drawn from the sun to the earth or any other planet at any point in its orbit, as SP in the figure, is called the planet's *radius vector*, and the angle ASP, reckoned from the perihelion point, in the direction of the planet's motion, is called its *anomaly*. The mean of the perihelion and aphelion distances is called the *mean distance*. It is equal to half the major axis.

160. Discovery of the Eccentricity of the Earth's Orbit by Hipparchus. The variations in the sun's diameter are too slight to be detected without a telescope, so that the ancients failed to perceive them. Hipparchus, however, about 120 B.C., discovered that the earth is not the center of the

circular orbit [1] which he supposed the sun to describe around it with uniform velocity.

Obviously the sun's *apparent* motion is not uniform, because it takes 186 days for the sun to pass from the vernal equinox to the autumnal, and only 179 days to return. Hipparchus explained this difference by the hypothesis that the earth is out of the center of the circle.

As a matter of fact, the earth's orbit is so nearly circular that the difference between the radius vector of the ellipse and that of an eccentric circle is everywhere so small that the method indicated in the preceding article would not practically suffice to discriminate between them. Other planetary orbits are, however, unmistakable ellipses, and the investigations of Newton show that the earth's orbit also is necessarily elliptical.

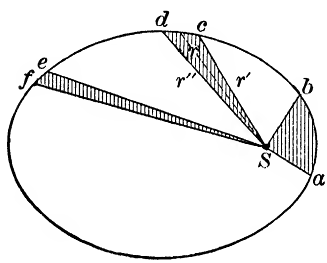


FIG. 59. Equable Description of Areas

The radius vector of a planet sweeps over equal areas in equal times. If *ab*, *cd*, and *ef* be portions of the orbit passed over by the earth in different weeks, then the shaded sectors are all of equal area

161. The Motion of the Earth in its Orbit. On comparing the positions of the earth in its orbit, as plotted by the method of section 158, or found still more accurately by calculation, with the times to which they correspond, it appears that the earth moves most rapidly near perihelion, not only in angle around the sun but also in distance along the orbit. The law which governs the motion was discovered by Kepler in 1609 and is that *the area swept out by the radius vector is always proportional to the time* (Fig. 59). Knowing this law and the date on which the earth passed perihelion, it is possible to calculate its direction from the sun at any time, and its distance in solar diameters.

162. Aberration of Light. A direct proof that the earth is moving in an orbit, and a determination of the size of the orbit in miles, can be obtained from the aberration of light. *Aberration* [2]

[1] Until the time of Kepler it was universally assumed, on metaphysical grounds, that the orbits of the celestial bodies must necessarily be circular and described with a uniform motion, "because," as was reasoned, "the circle is the only *perfect* curve, and uniform motion is the only motion proper to *heavenly* bodies."

[2] It was first discovered in 1725 (and later explained) by Bradley, who afterwards became the English astronomer royal.

is the apparent displacement of a heavenly body, due to the combination of the orbital velocity of the earth with the velocity of light.

The fact that light is not transmitted instantaneously, but with a finite velocity, causes a displacement of an object viewed from any moving station, unless the motion is directly toward or from that object. The direction in which we point our telescope to observe a star is usually not the same as if we were at rest, and the angle between the two directions is the star's *aberration* at the moment (not to be confused with the aberration of lenses, § 54).

We may illustrate this by considering what would happen in the case of falling raindrops observed by a person in motion. Suppose the observer standing with a tube in his hand while the drops are falling vertically. If he wishes to have the drops descend through the tube without touching the side, he must obviously keep it vertical so long as he stands still; but if he advances in any direction, the drops will strike his face and he will have to draw back the bottom of the tube (Fig. 60) by an amount which equals the advance he makes during the time while a drop is falling through it; that is, he must incline the tube forward at an angle, a, which depends both upon the velocity of the raindrop and the velocity of his own motion, so that when the drop, which entered the tube at B, reaches A', the bottom of the tube will be there also. This angle is given by the equation

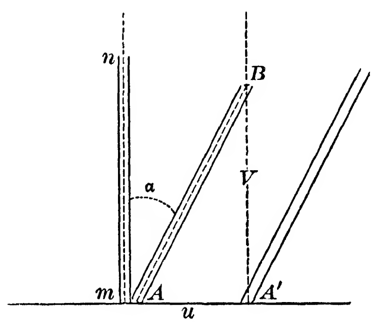


FIG. 60. Aberration of a Raindrop

$$\tan a = u/V,$$

in which V is the velocity of the drop, and u the velocity of the observer at right angles to V.

This illustration is not a demonstration, because light does not consist of *particles* but of *waves* transmitted through space; but it can be shown that the apparent direction of motion of a wave is affected in precisely the same way. A discussion based on the principles of relativity, though more difficult, leads to the same result.

163. The Constant of Aberration. By the discussion of thousands of observations upon stars it is found that the *maximum aberration* of a star — the same for all stars — is about 20″.47, which is called the *constant of aberration*. This maximum displacement occurs, of course, whenever the earth's motion is at right angles to the line drawn from the earth to the star, usually twice a year.

A star at the pole of the ecliptic is, however, permanently in a direction perpendicular to the earth's motion, and will therefore always be displaced by the same amount of 20″.5, but in a *direction continually changing*. It therefore appears to describe during the year, as its "aberrational orbit," a little circle 41″ in diameter.

A star *on the ecliptic* (*latitude* 0°) appears simply to oscillate back and forth in *a straight line 41″ long*.

Between the ecliptic and its pole the aberrational orbit is an *ellipse* having its major axis parallel to the ecliptic and *always 41″ long*, while its minor axis depends upon the star's latitude β, and always equals *41″ sin* β.

There is also a very slight *diurnal aberration* due to the rotation of the earth, its amount depending upon the observer's *latitude* and ranging from 0″.31 at the equator to zero at the pole.

164. Determination of the Earth's Orbital Velocity and the Mean Distance of the Sun by means of Aberration. From section 162, $\tan a = u/V$, which gives $u = V \tan a$, u in this case being the velocity of the earth in its orbit and V the velocity of light, while a is the constant of aberration. The recent experiments of Michelson (§ 556) make V equal 299,796 km./sec. (186,285 mi./sec.), with a probable error of about 3 miles. We have, therefore, u, the velocity of the earth in its orbit, equals 299,796 tan 20″.47 = 29.75 km./sec. (18.49 mi./sec.).

The circumference of the orbit, regarded as circular (which in the case of the earth involves no sensible error), is found by multiplying this velocity, 18.5, by the number of mean solar seconds in the *sidereal* year (§ 175). Dividing this circumference by 2π, we find the radius of the orbit, or the mean distance of the sun, to be very nearly **92,900,000** miles.

The uncertainty of the constant of aberration affects the distance proportionally, by perhaps 100,000 miles.

165. Precession of the Equinoxes. This is a slow westward motion of the equinoxes and was first discovered by Hipparchus about 125 B.C. He found that the "year of the seasons," from solstice to solstice (as determined by the gnomon), was shorter than that determined by the *heliacal rising and setting of the stars* (that is, the times when certain constellations rise and set with the sun), just as if the equinox "preceded," that is, "stepped forward" a little to meet the sun. The difference between the year of the seasons and the sidereal year is about twenty minutes. This difference of twenty minutes, since it is about one twenty-six-thousandth part of the year, can be accounted for by an annual westward motion of the equinox of about $50''$ of arc ($1/26{,}000 \times 360°$). The annual precession in 1925, according to Newcomb, is $50''.2619$.

Since the equinox is the point of intersection of the equator and the ecliptic, its motion must, of course, be interpreted as a motion of one or both of these circles and of their poles. As a matter of fact neither pole is stationary. That the motion of the pole of the equator (the celestial pole) contributes much the larger share of the precession is shown by the fact that the change in the latitudes of the stars in the last two thousand years has been very slight in comparison with the change in their longitudes, right ascensions, and declinations.

The motion of the *celestial pole* may be treated as partly periodic and partly progressive; that is, the actual pole may be considered as oscillating in a short period about a *mean pole* which moves steadily forward.[1]

The periodic motions of the celestial pole are known as *nutations*; the progressive motion of the mean pole, as (1) the *luni-solar precession.* The motion of the *ecliptic pole* produces (2) the *planetary precession*, and the sum of the two precessions is the *general precession.*

(1) *The luni-solar precession.* The mean pole moves around the pole of the ecliptic, regarded as fixed, in a circle at constant velocity. The equator continually shifts, therefore, so that its

[1] The distinction between progressive (often called secular) and periodic motions is not rigid, for the reason that the former, while apparently continuing indefinitely, may bring the object back to its initial position after the lapse of a very long period.

intersection with the ecliptic (the vernal equinox) moves *westward* at a uniform rate, while the angle between the two (the obliquity) remains constant (Fig. 61). The pole is always moving toward the position which the vernal equinox occupies at the time, while the equinox moves just fast enough to keep always *precisely* 90° from the pole.

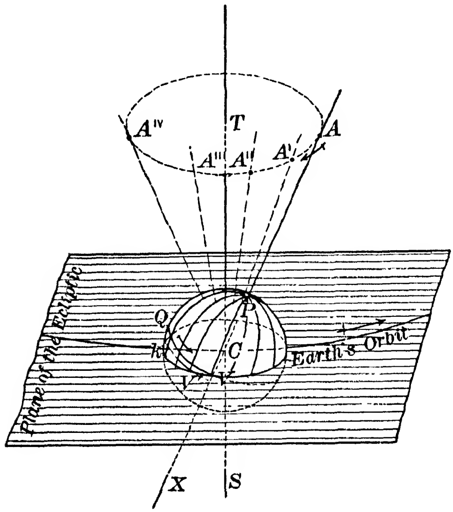


FIG. 61. Conical Motion of Earth's Axis

The earth is pictured as half immersed below the surface of the plane of the ecliptic in which it moves along its orbit. The line *SCT* points to the pole of the ecliptic. About this line the earth's axis *ACX* shifts conically (like the axis of a spinning top), taking up successively the positions *A'C*, *A''C*, etc., all the while keeping its inclination (the angle *ACT*) unchanged. The direction of the equinox is thus changed from *CV* to *CV'* and onward

(2) *The planetary precession.* The motion of the pole of the ecliptic, which is only about 1/40 as fast as that of the celestial pole, changes the direction of the line joining the two poles and causes the equinox to move *eastward* 0''.11 per year. It also diminishes the obliquity of the ecliptic by 0''.47 annually. This again causes a small change in the rate of the luni-solar precession from century to century. While the pole of the ecliptic has remained almost fixed among the stars, the pole of the equator has traveled many degrees since the earliest observations. The appearance of the heavens has consequently changed greatly. The present polestar was once far from the pole (Fig. 62).

Great changes have taken place in the apparent position of other constellations in the sky. Six thousand years ago the Southern Cross was visible in England and Germany, and Cetus never rose above the horizon there.

Another effect of precession is that the *signs* of the zodiac (§ 24) no longer correspond to their zodiacal constellations. The *sign* of

Aries is now in the *constellation* of Pisces, and so on. In the last two thousand years each sign has backed bodily, so to speak, into the constellation west of it.

The reader must again be warned against confusing the precessional motion of the *celestial* pole with the motion of the *terrestrial* pole which

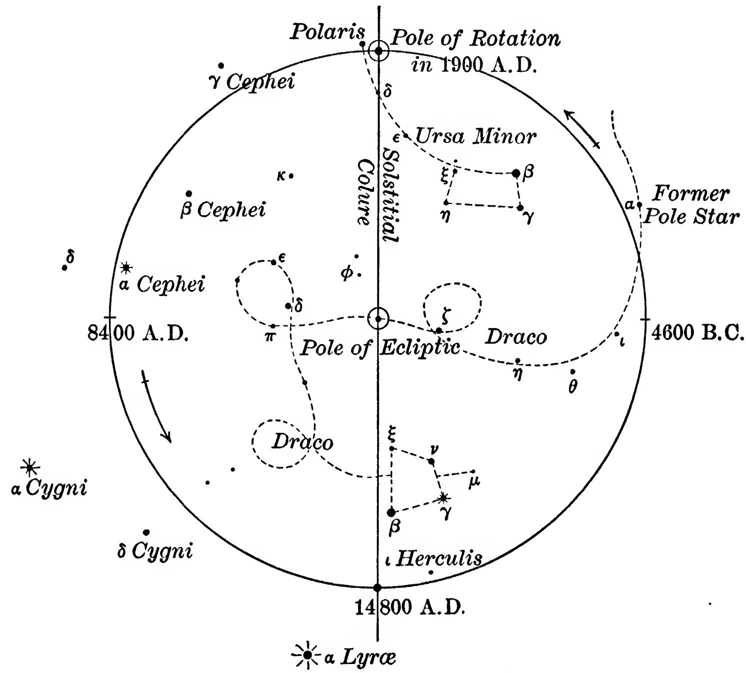


FIG. 62. Precessional Path of the Celestial Pole

Reckoning back about 4600 years, we see that α Draconis was then the polestar. About 5600 years hence α Cephei will take the office, and about 12,000 years from now *Vega* (α Lyræ) will be the polestar — a splendid one, but rather far from the pole. (Owing to the motion of the pole of the ecliptic the actual track of the celestial pole will not, however, be exactly circular, nor will it follow quite the same path in its next revolution)

causes the variation of latitude (§ 137). The former is a motion of the axis and the earth together; the latter is a motion of the axis within the earth. The latter involves a slight change in the bearing of one terrestrial object from another; the former does not, — a north-south line drawn on the earth's surface remains a north-south line notwithstanding the precession.

166. Physical Cause of Precession. The physical cause of this slow conical motion of the earth's axis was first explained by

Newton. The earth may be considered as consisting of a sphere encircled by a protuberant ring of matter, — the equatorial bulge. The sun and moon, acting on this ring, tend to change the inclination of the equator, — the one to draw it into coincidence with the plane of the ecliptic, the other to draw it into coincidence with the plane of the moon's orbit. As the plane of the moon's orbit is inclined only about 5° to the ecliptic, the two forces act nearly in the same plane and combine to produce the luni-solar precession. The moon, being nearer than the sun, is much the more effective in producing the precession.

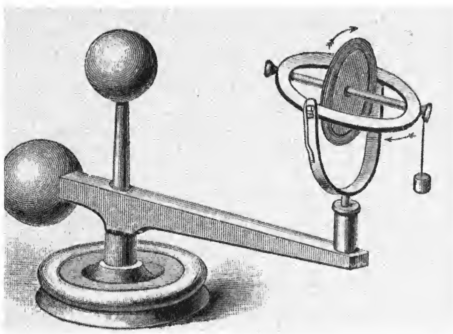


FIG. 63. Precession illustrated by the Gyroscope

If it were not for the earth's rotation, this action of the sun and moon would actually bring the two planes of the equator and ecliptic into coincidence; but since the earth is spinning on its axis, we get the same result as we do with the whirling wheel of a gyroscope by hanging a weight at one end of its axis (Fig. 63). We then have a *combination of two rotations at right angles to each other*, — one the whirl of the wheel, the other the tip which the weight tends to give the axis. The resultant effect (very surprising when the experiment is seen for the first time) is that the axis of the wheel, instead of tipping, maintains its inclination unchanged but *moves around conically* like the axis of the earth. Any force tending to change the direction of the axis of a whirling body produces a motion *exactly at right angles* to its own direction.

Compared with the mass of the earth and its momentum of rotation this disturbing force is very slight, and consequently the rate of precession is extremely slow. If the earth were spherical, there would be no precession. If it revolved on its axis more slowly, precession would be more rapid, as it would be also if the sun and moon were larger or nearer, or if the obliquity of the ecliptic were greater, not exceeding 45°. The cause of the

planetary precession is the alteration of the plane of the earth's orbit by the action of the other planets.

167. Nutations. The forces which tend to pull the equator toward the ecliptic continually vary. When the sun and moon are crossing the celestial equator the action becomes zero, — twice a year for the sun, twice a month for the moon. Moreover, as we shall see (§ 188), the moon's orbit is continuously changing its position in such a way that the maximum declination attained by the moon during the month varies by as much as 10°. As a consequence the actual pole follows a sinuous curve, oscillating about the mean pole (steadily advancing by precession) in an irregular curve not very different from a circle. This involves an alternate motion toward and from the pole of the ecliptic (a nodding, which gives the motion its name of "nutation"), as well as a periodic variation in the rate of advance, sometimes known as the equation of the equinox. The largest nutation (that depending on the motion (§ 188) of the moon's nodes) has a maximum amount of $9''.21$ and a period of a little less than nineteen years.

168. The Gyro-compass. This consists of a rapidly revolving gyro-wheel driven by an electric motor and so mounted that its axis is constrained to be horizontal but may move freely in a horizontal plane. If the axis points east and west, the earth's rotation tips one end down and the other end up (acting like the pull of the weight in Fig. 63) and produces a precession, which causes one end of the axis to seek the north and the other the south. If the axis overshoots the mark, the precessional force reverses in direction and brings it back. Since this instrument will work inside the armor of a battleship, which shields the ordinary compass against the earth's magnetic force, it is of great use in the navy. The directive force of the gyro-compass is many times that of the magnetic compass, and it points *true* north.

169. The Equation of Time. The equation of time at any moment is the difference between apparent and mean solar time, that is, the difference in hour angle of the sun and the fictitious mean sun (§ 36), and is therefore the difference of their right ascensions.

There are two principal causes of this difference:

(1) *The variable motion of the sun in the ecliptic, due to the eccentricity of the earth's orbit.* Near perihelion (about January 2)

the sun's eastward motion in the ecliptic is most rapid and is faster than that of the mean sun. It draws away, therefore, to the eastward of the mean sun (with which it coincides at perihelion) and comes to the meridian late. At aphelion it is once more in coincidence with the mean sun but is now moving more slowly, so that it begins at once to fall behind—to the westward—and comes to the meridian early. The daily differences sum up to a maximum difference of about $7\frac{3}{4}$ minutes in April and October.

(2) *The obliquity of the ecliptic.* Even if the sun's motion in *longitude* (that is, along the ecliptic) were uniform, its motion in *right ascension* would be variable. If the true and fictitious suns were together at the vernal equinox, one moving uniformly in the ecliptic and the other in the equator, they would indeed be together (that is, have the same right ascensions) at the two solstices and at the other equinox, because it is just 180° from equinox to equinox, and the solstices are exactly halfway between them; but at any point between the solstices and equinoxes their right ascensions would differ.

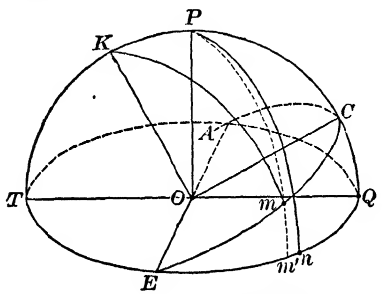


FIG. 64. The Sun's Variable Motion in R. A.

In this illustration the observer is supposed to be looking at the globe from the west, E (the vernal equinox) being at the west point of the horizon. ECA is the ecliptic, its pole being K, while $EQAT$ is the celestial equator, its pole (of diurnal rotation) being P

This is easily seen by taking a celestial globe and marking on the ecliptic the point m (Fig. 64) halfway between the vernal equinox E and the summer solstice C, and also marking a point n on the equator 45° from the equinox. It will be seen at once that the former point is west of n, the difference of right ascension being $m'n$, so that m, in the apparent diurnal revolution of the sky, will come first to the meridian.

Even if the sun moved uniformly in the ecliptic, it would, by reason of the obliquity, be west of the mean sun, which is supposed to move uniformly in the equator, between March 21 and June 21, and cross the meridian early; in the next quarter year it will be east of the mean sun and come late to the meridian, and

so on. The maximum difference (in February, May, August, and November) amounts to 10 minutes.

170. Combination of the Effects of the Two Causes. We can represent the two components of the equation of time and the result of their combination by a graphical construction (Fig. 65).

The central horizontal line is a scale of *dates* one year long, the months being indicated at the top. The *dotted curve* shows that component of the equation of time which is due to the eccentricity of the earth's orbit. In the same way the *broken-line curve* denotes the effect of the obliquity of the ecliptic. The *heavy-line curve* represents the combined effect of the two causes,

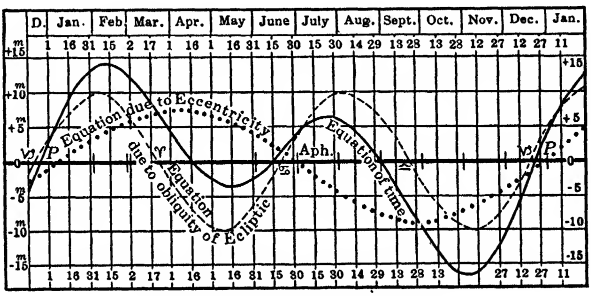


FIG. 65. The Equation of Time

Mean time minus apparent time. (The *American Ephemeris* tabulates as well apparent minus mean.) According to the convention of this diagram the mean time is obtained from the apparent time by adding algebraically the equation of time to the latter

its ordinate at each point being made equal to the algebraic sum of the ordinates of the other two curves. The equation of time can be read from this curve within a minute, which is as closely as the apparent time can be found from a sundial.

The two causes discussed above are only the principal ones. Every perturbation suffered by the earth slightly modifies the result, but all other causes combined never affect the equation of time by as much as ten seconds.

The equation of time becomes zero four times yearly, as will be seen from the figure, — about April 15, June 14, September 1, and December 24; but the dates vary a little from year to year.

171. The Seasons. The earth in its orbital motion keeps its axis parallel to itself, except for the minute effect of precession. Since this axis is not perpendicular to the plane of its orbit, the

poles of the earth vary in their presentation to the sun. On June 21 the earth is so situated that its *north* pole is inclined toward the sun by about $23\frac{1}{2}°$ (Fig. 66). The south pole is then in the unilluminated half of the globe, while the north pole receives sunlight all day long; and in all portions of the northern hemisphere the day is longer than the night, and vice versa in the southern hemisphere. At the time of the winter solstice these conditions are reversed and the south pole has perpetual sunshine. At the two equinoxes, March 21 and September 21, the plane of the earth's equator passes through the sun, so that the circle which divides

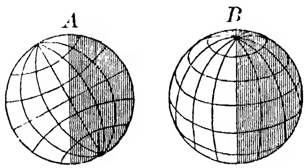


Fig. 66. Position of Pole at Solstice and Equinox

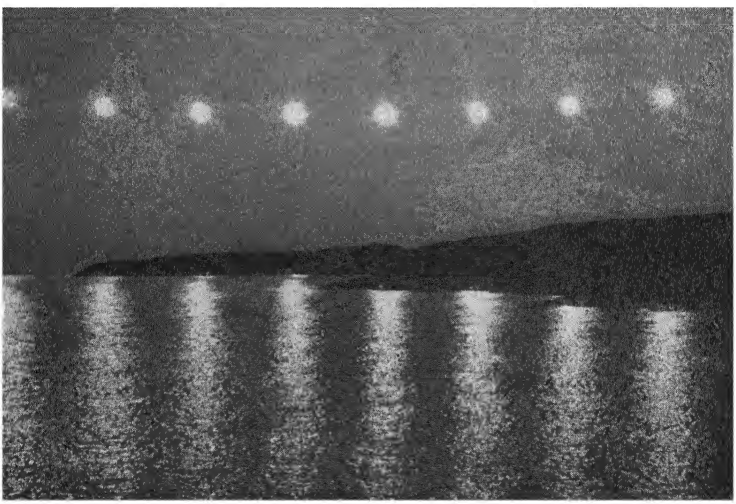


Fig. 67. The Midnight Sun at Etah, Greenland, in July

A series of exposures at intervals of twenty minutes shows the sun reaching minimum altitude at midnight. (From a photograph by Donald MacMillan)

day from night upon the earth passes through the pole, and day and night are then everywhere equal. On the equator, day and night are equal at all times of the year, and there are no seasons in the proper sense of the word.

At all places within the *torrid zone*, which extends $23\frac{1}{2}°$ north and south of the equator and is bounded by the *tropics* of Cancer and Capricorn, the sun passes, at some time in the year, through the zenith. In the *temperate zones*, which are each 43° wide, the sun is never seen in the zenith, nor does it fail to appear above the horizon at noon. The *frigid zones* extend $23\frac{1}{2}°$ from the poles and are bounded by the arctic and antarctic circles. In these regions one or more days elapse in winter without the appearance of the sun, while in summer the sun makes one or more complete circuits above the horizon (§ 32 and Fig. 67).

172. Effects on Temperature. The changes in the *insolation* (exposure to sunshine) at any place involve changes of temperature and of other climatic conditions which produce the seasons. Taking as a standard the average amount of heat received from the sun in twenty-four hours on the day of the equinox, it is clear that the surface of the soil at any place in the northern hemisphere will receive, every twenty-four hours, more than the average of heat whenever the sun is north of the celestial equator, and for two reasons:

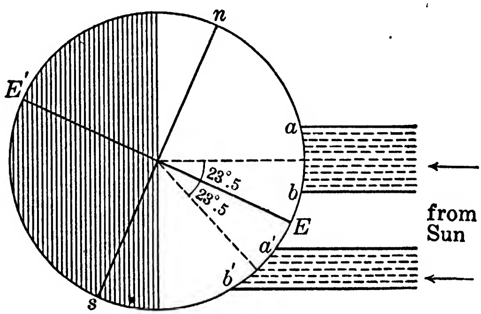


FIG. 68. Effect of Sun's Elevation on Amount of Heat imparted to the Soil

In June more heat from the sun reaches a given area (*a* to *b*) in the northern hemisphere than reaches an equal area (*a'* to *b'*) in the same latitude south of the equator

(1) Sunshine lasts more than half the day.

(2) The *mean altitude* of the sun while above the horizon is greater than at the time of the equinox.

Now the more obliquely the rays strike, the less heat they bring to each square inch of the surface (Fig. 68). A beam of sunshine of a certain cross-section is spread over a larger area when it strikes obliquely than when it strikes vertically, and its heating efficiency is in inverse ratio to the surface over which the heat is distributed.

For these two reasons, therefore, at a place in the northern hemisphere the mean temperature of the day rises rapidly as the sun comes north of the equator, thus causing summer.

173. Time of Highest Temperature. The northern hemisphere receives the most heat in twenty-four hours at the time of the summer solstice; but this is not the hottest time of the season, for the reason that the surface is receiving more heat than it loses and is therefore getting hotter. The maximum temperature will not be reached until the increase ceases, that is, not until the amount of heat *lost* in twenty-four hours equals that *received*, which occurs in our latitude about August 1. For similar reasons the minimum temperature of winter occurs about February 1.

Since the weather is not entirely "made on the spot where it is used," but is much influenced by winds and currents that come from great distances, the actual date of the maximum temperature at any particular place cannot be determined beforehand by astronomical considerations alone, but varies considerably from year to year.

174. Difference between Seasons in Northern and Southern Hemispheres. Since in December the distance of the earth from the sun is about 3 per cent less than it is in June, the earth as a whole receives, hourly, about 6 per cent more heat in December than in June, the heat received varying inversely as the *square* of the distance. For this reason the southern summer, which occurs in December and January, is hotter than the northern summer, and the southern winter is colder. In midsummer, when the sun shines at the pole continuously, the amount of heat received there per square mile during the twenty-four hours is 25 per cent greater than at the equator for the same area and time. If it were not for the great accumulation of ice at the pole, summers there would be hot.

175. The Sidereal and Tropical Years. The *sidereal year*, as its name implies, is the time occupied by the sun in apparently completing the circuit of the heavens *from a given star to the same star again*. Its length is $365^d\ 6^h\ 9^m\ 9^s.5$ of mean solar time ($365^d.25636$).

From the mechanical point of view this is the *true* year; that is, it is the time occupied by the earth in making one complete

revolution around the sun from a given direction in space to the same direction again.

The *tropical year* is the time included between two successive passages of the vernal equinox by the sun. On account of precession (§ 165) the equinox moves yearly $50''.3$ toward the west, so that the tropical year is shorter than the sidereal year, its length being $365^d\ 5^h\ 48^m\ 46^s.0$ ($365^d.24220$). Its length was determined by the ancients with considerable accuracy, as $365\frac{1}{4}$ days, by means of the gnomon (§ 90); they noted the dates at which the noonday shadow was longest (or shortest), that is, the dates of the solstices.

Since the *seasons* depend on the sun's place with respect to the equinox, the tropical year is the year of chronology and civil reckoning. Whenever a period of so many years is spoken of, we always understand tropical years unless the term is otherwise distinctly indicated.

A third kind of year is the *anomalistic year*, — the time between two successive passages of the perihelion. Since the line of apsides of the earth's orbit moves eastward about $11''$ a year (§ 328), this kind of year is nearly five minutes longer than the sidereal year, its length being $365^d\ 6^h\ 13^m\ 53^s.0$ ($365^d.25964$). It is very little used.

176. The Calendar. The natural units of time are the day, month, and year. The day is too short for convenience in dealing with considerable periods, — such as the life of a man, for instance, — and the same is true of the month, so that for chronological purposes the *tropical year* (the year of the seasons) is employed. At the same time so many religious ideas and observances have been connected with the changes of the moon that there used to be a constant struggle to reconcile the *month* (the period of the moon's orbital revolution) with the *year*. Since the two are incommensurable, no really satisfactory solution is possible, and the modern calendar of civilized nations entirely disregards the moon.

In ancient times the calendar was in the hands of the priesthood and was predominantly lunar, the seasons being either disregarded or kept roughly in place by the occasional intercalation or dropping of a month. The principal Mohammedan nations still use a purely lunar calendar for religious purposes, having a year of twelve lunar months, containing alternately 354

and 355 days. In their reckoning, therefore, the months and the religious festivals fall continually in different seasons, and their calendar gains on ours about one year in thirty-three.

177. The Julian Calendar. When Julius Cæsar came into power he found the Roman calendar in a state of hopeless confusion. He therefore sought the advice of the Alexandrian astronomer Sosigenes, and in accordance with his suggestions established (45 B.C.) what is known as the *Julian calendar*, which still, with a trifling modification, continues in use among all civilized nations. He discarded all consideration of the moon, and, adopting $365\frac{1}{4}$ days as the true length of the year, he ordained that every fourth year should contain 366 days, the extra day being inserted by repeating the sixth day before the kalends of March, whence such a year is called *bissextile*. He also transferred the beginning of the year to January 1; up to that time it had been in March, as is still indicated by the names of several of the months, as September, that is, the *seventh* month, etc.

Cæsar also took possession of the month Quintilis, naming it *July* after himself. His successor, Augustus, in a similar manner appropriated the next month, Sextilis, calling it *August*. The story that, to make his month as long as July, he added to it a day stolen from February, appears to be unfounded.

178. The Gregorian Calendar. The true length of the tropical year is not $365\frac{1}{4}$ days, but $365^d\ 5^h\ 48^m\ 46^s.0$, leaving a difference of $11^m\ 14^s.0$ by which the Julian year is too long. This amounts to a little more than three days in four hundred years. As a consequence, in the Julian calendar the date of the vernal equinox comes earlier and earlier as time goes on, and by A.D. 1582 it had fallen back to the eleventh of March instead of occurring on the twenty-first, as it did at the time of the Council of Nice, A.D. 325. Pope Gregory, therefore, under the advice of the distinguished astronomer Clavius, ordered that the calendar should be corrected by dropping *ten days*, so that the day following October 4, 1582, should be called the fifteenth instead of the fifth; and, further, to prevent any future displacement of the equinox, he decreed *that thereafter only such century years should be leap-years as are divisible by 400*. (Thus, 1700, 1800, 1900, 2100, and so on, are not leap-years, while 1600 and 2000 are.)

179. The change was immediately adopted by all Catholic countries, but the Greek Church and most Protestant nations refused to recognize the pope's authority. It was, however, finally adopted in England by an act of Parliament, passed in 1751, providing that the year 1752 should begin on January 1 (instead of March 25, as had long been the rule in England), and that the day following September 2, 1752, should be reckoned as the fourteenth instead of the third, thus dropping eleven days.

The change was bitterly opposed by many, and there were riots in various parts of the country in consequence, especially at Bristol, where several persons were killed. The cry of the people was, "Give us back our fortnight!" for they supposed they had been robbed of eleven days, although the act of Parliament was carefully framed to prevent any injustice in the collection of interest, the payment of rents, etc.

At present, since the years 1800 and 1900 were leap-years in the Julian calendar and not in the Gregorian, the difference between the two calendars is thirteen days. The Julian calendar was adhered to in Russia until 1918, and in Rumania until 1919, but both dates were customarily used for scientific purposes, for example, June 9/22, 1916.

When Alaska was annexed to the United States the official date had to be changed by only eleven days, one day being provided for in the alteration from the Asiatic reckoning to the American (§ 40).

180. The Julian Day. A system of chronological reckoning by days has many advantages in the simplification of calculations which involve long periods of time, and in the avoidance of ambiguity. According to the system proposed by J. Scaliger in 1582 a date is expressed as the number of days elapsed since the beginning of the arbitrary "Julian era," January 1, 4713 B.C. Thus, the date of the solar eclipse of January 24, 1925, is J.D. 2,424,175, and this is perfectly definite to every astronomer. The number of days between any two events, even centuries apart, is at once found by merely taking the difference between their Julian-day numbers. The *Nautical Almanac* gives the Julian-day number for January 1 of every year. By international agreement the Julian days still begin at noon.

EXERCISES

1. What is the meridian altitude of the sun at Princeton, New Jersey (Lat. 40° 21′), on the day of the summer solstice?

2. What is the sun's approximate right ascension at that time?

3. On about what days during the year will the sun's right ascension be an even hour (that is, 0 hours, 2 hours, 4 hours, etc.)?

4. On what days will it be an *odd* hour?

5. What is the (approximate) sidereal time at 10 P.M. on May 12?
Ans. $13^h\ 26^m$.

6. At what time will Arcturus (R.A. $= 14^h\ 10^m$) come to the meridian on August 1? *Ans.* About $5^h\ 26^m$ P.M.

7. About what time of night is Mizar (R.A. $= 13^h\ 20^m$) vertically under the pole on October 10? *Ans.* Midnight.

8. In what latitude has the sun a meridian altitude of 80° on June 21?
Ans. $+ 33°\ 27'$.

9. What are the longitude and latitude (celestial) of the north celestial pole? *Ans.* Long. 90°, Lat. 66° 33′.

10. What are the right ascension and declination of the north pole of the ecliptic? *Ans.* R.A. 18^h, Dec. 66° 33′.

11. What are the greatest and least angles made by the ecliptic with the horizon at New York (Lat. 40° 43′)?

Ans. $(90° - 40°\ 43') \pm 23°\ 27' = \begin{cases} \text{Max. } 72°\ 44'. \\ \text{Min. } 25°\ 50'. \end{cases}$

12. Does the sun always pass through the vernal equinox on the same day of the month? If not, why not? How much can the date vary?

13. Will the ephemeris of the sun for one year be correct for every other year, and, if not, how much can it be in error?

Ans. A difference of $1\frac{3}{4}$ days' motion of the sun is possible; as, for instance, between 1897 and 1903, the leap-year being omitted in 1900.

14. When the sun is in the *sign* of Cancer, in what *constellation* is it?

15. What obliquity of the ecliptic would reduce the width of the temperate zone to zero?

16. At what standard time will the sun come to the meridian on March 21 at Boston (Long. $4^h\ 44^m$ west of Greenwich), the equation of time being $+ 7^m\ 28^s$? *Ans.* $11^h\ 51^m\ 28^s$ A.M.

17. When the equation of time is 16 minutes, as it is on November 1, how does the forenoon from sunrise till 12 o'clock compare in length with the afternoon from 12 o'clock till sunset?

18. Why do the afternoons begin to lengthen about December 8, a fortnight before the winter solstice?

19. There were five Sundays in February, 1880. When did this happen again? When will it happen again?

20. If the weather were "made on the spot where it is used and at the time when it is used," what would be the hottest place on the earth?

Ans. The south pole on December 22.

REFERENCES

SIR JOHN HERSCHEL, *Outlines of Astronomy* (D. Appleton & Co., New York, 1876), gives a fuller explanation of the considerations on which the Julian-day system of reckoning is founded.

For those interested in the history of astronomy the following books are very pleasant and profitable reading:

MARY AGNES CLERKE, *Popular History of Astronomy during the Nineteenth Century.* A. & C. Black, London.

ARTHUR BERRY, *A Short History of Astronomy.* Charles Scribner's Sons, New York.

ROBERT GRANT, *History of Physical Astronomy to the Middle of the Nineteenth Century.* H. G. Bohn, London.

J. L. E. DREYER, *History of the Planetary Systems.* Cambridge University Press.

CHAPTER VI

THE MOON

THE MOON'S APPARENT MOTION · PHASES · THE MONTH · THE ORBIT · SYNODIC MOTION · THE HARVEST MOON · DISTANCE, MASS, DENSITY, AND GRAVITY · ROTATION AND LIBRATIONS · ATMOSPHERE, LIGHT, AND HEAT · TELESCOPIC APPEARANCE

181. Next to the sun, the moon is to us the most conspicuous and the most important of the heavenly bodies. If the stars and planets were all extinguished, our eyes would miss them, and that is all; but if the moon were annihilated, the interests of commerce would be seriously affected by the great diminution of the tides. It owes its conspicuousness and economic importance, however, solely to its nearness, for it is really a very insignificant body as compared with the stars and the planets.

As an inspiration to the development of astronomical theory it ranks high among the heavenly bodies. The very beginnings of the science seem to have originated in the study of its motions and phases and of the different phenomena which it causes, such as the eclipses and the tides; and in the development of modern theoretical astronomy the lunar theory, with the problems which it raises, has been perhaps the most fertile field of discovery and invention.

182. The Moon's Apparent Motion; Definition of Terms. One of the earliest observed of astronomical phenomena must have been the eastward motion of the moon with reference to the sun and stars, and the accompanying changes of phase. If we note the moon tonight as near some conspicuous star, we shall find it tomorrow night at a point about 13° farther east, and the next night as much farther still; it makes a complete circuit of the heavens, from star to star again, in about $27\frac{1}{3}$ days. In other words, it revolves around the earth in that time, while it accompanies us in our annual journey around the sun.

Since the moon moves eastward among the stars so much faster than the sun, it overtakes and passes the sun at regular intervals; and as its *phases* depend upon its apparent position with respect to the sun, this interval from new moon to new moon is especially noticeable and is what we ordinarily understand as the *month*, — technically, the *synodic month.*

The *elongation* of the moon is its angular distance from the sun at any time. When the moon has the same longitude as the sun, it is said to be in *conjunction* and is new; at full moon the

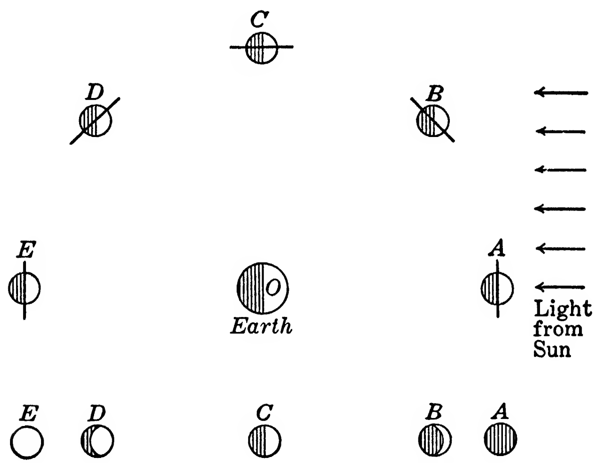


FIG. 69. Phases of the Moon

Sunlight from the right illumines one hemisphere of the revolving moon. The corresponding phases apparent to the observer *O*, on the earth, are pictured below

difference in longitude is 180°, and it is said to be in *opposition.* In both cases the moon is in *syzygy*; that is, the sun, moon, and earth are ranged nearly along a straight line. When the elongation is 90°, it is said to be in *quadrature.*

183. The Phases of the Moon. Since the moon is an opaque body shining merely by reflected light, we can see only that hemisphere of its surface which happens to be illuminated, and of this hemisphere only that portion which happens to be turned toward the earth (Fig. 69). When the moon is between the earth and the sun (at new moon), the dark side is presented directly toward us, and the moon is entirely invisible. A week later, at

the first quarter, half of the illuminated hemisphere is visible, just as it is a week after the full. Between the new moon and the half-moon, whether waxing or waning, we see *less* than half of the illuminated portion, and we then have the crescent phase. Between the half-moon and the full moon we see *more* than half of the moon's illuminated side, and we have then what is called

FIG. 70. The Gibbous Moon — before the Third Quarter

The rays from Tycho, near the south pole (at the top), and from Copernicus, near the equator, are conspicuous. (Photographed at the Yerkes Observatory)

the gibbous phase (Fig. 70). (The phases are illustrated by viewing from different sides a baseball lighted by a lamp.)

184. The Terminator. The line which separates the dark portion of the disk from the bright portion is called the *terminator*. Since it is a semi-circle viewed obliquely, the terminator is always a semi-ellipse. The illuminated portion of the moon's disk is therefore always a figure made up of a semi-circle plus or minus a semi-ellipse. At new or full moon, however, the semi-ellipse becomes a semi-circle, and at half-moon a straight line. The points of the crescent moon are known as the *cusps*.

It is to be noticed that the straight line joining the ends of the terminator is always perpendicular to a line from the moon to the sun (Fig. 72 *B*), so that the *horns* are *always turned away from the sun*. The precise position in which they will stand at any time is perfectly predictable from the geometrical relations of the earth, sun, and moon. Artists sometimes carelessly represent a crescent moon at night with its horns pointed downward.

185. Earth-Shine on the Moon. Near the time of new moon the whole disk is easily visible, the portion on which sunlight does not fall being illuminated by a pale light (Fig. 71). This light is *earth-shine*, the earth as seen from the moon being then nearly full.

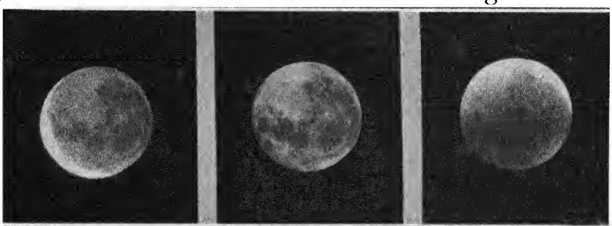


FIG. 71. The Earth-Lit, the Full, and the Totally Eclipsed Moon

That is, the moon illuminated by *reflected*, *direct*, and *refracted* sunlight. The picture on the left shows the "old moon in the new moon's arms": *sunlight* on the slender crescent, *earth-shine* on the rest of the moon. In the middle is the picture of the full moon. The picture on the right was taken during eclipse; although the moon is completely immersed in the earth's shadow, it is dimly illuminated by light *refracted* through the earth's atmosphere into the shadow. The exposures necessary to obtain good photographs in the three cases were very different. (From photographs by E. E. Barnard, Yerkes Observatory)

Seen from the moon, the earth would show all the phases that the moon does, the earth's phase being in every case exactly supplementary to that of the moon as seen by us at the time.

186. Sidereal and Synodic Months. The *sidereal month* is the time it takes the moon to make its revolution *from a given star to the same star again*, as seen from the center of the earth. It averages $27^d\ 7^h\ 43^m\ 11^s.47$ ($27^d.32166$), but it varies some seven hours on account of perturbations. The mean daily motion is $360° \div 27.32166$, or $13°\ 11'$. Mechanically considered, the *sidereal month* is the true month.

The *synodic month* is the time between two successive conjunctions or oppositions, that is, between successive new or full

moons. Its average value is $29^d\ 12^h\ 44^m\ 2^s.78$ ($29^d.53059$), but it varies nearly thirteen hours, mainly on account of the eccentricity of the lunar orbit. The synodic month is what we ordinarily mean when we speak of a month.

If M be the length of the moon's sidereal period, E the length of the sidereal year, and S that of the synodic month, the three quantities are connected by a very simple relation. $1/M$ is the fraction of a circumference moved over by the moon in a day. Similarly, $1/E$ is the apparent daily motion of the sun. The difference is the amount which the moon *gains* on the sun daily. Now it gains a whole revolution in one synodic month of S days, and therefore must gain daily $1/S$ of the circumference. Hence we have the important equation $1/M - 1/E = 1/S$, the equation of synodic motion; whence $S = \frac{EM}{E - M}$.

The moon moves about $12°.2$ ($360° \times 1/S$) to the *eastward of the sun* each day.

Another way of looking at the matter (leading, of course, to the same result) is this: In a sidereal year the number of sidereal months must be just one greater than the number of synodic months; the numbers are, respectively, 13.369+ and 12.369+.

The equation of synodic motion is of general application to all cases of a "stern chase" (§ 268).

187. The Metonic Cycle. Since 235 synodic months are very nearly equal to 19 Julian years, the phases of the moon recur after nineteen years, on the same days of the month, with perhaps a shift of one day, according to the number of leap-years intervening. This cycle was discovered by Meton about 433 B.C. It is used in the ecclesiastical calendar for finding the date of Easter. This is fixed as the first Sunday following the first full moon following the vernal equinox. Its date ranges through more than a month, as do also those of the other movable feasts which depend upon it.

188. The Moon's Path on the Celestial Sphere; the Nodes and their Motion. By observing the moon's right ascension and declination daily with suitable instruments we can map out its apparent path on the celestial sphere. It turns out to be very nearly a great circle, inclined to the ecliptic at an angle of about 5°.

The two points where the path cuts the ecliptic are called the *nodes*, the *ascending* node being the one where the moon passes from the south side to the north side of the ecliptic. The opposite node is called the *descending* node. (Ancient astronomers all lived in the northern hemisphere.)

On account of the so-called perturbations, due to the attraction of the sun, the moon at the end of the month never comes back exactly to the point of beginning.

One of the most important of these perturbations is the *regression of the nodes*. These slide westward on the ecliptic in the same manner as does the vernal equinox (§ 165), but much faster, completing their circuit in a little less than nineteen years instead of twenty-six thousand. The average time between two successive passages of the moon through the same node is called the *nodical* or *draconitic* month. It is 27.2122 days, — an important period in the theory of eclipses. The inclination also varies from $4^\circ\,59'$ to $5^\circ\,18'$, the mean being $5^\circ\,8'$.

When the *ascending node* of the moon's orbit coincides with the vernal equinox, the angle between the moon's path and the equator has its maximum value of $23^\circ\,27' + 5^\circ\,8'$, or $28^\circ\,35'$; nine and one-half years later, when the descending node has come to the same point, the angle is only $23^\circ\,27' - 5^\circ\,8'$, or $18^\circ\,19'$. In the first case the moon's meridian altitude will range, during the month, through $57^\circ\,10'$; in the second, through only $36^\circ\,38'$. The moon is much more effective in producing precession of the earth's axis in the first case than in the second. This accounts for the principal term in the nutation.

189. Interval between the Moon's Successive Transits; Daily Retardation of its Rising and Setting. Owing to the eastward motion of the moon it comes to the meridian *later* each day. If we call the average interval between its successive transits a lunar day, we see at once that, while in the synodic month there are 29.5306 mean solar days, there must be just one less of these lunar days, since the moon, in the synodic month, moves around eastward from the sun to the sun again, thus losing one complete relative rotation.

It follows, therefore, that the length of the lunar day must be $24^h \times \frac{29.5306}{28.5306}$, or $24^h\,50^m.47$, the average daily retardation of

transit being $50\frac{1}{2}$ minutes. It ranges, however, all the way from 38 minutes to 66 minutes, on account of the variations in the rate of the moon's motion in right ascension (due partly to perturbations but mainly to the elliptical form of its orbit and its inclination to the celestial equator), — variations precisely analogous to the inequalities of the sun's motion, which produce the equation of time (§ 169), but many times as great.

The average retardation of the moon's daily *rising* and *setting* is also the same, 50.47 minutes, but the actual retardation is much more variable than that of the transits, depending largely on the latitude of the observer. In latitude 40° the extreme range is from 13 minutes to 80 minutes. In higher latitudes it is still greater. Indeed, in latitudes above 61° 20′ the moon, when it has its greatest possible declination of 28° 47′, will become *circumpolar* for a certain time each month and will remain visible without setting at all for a whole day or more, according to the latitude of the observer. (There is also, in these higher latitudes, at least one day in the month on which the moon does not rise.)

190. Harvest and Hunter's Moon. The full moon which occurs nearest the time of the autumnal equinox is called the *harvest moon*; the next following one, the *hunter's moon.* The peculiarity of the harvest moon is simply that its *retardation of rising* (in the northern hemisphere) is smaller than that of any other full moon during the year; for several evenings the moon rises but little later (22 minutes on the average in latitude 40°) each night, so that there is moonlight in the early evening for an unusual number of evenings. The phenomenon is more striking in high latitudes.

In the autumn the *sun* is near the *autumnal equinox*; the *full moon*, rising in the east, is therefore near the *vernal equinox*; the ecliptic (with which the moon's orbit may, for the purpose of this discussion, be regarded as coincident) has its smallest inclination with the horizon. That half of the ecliptic which is above the horizon lies to the south of the equator. The daily motion of the moon along its orbit has therefore its minimum effect in delaying its rising (Fig. 72 *A*).

If the *ascending node* of the moon's orbit coincides with the first of Aries, then, when this node is rising, the moon's path will

lie still nearer the horizon than the ecliptic, and the phenomenon of the harvest moon will be especially noticeable.

191. Form of the Moon's Orbit. By observation of the moon's apparent diameter, combined with observations of its place in the sky, we can determine the *form of its orbit* around the earth in the same way as the form of the earth's orbit around the sun was worked out in section 158 (p. 136). The moon's apparent diameter ranges from 33′ 30″, when nearest, to 29′ 21″, when most remote.

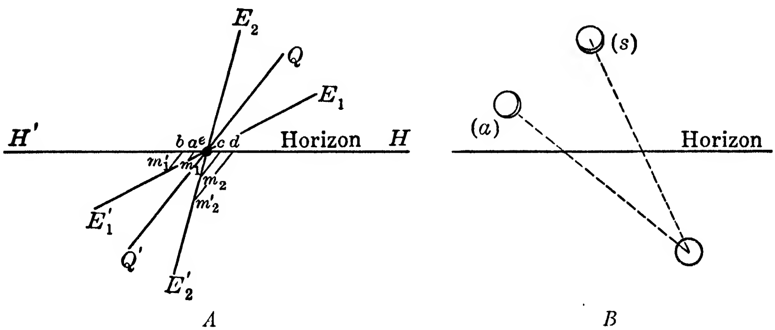


FIG. 72. The Harvest Moon (A) and the Crescent Moon (B)

In the diagram at the left we are looking toward the east. QQ' is the celestial equator, $E_1E'_1$ the moon's orbit (nearly coincident with the ecliptic) at the time of the full moon in September or early October. Suppose the moon rises at the east point e at 6 o'clock tonight. Tomorrow night at 6 o'clock it will be 12°.2 from e along its orbit at m_1. To reach the horizon it passes over the path m_1a (parallel to the equator), which is much shorter than m_1e. The moon will rise (latitude 40°) at about 6 : 22. The next night it rises from m'_1 at 6 o'clock, to b at about 6 : 44, and so on. In March, when the moon's orbit occupies the position $E_2E'_2$, the retardation of rising is a maximum. In the diagram at the right we are looking toward the west; the sun has set, but the crescent moon is still above the horizon. The line joining the cusps of the crescent is at right angles to the line from the moon to the sun (§ 184), since the sun illuminates the hemisphere of the moon which faces it. In spring (s) the moon "holds water," and sets late; in autumn (a) it does not "hold water," and sets early. Of course these configurations are of no value in predicting whether the weather will be rainy or fair. There is no scientific evidence that the moon influences either the weather or the growth of crops (§ 206)

The orbit turns out to be an ellipse like that of the earth around the sun, but of much greater eccentricity, averaging about 1/18. We say "averaging" because it varies from 1/15 to 1/23 on account of perturbations.

The point of the moon's orbit nearest the earth is called the *perigee* (περί + γῆ); that most remote, the *apogee* (ἀπό + γῆ). On account of perturbations the line of apsides is in continual

motion like the line of nodes, but it moves *eastward* instead of westward, completing its revolution in about nine years.

In its motion around the earth the moon also nearly observes the same "law of areas" that the earth does in its orbit around the sun.

192. Method of Determining the Size of the Moon's Orbit, that is, its Distance and Parallax. In the case of any heavenly body one of the first and most fundamental inquiries relates to its distance; until this has been measured we can get no knowledge of the real dimensions of its orbit, nor of the size, mass, etc. of the body itself. The problem is usually solved by measuring the parallactic displacement (§ 111) due to a known change in the position of the observer. Many methods are applicable in the case of the moon. We limit ourselves here to a single one, the simplest, though perhaps not the most accurate.

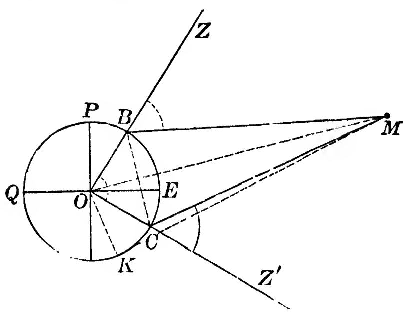


FIG. 73. Determination of the Moon's Parallax

At each of two observatories, B and C (Fig. 73), on or very nearly on the same meridian and very far apart (Berlin and Cape of Good Hope, for instance), the moon's zenith distance, ZBM and $Z'CM$, is observed simultaneously with the meridian circle. This gives, in the quadrilateral $BOCM$, the two angles OBM and OCM. The angle BOC, at the center of the earth, is the difference of the *geocentric* latitudes of the two observatories (numerically their sum). Moreover, the sides BO and CO are known, being radii of the earth.

The quadrilateral can therefore be solved by a simple trigonometrical process, and the *moon's distance* from the center of the earth, OM, found. When OM is determined, we at once find the *horizontal parallax* from the equation

$$\sin p_h = \sin OMK = \frac{OK}{OM} = \frac{r}{R}.$$

Knowledge of the law of the moon's orbital motion gives the ratio of the distance OM at this moment to the mean distance, which may be determined. The moon's parallax can also be deduced by means of occultations of stars observed at widely separated points on the earth and, most accurately of all, by gravitational theory (§ 309).

193. Parallax, Distance, and Velocity of the Moon. The moon's *equatorial horizontal parallax* (at mean distance) is found to be 57′ 2″.7, according to Brown, but it varies considerably from day to day on account of the eccentricity of the orbit.

The corresponding mean distance of the moon from the earth is 238,857 miles, or 384,403 kilometers, or 60.267 times the earth's equatorial radius. The distance ranges between 252,710 and 221,463 miles.

Knowing the size and form of the moon's orbit, we can easily compute the mean velocity of its motion. It averages 2287 miles an hour, or about 3350 feet per second. The mean angular velocity in the celestial sphere is about 33′ an hour, just a little greater than the apparent diameter of the moon itself.

194. Form of the Moon's Orbit with Reference to the Sun. While the moon moves in a small elliptical orbit around the earth, it also moves around the sun in company with the earth. This common motion of the moon and earth does not, of course, affect their relative motion, but to an observer outside the system, looking down upon moon and earth, the moon's motion around the earth would be a very small component of the moon's whole motion as seen by him.

The distance of the moon from the earth is only about 1/390 of the distance of the sun. The speed of the earth in its orbit around the sun is also more than thirty times as great as that of the moon around the earth; for the moon, therefore, the resulting path *in space* is one which deviates very slightly from the orbit of the earth and is *always concave toward the sun.*

If we represent the orbit of the earth by a circle with a radius of 100 inches, the moon will deviate from it by only one fourth of an inch on each side, crossing it twenty-four or twenty-five times in one revolution around the sun, that is, in a year.

195. Diameter, Area, and Volume of the Moon. The mean apparent diameter of the moon is 31′ 5″. Knowing its mean distance, we easily compute from this (§ 109) its real diameter, **2160 miles** (2159.86 ± 0.08 miles, or 3475.9 km., according to Ross). This is 0.273 of the earth's diameter, — somewhat more than one quarter.

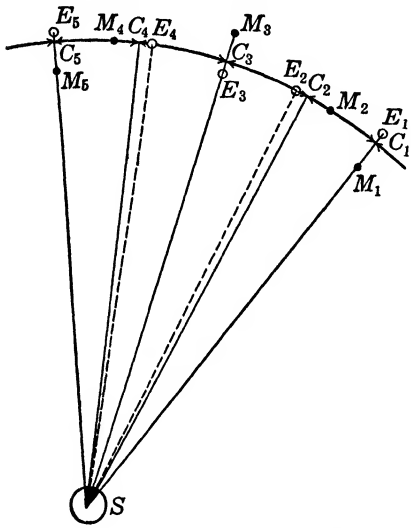


FIG. 74. The Common Center of Mass of Earth and Moon

E_1, $E_2, \cdots E_5$, are successive positions of the earth; $M_1 \cdots M_5$, of the moon; and $C_1 \cdots C_5$, of the center of mass of the two. C moves along the orbit in accordance with the law of areas, but E gets ahead at the moon's first quarter (E_2) and falls behind at the last quarter (E_4). By comparison of the observed longitudes of the sun with those calculated from the law of areas the distance CE can be found. The distances CM and CE (especially the latter) are much exaggerated in the figure. C is actually inside the earth

Since the surfaces of globes vary as the squares of their diameters, and their volumes as the cubes, this makes the *surface area* of the moon equal to 0.0744 (about 1/14) of the earth's, and the *volume*, or bulk, 0.0203 (almost exactly 1/49) of the earth's.

No other satellite is nearly so large as the moon in comparison with its primary planet. The earth and moon together, as seen from a distance, are really in many respects more like a *double planet* than a planet and satellite of ordinary proportions.

196. Mass, Density, and Superficial Gravity of the Moon. The accurate determination of the moon's *mass* is a very difficult problem. Though it is the nearest of all the heavenly bodies, it is far more difficult to weigh it than to determine the mass of Neptune, the remotest of the planets. There are two good methods of dealing with the problem.

(1) One consists in determining the position of the *center of gravity*, or *center of mass*, of earth and moon. It is this point, and not the earth's center, which describes around the sun what is called the "orbit of the earth." Now the earth and the moon

revolve together around this common center of gravity every month in orbits exactly alike in form but differing greatly in size, the earth's orbit being as much smaller than the moon's as its mass is greater.

The necessary result of this monthly motion of the earth's center is a "lunar equation," that is, a slight alternate eastward and westward displacement in the heavens of every object viewed from the earth as compared with the place the object would occupy if the earth had no such motion (Fig. 74). In the case of the stars or the remoter planets the displacement is not sensible, but this motion of the earth can be measured by observing through the month the apparent motion of the sun or, better, of one of the nearer planets, as Mars or Venus, or the recently discovered Eros, when nearest the earth.

From such observations it is found that the radius of the monthly orbit of the earth's center (that is, the distance from the earth's center to the common center of gravity of earth and moon) is 2880 miles. This is just about 1/82.5 of the distance from the earth to the moon, and by elementary principles of mechanics the conclusion follows that the *mass* of the moon is 1/81.5 that of the earth.

(2) The moon's mass may be found from the constants of precession and nutation; the mathematical analysis is difficult, but the results are accurate.

The most accurate determination of the moon's mass yet made is that derived by Hinks from observations of the small planet Eros, on the principle of method (1). He summarizes the results of various determinations of the ratio of the mass of the earth to that of the moon as follows:

Newcomb, from observations of the sun and planets, 81.48 ± 0.20
Newcomb, from the constants of precession and nutation, 81.62 ± 0.20
Gill, from observations of minor planets, 81.76 ± 0.12
Hinks, from observations of Eros, 81.53 ± 0.05
The weighted mean of these is **81.56 ± 0.04**.

Since the density of a body is equal to its mass divided by its volume, the density of the moon, compared with that of the earth, is found by dividing 1/81.56 by 0.0203. According to Ross the exact result is 0.6043 ± 0.0003 times the earth's den-

sity, or 3.33 ± 0.01 times that of water, almost exactly that of the basic rocks which underlie the thin surface crust of the earth. This is just what might be expected if the moon once formed a part of the earth and if, as is probable, the dense iron core remained with the larger mass when they separated.

The *superficial gravity*, or the attraction of the moon for bodies at its surface, is *mass* $\div$ (*radius*)^2, that is, 1/81.56 divided by $(0.273)^2$, and comes out about *one sixth* of gravity at the earth's surface. That is, a body weighing six pounds on the earth's surface would, at the surface of the moon, weigh only one pound (by a spring balance). A man who can throw a baseball 400 feet here would be able, on the moon, to throw it nearly half a mile.

This is a point that must be borne in mind in connection with the enormous scale of the surface structure of the moon. Volcanic forces on the moon would throw ejected materials to a vastly greater distance than on the earth (§ 149).

197. Rotation of the Moon. The moon rotates on its axis once a sidereal month, that is, *in exactly the same time as that occupied by its revolution around the earth*; *and it keeps the same side, almost exactly, always toward the earth.* We see today the same aspect of the moon as Galileo did in the days when he first turned his telescope upon it.

Many find difficulty in seeing why a motion of this sort should be called a rotation of the moon, since it is much like the motion of a ball fixed on a revolving crank (Fig. 75). "Such a ball," they say, "revolves around the shaft but does not rotate on its own axis." It does rotate, however; for if we mark one side of the ball, we shall find the marked side presented successively to every point of the compass as the crank turns, so that the ball turns on its own axis as really as if it were whirling upon a pin fastened to the table.

By virtue of its connection with the crank the ball has two distinct motions: (1) the *motion of translation*, which carries its center in a circle around the axis of the shaft; (2) an additional *motion of rotation* [1] around a line drawn through its center parallel to the shaft. The pin *A* (in the figure) and the hole in which it fits both rotate at the same rate, so that the ball, while it turns on its axis (an imaginary line), *does not turn on the pin, nor the pin in the hole.*

[1] Rotation consists essentially in this: that a line connecting any two points, and not parallel to the axis of the rotating body, will sweep out a circle on the celestial sphere if produced to it.

198. Geometrical Librations. While in the long run the moon keeps the same face toward the earth, this is not so in the short run; there is no crank connection between the earth and the moon, and the moon in different parts of a single month does not keep *exactly* the same face toward the earth, but rotates with perfect independence of her orbital motion. With reference to the center of the earth the moon's face is continually oscillating slightly, and these oscillations constitute what are called *librations*, (discovered by Galileo). We distinguish three, namely, the libration in *latitude*, the libration in *longitude*, and the *diurnal* libration.

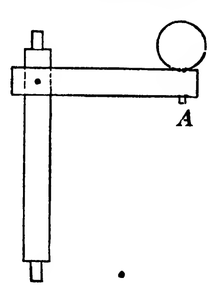


FIG. 75. The Moon's Rotation

(1) The *libration in latitude* is due to the fact that the moon's equator does not coincide with the plane of its orbit, but makes with it an angle of about $6\frac{1}{2}°$. This inclination of the moon's equator causes its north pole at one time in the month to be tipped $6\frac{1}{2}°$ toward the earth, while a fortnight later the south pole is similarly inclined to us, just as the north and south poles of the earth are alternately, for periods of six months, presented to the sun, causing the seasons.

(2) The *libration in longitude* depends on the fact that the moon's angular motion in its elliptical orbit is *variable*, while the motion of the rotation is *uniform*, like that of any other undisturbed body; the two motions, therefore, do not keep pace exactly during the month, and we see alternately a few degrees around the eastern edge and around the western edge of the lunar globe. This libration amounts to about $7\frac{3}{4}°$ each way.

(3) The *diurnal libration.* Again, when the moon is rising we look over its upper edge, which is then its *western* edge, seeing a little more of that part of the moon than if we were observing it from the center of the earth; and vice versa when it is setting. This constitutes the so-called *diurnal libration* and amounts to about one degree. Strictly speaking, this diurnal libration is not a libration of the moon but of the observer. The telescopic effect is the same, however, as that of a true libration.

On the whole, taking all three librations into account, we see considerably more than half the moon, the portion that never disappears being about *41 per cent* of the moon's surface; that never visible, also *41 per cent*, while that which is alternately visible and invisible is *18 per cent.*

199. Physical Libration. Besides these geometrical librations, which arise from the lack of uniformity of the motion of the observer relatively to the

moon, there is a minute *physical libration* due to real irregularities in its rotation. The moon is not spherical, the greatest diameter being directed toward the earth, while the equatorial diameter at right angles to this is shorter, and the polar diameter shorter still. On account of the geometrical librations the longest diameter does not always point directly toward the earth. The attraction of the earth on this protuberance swings the moon a little in various directions. The maximum deviation from a mean position is about a mile on the moon's surface, but can be detected by careful observations. From Hayn's discussion it appears that the equatorial diameters of the moon differ by about one third of a mile, while the polar diameter is fully a mile less than the shorter of these.

The exact long-run agreement between the moon's time of rotation and its orbital revolution cannot be accidental. If the moon were ever plastic, the earth's attraction must necessarily have produced a huge tidal bulge upon its surface, and the effect would have been ultimately to force an agreement between the lunar sidereal day and the sidereal month. The subject is resumed in connection with tidal evolution (§ 359).

PHYSICAL CHARACTERISTICS OF THE MOON

200. The Moon's Atmosphere. The moon has no atmosphere, or, at least, if any exists, it probably produces a barometric pressure not more than 1/100,000 of the atmospheric pressure at the earth's surface. The principal evidence on this point is found in the *telescopic appearance.* The parts of the moon near the edge of the disk, or limb, which, if there were any atmosphere, would be seen through its greatest possible depth, are visible without the least obscuration. There is no haze, and all the shadows are perfectly black; there is no evidence of clouds or storms, or of anything like the ordinary phenomena of the terrestrial atmosphere.

Most important of all, there is no twilight at the cusps of the crescent. An atmosphere 10,000 times thinner than the earth's, illuminated by full sunlight, would be more conspicuous than the dark part of the moon when lighted by the full earth. This test is therefore a very delicate one.

There is also *no evidence of refraction* at the moon's limb when the *moon intervenes between us and any more distant object.* At an eclipse of the sun there is no distortion of the sun's limb where

the moon cuts it. Further evidence of this sort comes from occultations of the stars. The star retains its full brightness in the field of the telescope until, all at once, without the least warning, it simply is not there, the disappearance generally being absolutely instantaneous. Its reappearance at the dark limb is of the same sort, and still more startling. Now if the moon had any perceptible atmosphere (or the star any sensible diameter) the disappearance would be gradual. The star-image would change color, become distorted, and fade away more or less gradually.

201. What Has Become of the Moon's Atmosphere? If the moon ever formed a part of the same mass as the earth, she probably once had an atmosphere. Its disappearance is explained on the basis of the kinetic theory of gases, according to which the molecules of a gas are continually flying in all directions with high velocities, colliding with one another and rebounding like perfectly elastic spheres. The mean-square velocity of the molecules (that is, the velocity whose square is equal to the mean of the squares of the individual velocities) varies inversely as the square root of the molecular weight of the gas, and directly as the square root of the absolute temperature. The value of this velocity at 0° centigrade is 1.84 km./sec. for hydrogen, 1.31 for helium, 0.62 for water vapor, 0.49 for nitrogen, 0.46 for oxygen, and 0.39 for carbon dioxide. At 100° C. these velocities are increased by 17 per cent.

Now, at any given distance from a body there is a so-called parabolic velocity, or velocity of escape, depending on the mass of the body (§ 314); and if a particle at this distance has a velocity, relative to the body, which is greater than the velocity of escape, it cannot be retained by the gravitational attraction but will fly off into space. At the surface of the earth the velocity of escape is 11.188 km./sec.; at the moon's surface, only 2.38 km./sec.; and at the sun's, 617 km./sec.

Even if the mean velocity of the molecules is considerably less than the parabolic velocity, the atmosphere will be gradually lost by the escape of fast-moving molecules from its extreme upper regions, where the free paths of the molecules are so long that they stand a chance of getting away without being stopped by collisions. It appears from the calculations of Jeans that if

the mean molecular velocity is one third of the velocity of escape, the atmosphere will be reduced to one half its original amount in a few weeks. For one fourth the velocity of escape, the corresponding time is several thousand years; for one fifth, it is hundreds of millions of years.

It follows that all known gases, even hydrogen, should be retained by the earth, and, a fortiori, by the sun, for a practically indefinite period. From the moon, on the other hand, hydrogen and helium would diffuse away at once; water vapor would go more slowly, but would disappear entirely in a (geologically speaking) very short time. At a temperature of 100° C., oxygen and nitrogen would slowly but steadily escape; and if ever in its history the moon was really hot, it must have lost the heavier gases as well.

This theory is evidently applicable to any heavenly body for which we can compute the velocity of escape, and the results of its application to the planets are also in accord with the observed facts. The earth's atmosphere contains a small proportion of helium.

202. Water on the Moon's Surface. Observations with the naked eye indicate that there can be no oceans, or even lakes of any size, in the equatorial regions of the moon; for if any such bodies of standing water existed, they would be at times in such a position as to reflect the sunlight to us. Such a reflection would be very conspicuous if it existed, for it is a matter of everyday observation that the reflection of sunlight from water is by far the most brilliant object in a landscape.

If there is no atmosphere on the moon, there can of course be no water on its surface, or even any moisture in the ground, for it would immediately evaporate and form an atmosphere of water vapor. Any free water which the moon ever possessed must have evaporated in this fashion and escaped into space, molecule by molecule, as described above. Water of hydration may, however, be present as a chemical constituent of the rocks.

203. The Moon's Light. As to *quality* the moon's light is simply sunlight, showing a spectrum identical in every detail with that of light coming directly from the sun itself.

The *brightness*, compared with that of sunlight, is difficult to measure accurately, and different investigators have found results for the ratio of sunlight to full moonlight ranging all the way from 375,000 to 630,000. The mean of the best determinations is 465,000, with a probable error of about 10 per cent. According to this, if the whole visible hemisphere of sky were packed with full moons, we should receive from it about one fifth of the light of the sun. The light of the full moon varies nearly 30 per cent with the changes in its distance. In comparison with artificial standards it is found that the light of the full moon is about one quarter as bright as that of a standard candle at a distance of one meter, or, in other words, that the intensity of full moonlight is 0.24 meter-candle (§ 568).

Photographically, full moonlight is only about 1/650,000 as bright as sunlight, which indicates that the moon's surface is yellowish.

After full moon the light falls off rapidly, the mean results of several observers being as follows:

Elongation	180°	160°	140°	120°	100°	80°	60°	40°	20°
Light	100	65	41	26	15	7.5	3.2	1.0	0.1

The waxing moon, shortly after the first quarter, is 20 per cent brighter than the waning moon at the corresponding phase before the third quarter, obviously because the region illuminated by the sun in the latter case contains more of the dark areas conspicuous to the eye.

The half-moon, though apparently of half the area of the full moon, is only one ninth as bright. Part of this difference arises from the fact that in the region near the terminator of the half moon the sun's rays strike the surface very obliquely, and therefore illuminate it feebly; but most of it must be due to the rough character of the lunar surface, which causes it to be more or less darkened, except at the full, by the shadows cast by its own irregularities. The shadows of the mountains which are visible with the telescope are probably of less importance than those of innumerable small irregularities, perhaps no bigger than bowlders or even pebbles. A homely illustration of the same principle is that a broken road of rough but white snow appears darker than

the surrounding smooth snow if one looks toward the sun, and brighter if one looks the other way.

204. The Albedo of the Moon. The *albedo* of a spherical body may be defined, for astronomical purposes, as the ratio of the total amount of sunlight reflected from the body, *in all directions*, to the amount that falls upon the body. It therefore cannot be determined accurately from observations at the full phase alone. The average albedo, or reflecting power, of the moon's surface, is 0.073; that is, the moon reflects only a little over 7 per cent of the sun's light, the remainder being absorbed and going to heat the surface. If the irregularities of the surface could be smoothed out, so that it would no longer be darkened by their shadows, the reflecting power would be a little greater, — perhaps as much as 10 per cent. This is comparable with the albedo of rather dark-colored rocks, so that it appears that the moon's surface is of a rather dark brown (*brown* rather than *gray*, because of its still lower photographic albedo). There are, however, great variations in the reflecting power of different portions of the moon's surface, some spots being probably as bright as white sand, and others as dark as slate.

205. Heat of the Moon and Temperature of the Surface. For a long time it was impossible to detect the moon's heat by observation, but with modern apparatus it is easy to detect and to measure.

A considerable percentage of the lunar heat is heat simply reflected like light, while the rest, about four fifths of the whole, is obscure heat, that is, heat which has first been absorbed by the moon's surface and then radiated, like the heat from a brick surface that has been warmed by sunshine. This is shown by the fact that a water-cell cuts off a large percentage of the moon's heat (§ 618).

The lunar rocks are exposed to the sun's rays in a cloudless sky for fourteen days at a time, and must become very hot.

Dietzius has recently calculated that the temperature at the moon's equator should rise to about 110° C. at noon, drop to about – 10° C. at sunset, reach a minimum of – 80° C. after the surface has cooled off during the two weeks of night, and rise abruptly as the sunlight returns. It would fall much lower if it were not for the heat stored in the rocks during the preceding

lunar day. Observation (§ 618) of the moon's heat confirms the high midday temperature, placing it at about 120° C.

206. Lunar Influences on the Earth. The moon's *attraction* coöperates with that of the sun in producing the *tides* (§ 345) and slight changes in the pressure of the atmosphere.

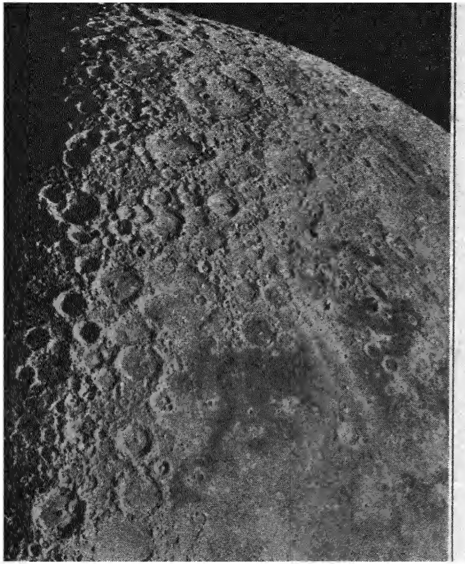
There are also certain distinctly ascertained disturbances of terrestrial magnetism connected with the approach and recession of the moon at perigee and apogee, but this ends the list of ascertained lunar influences.

The multitude of current beliefs as to the controlling influence of the moon's phases and changes upon the weather and the various conditions of life are unfounded, or at least unverified. It is quite certain that if the moon has any influence at all of the sort imagined, it is extremely slight, — so slight that it has not yet been demonstrated, though numerous investigations have been made expressly for the purpose of detecting it. It is not certain, for instance, whether it is warmer or not, on the average, or less cloudy or not, at the time of full moon.

207. The Moon's Telescopic Appearance and Surface. Even to the naked eye the moon is a beautiful object, diversified with markings which are associated with numerous popular myths. In a powerful telescope most of these markings vanish and are replaced by a multitude of smaller details which make the moon, on the whole, the finest of all telescopic objects, — especially so for instruments of moderate size (say from 6 to 10 inches in diameter), which generally give a more pleasing view of our satellite than instruments either much larger or much smaller.

An instrument of this size, with magnifying powers between 250 and 500, brings the moon optically within a distance ranging from 1000 to 500 miles; and since an object half a mile in diameter on the moon subtends an angle of about $0''.43$, it would be distinctly visible. A long line, or streak, even less than a quarter of a mile across can probably be seen. With larger telescopes the power can now and then be carried very much higher, and correspondingly smaller details made out, *when the seeing is at its best*, not otherwise.

For most purposes the best time to look at the moon is when it is between six and ten days old. At the time of full moon few



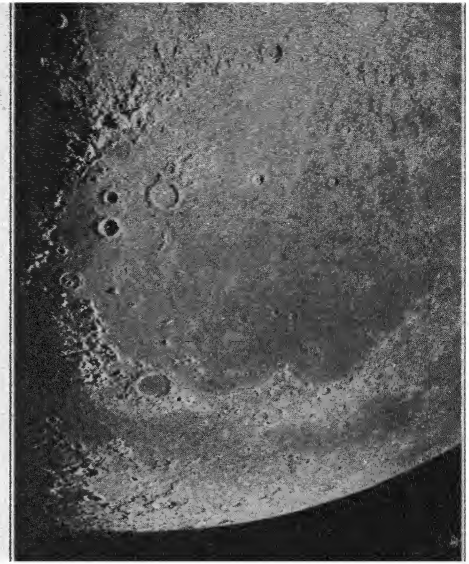


FIG. 76. Southern Portion of the Moon at Last Quarter

This photograph, as well as the one at the right, was taken with the 100-inch reflector. The region around the south pole is very rough. Tycho is a little above the center. The crater in the lower left corner, with an isolated mountain in the center, is Albategnius, sixty-four miles in diameter. Note the "straight wall" (No. 9, Fig. 80). (From photograph by Mt. Wilson Observatory)

FIG. 77. Northern Portion of the Moon at Last Quarter

In the upper right corner is Copernicus; at the left is Eratosthenes, at the end of the Apennines. The Alps, on the left below the center, continue the boundary of the great plain known as Mare Imbrium. It is sunset along the terminator, and the high peaks throw long shadows. (South is at the top, following the usual astronomical convention.) (From photograph by Mt. Wilson Observatory)

objects on the surface are well seen, as there are then no shadows to give relief.

It is evident that while with the telescope we should be able to see such objects as lakes, rivers, forests, and great cities, if they existed on the moon, it would be hopeless to expect to distinguish any of the minor indications of life, such as buildings (except perhaps the very largest) or roads.

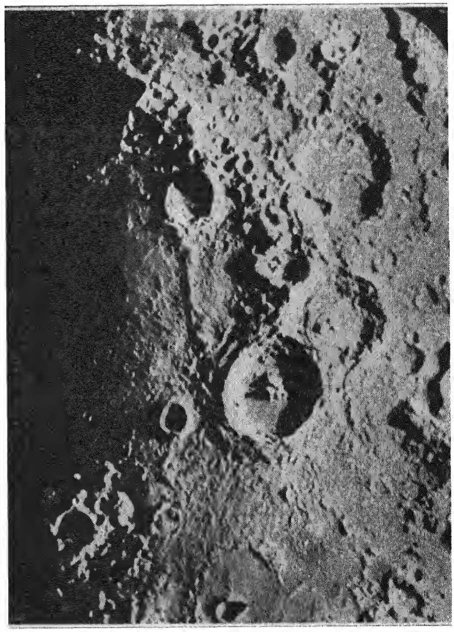


FIG. 78. Catharina, Cyrillus, and Theophilus

The great crater just below the center is Theophilus. It is 64 miles in diameter, and the surrounding wall rises nearly 19,000 feet above the interior. The central mountain is 5000 or 6000 feet high. The floor lies several thousand feet below the level of the surrounding plain. (From photograph by Yerkes Observatory)

208. The Moon's Surface Structure. The moon's surface is for the most part extremely broken. On earth the mountains are mostly in long ranges, like the Andes and Himalayas. On the moon the ranges are few in number; but, on the other hand, the surface is pitted all over with great *craters*, which closely resemble the volcanic craters on the earth's surface, though on an immensely larger scale. The largest terrestrial craters do not exceed 6 or 7 miles in diameter; many of those on the moon are 50 or 60 miles across, and some have a diameter of more than 100 miles, while smaller ones from 5 to 20 miles in diameter are counted by the hundred.

A typical lunar crater is nearly circular; the circumference is formed by a ring of mountains which rise anywhere from 1000 to 20,000 feet above the surrounding country. The floor within the ring may be either above or below the outside level; some craters

are deep, and some are filled nearly to the brim. In a few cases the surrounding mountain ring is entirely absent, and the crater is a mere hole in the plain. Frequently in the center of the crater there rises a group of peaks which attain about the same elevation as the encircling ring, and these central peaks sometimes show holes or craters in their summits.

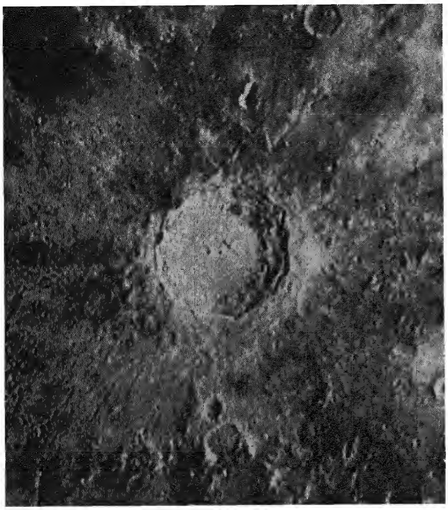


FIG. 79. Copernicus

Enlarged from a photograph made at the 134-foot Cassegrain focus (§ 61) of the 100-inch telescope of the Mt. Wilson Observatory. This crater is remarkable for the number of surrounding ridges and for the craterlets with which the neighboring region is thickly sown

On certain portions of the moon these craters stand very close together; older craters have been encroached upon, or more or less completely obliterated, by the newer, so that the whole surface is a chaos of which the counterpart is hardly to be found on the earth, even in the roughest portions of the Alps. This is especially the case near the moon's south pole. It is not surprising, since the forces of denudation and erosion which are continually wearing down the surface features of the earth are not operative on the moon.

The *height of a lunar mountain* or depth of a crater can be measured with considerable accuracy by means of its shadow, or, in the case of a mountain, by the measured distance between its summit and the terminator at the time when the top first catches the light, looking like a star quite detached from the bright part of the moon (Fig. 70).

209. The striking resemblance of these formations to terrestrial volcanic structures, like those exemplified by Vesuvius and

others, makes it natural to assume that they had a similar origin. This, however, is not absolutely certain, for there are considerable difficulties in the way, especially in the case of what are called the great Bulwark Plains. These lunar plains are so extensive that a person standing in the center could not see even the summit of the surrounding ring at any point; and yet there is no line of discrimination between them and the smaller craters; the series is continuous.

It is obvious, that if these lunar craters are the result of volcanic eruptions, they must be ancient formations, for it is quite certain that there is *no evidence of present volcanic activity.* The great plains, or *maria*, appear in some places to have invaded craters and broken down their walls; and it has been suggested that they were once actual seas of lava which melted their way into the adjacent mountains, the scattered craters within them being of subsequent formation.

210. Other Lunar Formations. The craters and mountains are not the only interesting formations on the moon's surface. There are a few long, straight lines of cliff of moderate height, which are evidently fault scarps, produced by motion along a crack in the crust, and not worn down by erosion like the thousands of similar features on the earth (Fig. 80, No. 9). There are many deep, narrow, crooked valleys called rills. Then there are numerous straight clefts, half a mile or so wide and of unknown depth, running in some cases several hundred miles, straight through mountain and valley, without any apparent regard for the accidents of the surface. They seem to be deep cracks in the crust of our satellite. Most curious of all are the light-colored streaks, or rays, which radiate from certain of the craters, extending in some cases a distance of many hundred miles. These are usually from 5 to 10 miles wide and neither elevated nor depressed to any considerable extent with reference to the general surface. Like the clefts, they pass across valley and mountain, and sometimes through craters, without any change in width or color. They have been doubtfully explained as a staining of the surface by vapors ascending from rifts too narrow to be visible.

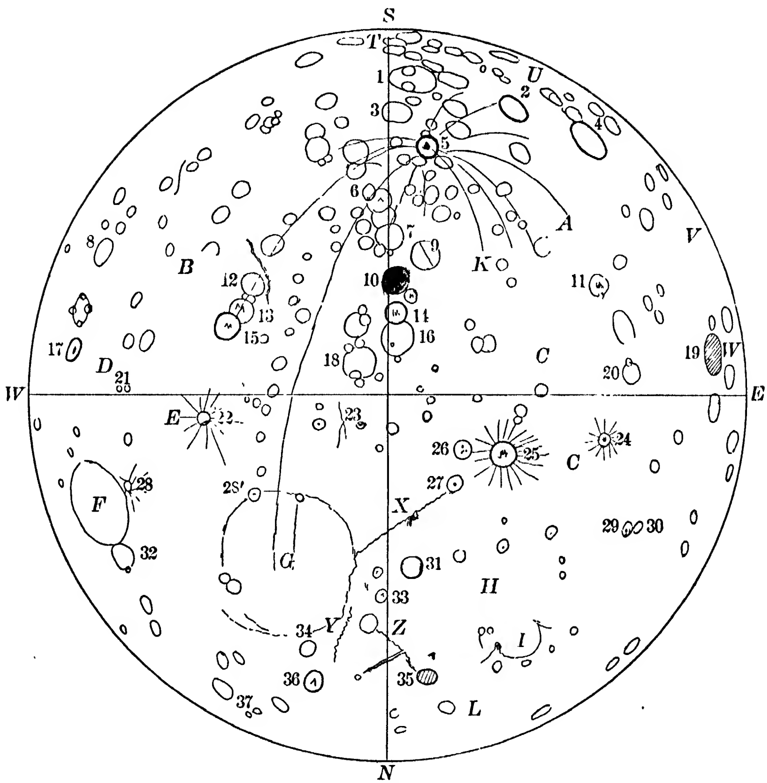


FIG. 80. Map of the Moon. (Reduced from Neison)

Key to the principal objects indicated in Fig. 80: *A*, Mare Humorum; *B*, Mare Nectaris; *C*, Oceanus Procellarum; *D*, Mare Fecunditatis; *E*, Mare Tranquilitatis; *F*, Mare Crisium; *G*, Mare Serenitatis; *H*, Mare Imbrium; *I*, Sinus Iridum; *K*, Mare Nubium; *L*, Mare Frigoris; *T*, Leibnitz Mountains; *U*, Doerfel Mountains; *V*, Rook Mountains; *W*, D'Alembert Mountains; *X*, Apennines; *Y*, Caucasus; *Z*, Alps. 1, Clavius; 2, Schiller; 3, Maginus; 4, Schickard; 5, Tycho; 6, Walther; 7, Purbach; 8, Petavius; 9, The "Straight Wall"; 10, Arzachel; 11, Gassendi; 12, Catherina; 13, Cyrillus; 14, Alphonsus; 15, Theophilus; 16, Ptolemy; 17, Langrenus; 18, Hipparchus; 19, Grimaldi; 20, Flamsteed; 21, Messier; 22, Maskelyne; 23, Triesnecker; 24, Kepler; 25, Copernicus; 26, Stadius; 27, Eratosthenes; 28, Proclus; 28′, Pliny; 29, Aristarchus; 30, Herodotus; 31, Archimedes; 32, Cleomedes; 33, Aristillus; 34, Eudoxus; 35, Plato; 36, Aristotle; 37, Endymion

The most remarkable of these ray systems is the one connected with the great crater Tycho, not very far from the moon's south pole, well shown in Fig. 81, which is a nearly full-moon photograph. The rays are not very conspicuous until within a few

FIG. 81. The Nearly Full Moon

With the surface in full sunlight there are no shadows to show the elevations, only a contrast in brightness. Mountains and craters are brighter than the great plains. Aristarchus is the brightest feature and Grimaldi the darkest. The bright rays diverging from Tycho extend far round the globe; those about Copernicus are more intricate. (Photographed at the Yerkes Observatory by Slocum)

days of full moon, but at that time they and the crater from which they diverge constitute by far the most striking feature of the whole lunar surface.

211. Lunar Photography. The moon was first successfully photographed by Bond in 1850, by means of the old daguerreotype process. Good results were obtained later by De la Rue in England and by Draper and Rutherfurd in this country; and further great advances have been made since 1890.

The best photographs of the moon are obtained with large telescopes of long focus; these give images which are large, so that the small details are not obscured by the grain of the plate, and at the same time bright, so that the exposures are short and the influence of atmospheric tremors (§ 118) is at a minimum. Magnificent negatives have been secured at many observatories, which bear enlargement to several times their original diameter, and show all but the very finest details as well as they can be seen directly with the telescope. Atlases of the moon, consisting of enlargements of such photographs, have been published by the Paris, Lick, and Harvard observatories.

The photographs have the further advantage that the positions of all the well-defined lunar markings may be measured upon them with great accuracy. Saunder and Franz have in this way determined the positions of more than 3000 points, with an average probable error of $\pm 0''.15$, — which corresponds to only 1000 feet on the moon's surface. By comparing photographs taken at widely different librations it is possible to find the absolute altitudes of the measured points above or below a mean sphere. In this way Saunder found altitudes for 34 points, varying from 16,000 feet below the mean sphere to 9000 feet above it, and concluded that the moon is not perfectly spherical, but that the radius which extends directly toward the earth is some 3000 feet longer than the polar radius.

212. Changes on the Moon. It is certain that there are no conspicuous changes; there are no such transformations as would be presented by the earth viewed telescopically, — no clouds, no storms, no snow of winter, and no spread of vegetation in the spring. At the same time it is confidently maintained by some observers that here and there alterations do take place in details of the lunar surface, while others as stoutly dispute it.

The difficulty in settling the question arises from the great changes in the appearance of a lunar object under varying illumination. To insure certainty in such delicate observations, comparisons must be made between the appearance of the object in question, as seen at *precisely the same phase of the moon*, with telescopes (and eyes too) of equal power, and under substantially

the same conditions in other respects, such as the height of the moon above the horizon and the clearness and steadiness of the air. It is, of course, very difficult to secure such identity of conditions. The disputed question whether short-lived changes, dependent on the phase of illumination, actually occur is obviously still more difficult to settle. No larger changes, such as might be caused by volcanic eruptions or landslides, have been detected since the advent of photography. The earlier drawings are not accurate enough to be good evidence.

213. Lunar Nomenclature. The great plains on the moon's surface were called by Galileo seas (*maria*), for he supposed that these grayish surfaces, which are visible to the naked eye and conspicuous in a small telescope, though not with a large one, might be covered with water.

Most of the ten *mountain ranges* on the moon are named after terrestrial mountains, as Caucasus, Alps, Apennines, though two or three bear the names of astronomers, like Leibnitz, Doerfel, etc.

The conspicuous *craters* bear the names of eminent ancient and medieval astronomers and philosophers, as Plato, Archimedes, Tycho, Copernicus, Kepler, and Gassendi, while hundreds of smaller and less conspicuous formations bear the names of more modern astronomers.

This system of nomenclature seems to have originated with Riccioli, who in 1650 made the first map of the moon.

214. Fig. 80 is reduced from a skeleton map of the moon by Neison and, though not large enough to exhibit much detail, will enable a student with a small telescope to identify the principal objects by the help of the key. (Compare with Fig. 81.)

215. Lunar Maps. A number of maps of the moon have been constructed by different observers. A small map, such as Fig. 80 or the one in Schurig's *Himmels-Atlas*, is convenient for the identification of the formations. For precise purposes large-scale photographs completely supplant the older drawings. Our maps of the visible part of the moon are on the whole as complete and accurate as our maps of the earth, taking into account the polar regions and the interior of the continents of Asia and Africa.

216. By photographing the moon with light of different colors, information can be obtained concerning the nature of the surface. The photographs made by R. W. Wood with color screens and plates sensitive to various regions of the spectrum show decided differences in details of the lunar surface. The most conspicuous difference is in a spot close to the crater

Aristarchus, which is very dark in a photograph taken with ultra-violet light, darkish in one taken with the violet, and invisible in one taken with orange light.

If a thin deposit of sulphur is made on a piece of volcanic rock and photographed with the same three kinds of light, the phenomena can be almost exactly reproduced. It seems probable, therefore, that the spot near Aristarchus is composed of rocks stained with sulphur, — a plausible hypothesis in view of the fact that sulphur is a frequent constituent of terrestrial volcanic ejecta.

EXERCISES

1. If the moon's sidereal period were 60 days, what would be its synodic period?

Ans. 71.7932 days.

2. In that case, what would be the mean interval between its meridian transits?

Ans. $24^h\ 20^m.34$.

3. Does the moon rise every day?

4. If the moon rises at $11^h\ 45^m$ P.M. on Wednesday, when (approximately) will it rise next?

5. What is the lowest latitude on the earth where the moon can remain above the horizon for 48 consecutive hours?

Ans. $90° - (23°\ 27' + 5°\ 8') = 61°\ 25'$.

6. At what time of the year does the full moon remain longest above the horizon?

7. How many times does the moon turn on its axis in a year?

8. Does the earth rise and set for an observer on the moon?

9. What determines the direction of the horns of the crescent moon?

10. Can a star ever be seen between the horns of the moon?

11. What point describes the "orbit of the earth" around the sun?

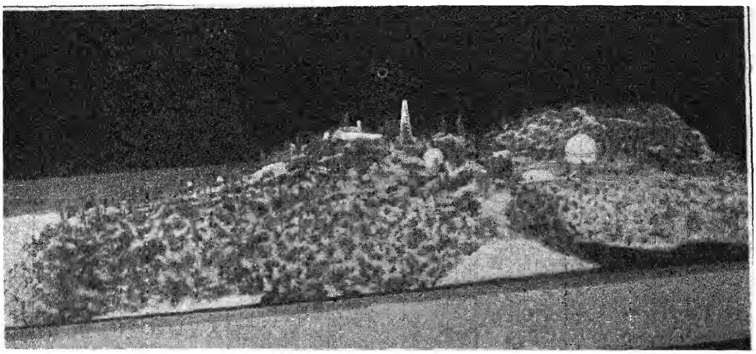
12. What may the elongation of the moon be when it is in opposition? in conjunction? in quadrature?

Ans. From 175° to 180°;
from 0° to 5°;
exactly 90°.

13. When can the crescent moon be observed with its horns turned down?

14. Under what circumstances is the retardation of rising of the harvest moon the least possible?

Ans. The moon must be simultaneously at the vernal equinox, at the ascending node, and in apogee.



Mt. Wilson Observatory

From a model showing how the instruments are scattered on the forested top of the mountain. The vegetation is carefully preserved to avoid heating of the soil and resulting air currents

CHAPTER VII

THE SUN

ITS DISTANCE, DIMENSIONS, MASS, AND DENSITY · ITS ROTATION AND EQUATORIAL ACCELERATION · METHODS OF STUDYING ITS SURFACE · THE PHOTOSPHERE · SUN-SPOTS · THEIR NATURE, DIMENSIONS, DEVELOPMENT, AND MOTIONS · THEIR DISTRIBUTION AND PERIODICITY

The sun is the nearest of the *stars*, — a hot, self-luminous globe. Though probably only of medium size compared with other stars, it is enormous as compared with the earth and the moon; and to the earth and the other planets which circle around it, it is the most magnificent and important of all the heavenly bodies. Its attraction controls their motions, and its rays supply the energy which maintains nearly every form of activity upon their surfaces.

217. Solar Parallax and the Distance of the Sun; the Astronomical Unit. The problem of finding accurately the sun's distance is one of the most important and difficult presented by astronomy, — important because this distance (that is, the radius of the earth's orbit) is the fundamental unit to which all measurements of celestial distance are referred; difficult because the measurements which determine it are so delicate that any minute error of observation is enormously magnified in the result.

Without a knowledge of the sun's distance we cannot form any idea of its real dimensions, mass, and density, and the tremendous scale of solar phenomena.

The *solar parallax*, or the sun's mean equatorial horizontal parallax, is a constant obtained from observation and is a measure of the sun's mean distance in terms of the earth's equatorial radius (§ 113). The mean distance (§ 159) derived from the parallax is known as the *astronomical unit* and is the semi-major axis of the earth's orbit, — the average of the greatest and least distance from the earth to the sun. It is often more convenient in astronomical calculations to use the parallax as a measure of distance.

218. Methods of Determining the Solar Parallax and the Sun's Distance. There are several very different ways of attacking this problem. We may classify under three general heads the methods that are capable of giving reliable results, simply referring here to the sections in which they are explained.

(1) Geometrical methods, which involve the direct measurement of the parallax of some planet or asteroid whose distance is known in terms of that of the sun (§ 287).

(2) Gravitational methods, which involve essentially the determination, from perturbations, of the ratio of the mass of the earth to that of the sun (§ 327).

(3) Methods depending upon the velocity of light, which give, directly, the earth's orbital velocity and the radius of the orbit (§ 164).

The best methods of each of the three classes yield very precise results.

VALUES OF THE SOLAR PARALLAX

METHOD	PARALLAX	PROBABLE ERROR
GEOMETRICAL METHOD		
Heliometer Observations of Asteroids, 1889–1890 (Gill) (§ 420)	8″.802	± 0″.005
Visual Observations of Eros, 1900–1901 (Hinks) (§ 420)	8″.806	± 0″.004
Photographic Observations of Eros, 1900–1901 (Hinks) (§ 420)	8″.807	± 0″.0027
Photographic Observations of Mars, 1924 (Jones and Halm) (§ 287)	8″.809	± 0″.005
GRAVITATIONAL METHOD		
Parallactic Inequality of the Moon's Motion, 1924 (Jones) (§ 337)	8″.805	± 0″.005
Perturbations of Eros, 1921 (Noteboom) (§§ 420, 327)	8″.799	± 0″.001
VELOCITY OF LIGHT		
Radial Velocities of Stars, 1912 (Hough) (§ 732)	8″.802	± 0″.004

The general mean of all these determinations is **8″.803 ± 0.001.**

Other methods give closely accordant values. The results by the aberrational method (§ 164) are not included here, because this method necessarily involves the comparison of observations

at opposite seasons of the year and is liable to systematic errors peculiar to the seasons.

The mutual agreement of the values arrived at by very different methods (none differing from the mean value by more than 1 in 1500) is very striking.

With Hayford's value of the radius of the earth (§ 138) the solar parallax corresponds to a mean distance of

149,450,000 ± 17,000 kilometers, or 92,870,000 miles.

The value $8''.80$ of the solar parallax has been used, by international agreement, in all the Nautical Almanacs since 1900. This value is evidently so near the truth that there is no likelihood that, for almanac purposes, it will ever have to be changed.

The actual distance of the earth from the sun varies, on account of the eccentricity of the earth's orbit, to the extent of about 3,000,000 miles. The distance is least at the beginning of the year and greatest early in July.

The mean *orbital velocity* of the earth, found by dividing the circumference of the orbit by the number of seconds in a year, is $18\frac{1}{2}$ miles a second, as already determined by aberration (§ 164). (Compare this velocity with that of a cannon shot, which seldom exceeds 3500 feet per second.)

Perhaps one of the most impressive illustrations of the distance of the sun is that such a shot would require four and a half years to reach the sun, traveling without change of speed. A railroad train running at 60 miles an hour, without stopping or slackening, would require 175 years, and the fare one way, at four cents a mile, would be $3,720,000. *Light* makes the journey in 499 seconds.

219. Dimensions of the Sun. The sun's *mean apparent diameter* is $31'\ 59''.3 \pm 0''.1$ (according to G. F. Auwers). Since at the distance of the sun one second of arc corresponds to 450.23 miles (92,870,000 ÷ 206,265), its real diameter is 864,100 miles (1,390,600 kilometers), or 109.1 times the mean diameter of the earth, or 1/107.5 of the mean distance.

If we suppose the sun to be hollowed out, and the earth placed at the center, the sun's surface would be 432,000 miles away. Now, since the distance of the moon from the earth is about 239,000 miles, it would be only a little more than halfway out from the earth to the inner surface of the hollow globe, which would thus form a sky background for the study of

the lunar motions. Fig. 82 illustrates the size of the sun, and of such objects upon it as the sun-spots and prominences, compared with the size of the earth and the moon's orbit.

If we represent the sun by a globe 2 feet in diameter, the earth on the same scale would be 0.22 of an inch in diameter, the size of a very small pea, at a distance from the sun of 215 feet; *and the nearest star, still on the same scale, would be 11,000 miles away.*

Since the *surfaces* of globes are proportional to the *squares* of their radii, the surface of the sun exceeds that of the earth in the

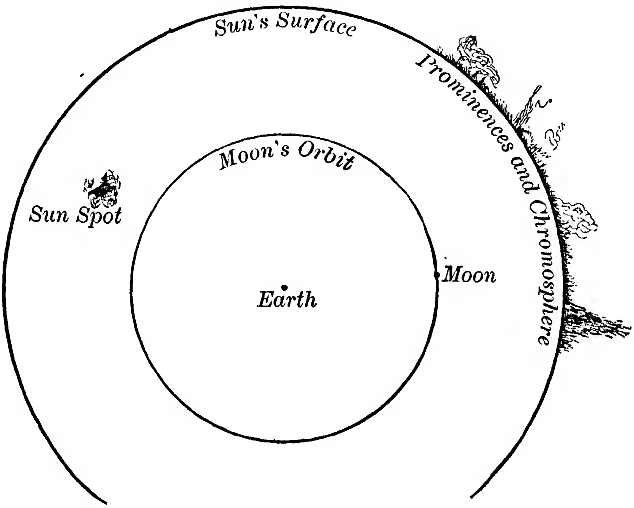


FIG. 82. Size of the Sun compared with that of the Moon's Orbit

ratio of $109.1^2 : 1$; that is, the area of the sun's surface is about 12,000 times that of the surface of the earth, or 6.075×10^{12} km.^{2}

The *volumes* of the spheres are proportional to the cubes of their radii. Hence, the sun's volume (or bulk) is 109.1^3, or 1,300,000 times that of the earth.

220. The sun's mass, like that of any other heavenly body, can be determined only by means of the effects of its gravitational attraction on some other body. It is obviously impossible to hold such a body fixed and measure the force with which the sun attracts it, but the acceleration in its motion (§ 149), produced by the sun's attraction, can be measured. The most convenient test object is the earth itself.

If A is the mean radius of the earth's orbit, and T the length of the year, its average acceleration is $a = 4\pi^2 A/T^2$.

Introducing $A = 1.4945 \times 10^{13}$ cm., $T = 3.1558 \times 10^7$ sec., we find $a = 0.59243$ cm./sec.^{2} Now at a point on the earth's surface for which the distance from the center equals the mean radius r of the earth (6.3712×10^8 cm.) the acceleration, g, of gravity (if there were no centrifugal force) is known to be 981.993 cm./sec.^{2}

By the law of gravitation (§ 146)

$$a = G\frac{S}{A^2} \quad \text{and} \quad g = G\frac{E}{r^2},$$

where S and E are the masses of the sun and the earth. Hence

$$\frac{S}{E} = \frac{aA^2}{gr^2} = \frac{4\pi^2 A^3}{gT^2 r^2}. \tag{1}$$

Introducing the numerical values gives $S = 331{,}950\ E$. A small error in the parallax produces three times as great a percentage error in A^3 and hence in S. A change in the solar parallax of $0''.001$ (its probable error) will change S by 113 E. The value of E/S may be obtained in quite a different way, by intricate calculation, from the perturbations of the nearer planets. Then equation (1), worked backward, leads to a very accurate value of A (compare § 327).

221. The Curvature of the Earth's Orbit. The distance which the earth would fall toward the sun in a second, if its orbital motion were arrested, is $\frac{1}{2}a$, or 0.117 inches, just as $\frac{1}{2}g$ (16.1 feet) is the distance which a body falls toward the earth in the first second; and this 0.117 inches is the amount by which the earth deviates from a tangent to its orbit in a second. In other words, the earth in traveling 18.5 miles is deflected toward the sun only a little more than *one ninth of an inch.*

222. Superficial Gravity, or Gravity at the Surface, of the Sun. This is found by dividing the sun's *mass* by the *square of its radius*, both compared with the earth, that is, $\frac{331{,}950}{(109.1)^2}$, which gives 27.89. Thus a mass of ten pounds would weigh 279 pounds on the sun. A body would fall 450 feet in the first second, and a pendulum that vibrates in a second here would vibrate in less than one fifth of a second there. But (putting temperature out

of consideration) *a watch* would go no faster there than here, since neither the *inertia* of the balance-wheel nor the *elasticity* of the spring would be affected by the increased gravity.

223. The Sun's Density. Its mean *density* as compared with that of the earth may be found by simply dividing its *mass* by its *volume*, or by the cube of its radius (both as compared with the earth); that is, the sun's density equals $331{,}950 \div (109.1)^3 = 0.256$, which is a little more than *one quarter* of the earth's density.

To get *its density compared with water* we must multiply this by the earth's mean density, 5.52, which gives 1.41; that is, *the sun's mean density is less than one and one-half times that of water.*

This is a most remarkable and significant fact, considering that the sun has a tremendous force of gravity and that a considerable portion of its mass is composed of metals, as indicated by the spectroscope. The obvious and only possible explanation is that the temperature of the sun is so high that its materials are almost wholly in the gaseous state, — not solid or even liquid.

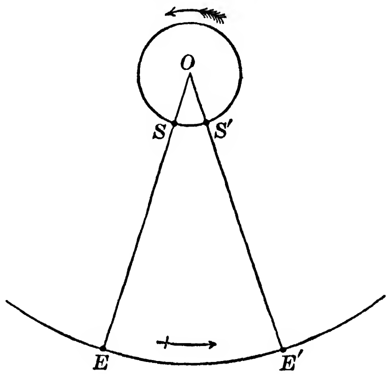


FIG. 83. The Synodic Period of Rotation is Longer than the Sidereal

An observer on the earth at E sees a spot on the central meridian of the sun at S; while the sun rotates, the earth also moves forward in its orbit, and when he next sees the spot in the same position on the disk of the sun he will be at E'. The spot has gone around the whole circumference plus the arc SS'

The pressure at the sun's center must be exceedingly great. For a sphere of uniform density of the sun's size and mass this pressure would exceed a billion atmospheres. If the density at the center is greater than the average, which is undoubtedly the case, the pressure must be still greater. To maintain the actual density at so great a pressure would demand a central temperature of many millions of degrees.

224. The Sun's Rotation and the Inclination of the Axis. The rotation of the sun is readily apparent to one who observes, from day to day, the positions of sun-spots on the solar disk. They

cross from east to west at a rate which gives about four weeks for the period of the sun's rotation as seen from the earth, — that is, the *synodic period.* The *true*, or *sidereal*, period may be calculated from the *observed synodic* period by the usual formula, already given in section 186 (p. 160).

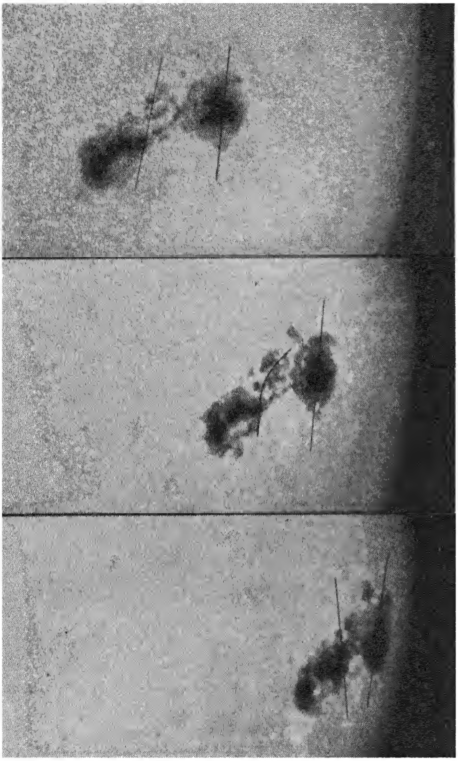


FIG. 84. The Rotation of the Sun

This causes the apparent motion of the spots across the disk. As the spot approaches the limb, the faculæ become more prominent. (The straight lines were drawn on the plate in connection with an investigation of the magnetic field of the spots.) These photographs were taken February 12, 13, 14, 1917. (Photographs by Mt. Wilson Observatory)

The paths of the spots across the sun's disk usually appear elliptical, and are most curved early in March and in September. Twice a year — early in June and in December — they become straight lines (Fig. 85). This indicates that the sun's axis is not perpendicular to the ecliptic (in which the earth travels). From a great number of careful measurements it is found that the sun's equator is inclined 7° 10′.5 to the ecliptic, and that its ascending node is in longitude 73° 47′.

225. The Equatorial Acceleration. It was noticed quite early that different spots give different results for the period of rotation, but the researches of Carrington, about 1860, first brought out the fact that the differences are systematic, and that *at the solar equator the time of rotation is shorter than on either side of it.*

According to Mr. and Mrs. Maunder, who have investigated the motions of nearly 1900 spots shown on the Greenwich photographs of the years 1879 to 1904, the mean sidereal rotation period for spots on the equator is 24.65 days; in latitude 20°, 25.19 days; in latitude 30°, 25.85 days; and for the few in latitude 35° (beyond which there are hardly any spots), 26.63 days.

Individual spots show great deviations from these mean rates of motion, as if they were drifting backward or forward over the sun's surface; and the differences between the rotation periods derived from different spots in the same latitude extend over a range which considerably exceeds the difference of the mean periods for the highest and lowest latitudes. When, however, a large number of spots are considered, there is no doubt of the steady lengthening of the mean period on both sides of the solar equator.

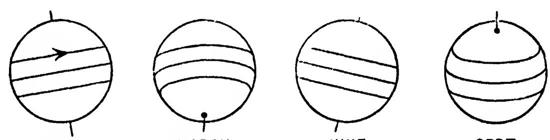


FIG. 85. Paths of Sun-Spots across the Sun's Disk

In September we see 7° beyond the north pole of the sun: the sun-spot paths are curved to the south. In March the south pole is tipped toward the earth and the sun-spot paths are curved to the north. In June and December the earth comes into the plane of the sun's equator and the spots seem to cross the disk in straight lines

226. If this remarkable equatorial acceleration were confined to the spots, it might be attributed to a drift of these over the sun's surface, like that of clouds over the earth; but there are several other ways of observing the sun's rotation, and the results of all agree closely. Long series of observations have been made on the *faculæ*, or bright regions on the sun's surface; on the *flocculi*, or regions of incandescent calcium vapor, revealed by the spectroheliograph (§ 584); and on the *gases of the lower portions of the solar atmosphere*, whose motion toward or from us may be measured spectroscopically (§ 579). The faculæ are confined to about the same latitudes as the sun-spots, and the flocculi extend but little farther from the solar equator; but the spectroscopic measures may be made at all solar latitudes, and show that the rotation grows steadily slower all the way to the pole. Thus Adams (1908) finds that the rotation period in latitude 40° is 27.48 days;

in latitude 60°, 30.93 days; and in latitude 75°, 33.15 days. The rotation period near the pole appears to be about 34 days.

227. Thus far all the formulæ with which attempts have been made to represent the velocity of the sun's surface in different latitudes are simply *empirical*; that is, they are deduced from the observations without being based on any satisfactory physical explanation. Recent work (1926) affords hope that such an explanation of the equatorial acceleration may soon be found.

The questions whether different layers in the sun's atmosphere rotate at different rates, and whether the rotation is subject to periodic variation, must be left unsettled until the small systematic errors of the spectroscopic observations have been traced to their sources and eliminated. The outstanding differences arise partly from errors of observation and partly from the turbulent condition of the solar surface, upon which the various markings are continually in motion, with varying rates and directions.

Various theories have been advanced to explain the equatorial acceleration. The latest attribute it to effects produced by the outpour of heat from the sun's interior, while others regard it as a survival from older conditions (compare § 541). It seems certain, at any rate, that even if there is now no force whatever at work to maintain the more rapid equatorial rotation, it must continue for many thousands, if not millions, of years before internal friction can slow it down to the average rate for the whole mass. If we assume, for example, that the main mass of the sun rotates at the rate found spectroscopically for the poles, there must be an equatorial current, of unknown but probably great depth, running eastward with a velocity of 550 meters per second, or 1230 miles an hour. This great current has, however, no definite banks, and its speed diminishes so gradually with increasing solar latitude that the maximum relative velocity of two points 1°, or 7500 miles, apart on the sun's surface is only 25 miles per hour. The fluid friction must therefore be so small, and the momentum of the current so great, that it would continue to circulate almost indefinitely.

228. Arrangements for the Study of the Sun's Surface. The heat and light of the sun are so intense that we cannot look

directly at it with a telescope. A very convenient method of exhibiting the sun to a number of persons at once is simply to attach to a telescope a small frame carrying a screen of white paper at a distance of a foot or more from the eyepiece (Fig. 86). When the focus is properly adjusted, a distinct image appears, which shows the sun's principal features very fairly, — indeed, with proper precautions, almost as well as could be done with elaborate apparatus. Still, it is generally more satisfactory to look at the sun directly with a suitable eyepiece.

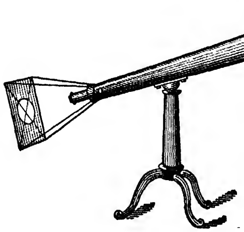


FIG. 86. Viewing the Sun

An image of the sun may be projected through the eyepiece upon a sheet of white paper. A screen should also be used at the object end, in order to shade the paper upon which the image is formed

With a small telescope (not more than $2\frac{1}{2}$ or 3 inches in diameter) a simple shade glass is often used between the eyepiece and the eye, but the dark glass soon becomes very hot and is apt to crack. With larger instruments it is necessary to use eyepieces especially designed for the purpose, and known as *solar eyepieces* or *helioscopes*, which reject most of the light coming from the object-glass and permit only a small fraction of it to enter the eye. It is not a good plan to cap the object-glass in order to reduce the light. To cut down the aperture is to sacrifice the definition of delicate details (§ 53).

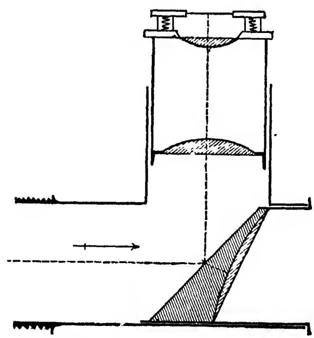


FIG. 87. The Herschel Solar Eyepiece

The simplest solar eyepiece, and a very good one, is known as Herschel's, in which the sun's rays are reflected at right angles by a plane of unsilvered glass. The reflector is made wedge-shaped, as shown in Fig. 87, in order that the reflection from the back surface may not interfere with the image. Most of the light passes through the open end of the eyepiece, but the reflected light is still too intense for the unprotected eye. Only a thin shade glass is required, however, which does not become very much heated. Polarizing eyepieces are also used.

In the study of the sun's surface, photography is very advantageous for some purposes. Arrangements must be made to secure an extremely rapid exposure, and it is best to use special slow plates. The size of the image of the sun on the negative depends on the focal length of the telescope (§ 50), averaging 1/107 of this, but being slightly larger in winter than in summer. The 150-foot tower telescope of the Mt. Wilson Observatory forms a 17-inch image of the sun.

Photographs do not show such fine detail as can be observed visually in moments of good seeing, but they secure in an instant a better picture of the sun's surface than a skillful draftsman could produce in hours. The great majority of solar observations are now made photographically.

The importance of the study of the sun has been recognized by the establishing of several observatories mainly for solar research. Among the most important of these solar observatories may be mentioned those at Mt. Wilson in California, Meudon in France, and Kodaikanal in India. Solar observations are also regularly made at the Yerkes Observatory, at Greenwich, and in many other places.

229. The Telescopic Appearance of the Sun. Before passing to a discussion of the details of the solar phenomena it will be well to mention the names that have been given to the various parts of the sun.

(1) *The photosphere*, the luminous surface of the sun directly visible in the telescope as a disk, with the *spots* and other markings observable upon it.

(2) *The sun's atmosphere*, lying above the photosphere and consisting of luminous but very nearly transparent gases. This can be directly observed with the telescope only when the photosphere is hidden during a total eclipse; but almost all its phenomena may now be studied at any time with the aid of the spectroscope, which, in addition, gives us extensive information concerning its composition and physical condition. This atmosphere may be divided into two regions:

(*a*) *The reversing layer*, extending to a height of a few hundred miles above the photosphere and composed of the vapors of many of the familiar terrestrial elements. This merges gradually into

(*b*) *the chromosphere* (composed mainly of the lighter gases, hydrogen and helium), which extends to a height of several thousand miles, and from which rise the *prominences* of various kinds. These are also masses of luminous gas, and rise sometimes to a height of hundreds of thousands of miles above the photosphere.

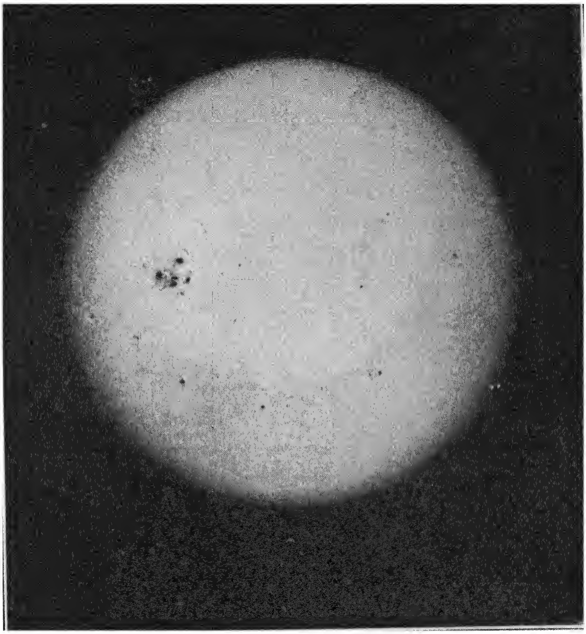


FIG. 88. Sun-Spot Maximum

A large spot-group and many smaller spots. Faculæ are visible near the limb. (From photograph by Yerkes Observatory)

(3) *The corona*, an outer envelope, of very great height and exceedingly small density, heretofore observable only during total eclipses of the sun.

230. The Photosphere. The sun's visible surface is called the *photosphere*, that is, the "light sphere." The general opacity (§ 660) of the gases in this region increases rapidly with depth and prevents us from seeing farther into the sun.

When studied with a telescope under favorable conditions and with a rather low power, it appears not smoothly bright, but mottled, looking much like rough drawing-paper. With a high

power and the best atmospheric conditions the surface is shown to be made up, as seen in Fig. 89, of a rather darkish background sprinkled over with grains, or nodules, as Herschel calls them, of something much more brilliant, — "like snowflakes on gray cloth," according to Langley. These nodules, or "rice grains,"are from 400 to 600 miles across, and when the seeing is best they

FIG. 89. "Rice Grains" and the Intricate Details of a Sun-Spot

From drawing by Langley

themselves break up into granules still more minute. Generally the nodules are about as broad as they are long, though irregular; but here and there, especially in the neighborhood of the spots, they are drawn out into long streaks, or filaments. They appear and disappear, and change in form continually.

The edge of the photosphere at the sun's limb appears perfectly sharp, as do many details on its surface, but it should be remembered that a transition, taking place gradually through a distance of 50 kilometers, would appear sharp in the best telescopes.

Certain bright streaks and patches called *faculæ* are also usually visible here and there upon the sun's surface, and, though not very obvious near the center of the disk, they become conspicuous near the limb, especially in the neighborhood of spots.

Near the sun's limb the photosphere appears much less brilliant than at the center. The variation is slight until near the limb,

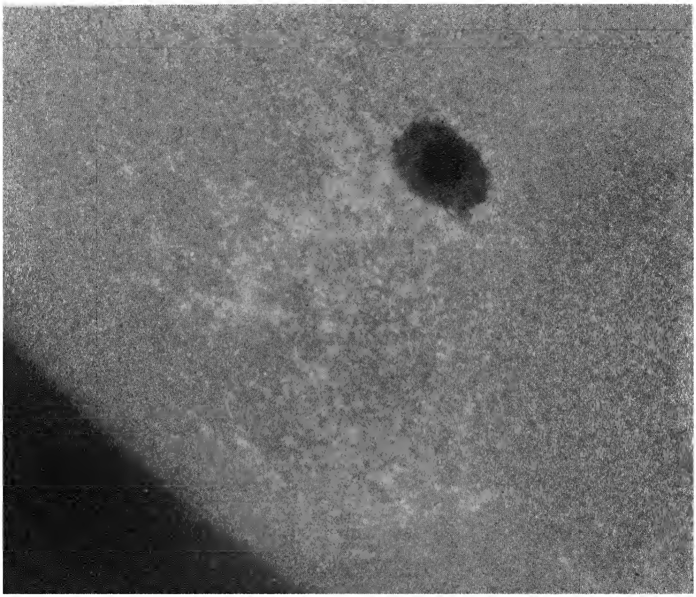


FIG. 90. Single Normal Spot, with Faculæ

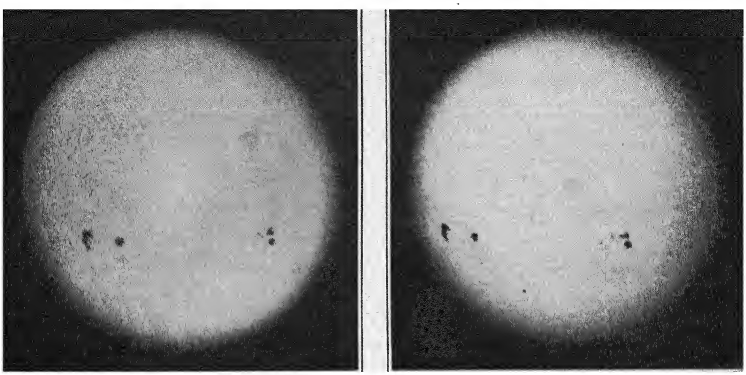
From photograph by Mt. Wilson Observatory

where the brightness falls off rapidly and, at the very edge, is not more than one third of the brightness at the center.

The light from the limb is not merely fainter but also redder than that from the center, so that on a projected image of the sun the limb appears brownish. For this reason the darkening at the limb is much more conspicuous on photographs than visually. When the sun is viewed with the naked eye through a shade glass, the contrast in brightness between the limb and the dark sky outside makes it appear relatively too bright and almost entirely conceals the darkening.

231. The Sun-Spots. Sun-spots, whenever visible, are the most conspicuous and interesting objects on the solar surface. The appearance of a normal sun-spot (Fig. 90), fully formed and not yet beginning to break up, is that of a dark central *umbra*, with a fringing *penumbra* composed of converging filaments. The umbra itself is not uniformly dark throughout but is overlaid with filmy clouds, which usually require a good helioscope to show them to any advantage.

Very few spots are strictly normal. They are often gathered in groups within a common penumbra, which is partly covered with



March 4 March 5

FIG. 91. Solar Photographs

Darkening at the limb is shown and displacement of spots by rotation. The south is at the top. (From photographs by Yerkes Observatory)

brilliant "bridges" extending across from the outside photosphere. Frequently the umbra is not in the center of the penumbra, or has a penumbra on one side only; and the penumbral filaments, instead of converging regularly toward the nucleus, are often distorted in every conceivable way.

At its inner edge the penumbra, from the convergence of these filaments, is usually brighter than at the outer edge. The inner ends of the filaments are ordinarily club-shaped, but sometimes are drawn out into fine points. The outer edge of the penumbra is usually rather sharply defined, and there the penumbra is darkest. In the neighborhood of the spot the surrounding

photosphere is usually much disturbed and brightened into faculæ, which ordinarily appear before the spot is formed and continue after it disappears.

Even the darkest portions of the sun-spot are dark *only by contrast*. This is directly observable when Venus or Mercury passes in front of the sun and transits across its disk. The planet (which, if there were no scattered light from the sky, would of course appear perfectly black) is conspicuously darker than the sun-spots. Further evidence that the spots give out light of their own is found in the fact that they are decidedly *redder* than the photosphere, and that their spectrum shows distinctive peculiarities.

Several lines of investigation indicate that the umbra of an average spot is rather less than one tenth as bright as the photosphere in yellow light, and relatively much fainter in the violet. Nevertheless, if we could see the spot on a dark background, instead of on the dazzling photosphere, it would appear brilliantly luminous, — brighter, indeed, for equal areas, than almost all terrestrial sources of light.

232. Dimensions of Sun-Spots. The diameter of the *umbra* of a sun-spot varies all the way from 500 miles, in the case of a very small one, to 40,000 or 50,000 miles, in the case of the largest. The *penumbra* surrounding a group of spots is sometimes 150,000 miles across, though that is exceptional. Not infrequently sun-spots are large enough to be visible with the naked eye and can actually be thus seen at sunset or through a fog or by the help of a shade glass.

The Chinese have many records of such objects, but interest in them dates from 1610, as an immediate consequence of Galileo's study with the telescope. Fabricius and Scheiner, however, share the honor of discovery with him, as independent observers.

233. Duration of the Spots. Most sun-spots are very short-lived phenomena. One fourth of all those shown on the Greenwich photographs lasted but a single day, and as many again, from two to four days. These, as might be expected, were small spots; the larger ones are far from permanent. Out of some 6000 groups observed in thirty-three years, only 468 were observed to have a continuous existence into a second rotation of the sun, 115 into

a third, 25 into a fourth, 12 into a fifth, and but one into a sixth. In a single recorded instance (1840–1841) a spot persisted for eighteen months.

The faculæ in the surrounding region generally endure much longer than the spots, and not infrequently a new group of spots breaks out in the same region where one has disappeared some time before, as if the *local disturbance which caused the spots and faculæ still continued deep below the visible surface.*

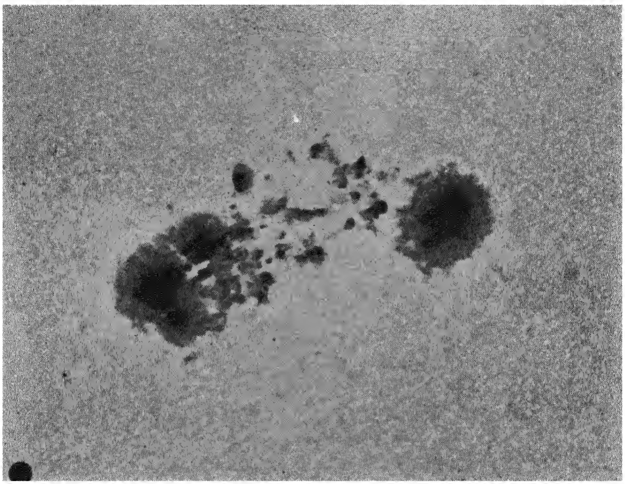


FIG. 92. The Great Sun-Spot Group of February 8, 1917

The earth, in its relative size, is represented by the black disk in the corner. (From photograph by Mt. Wilson Observatory)

The *development* of a spot or spot-group usually begins, according to Secchi, with the formation of faculæ interspersed with small, dark points, or pores. These pores grow rapidly larger and coalesce, and the neighboring granules of the photosphere are transformed into the filaments of the penumbra, converging toward the umbra. Ordinarily this process takes several days, but sometimes only a few hours.

According to Cortie the irregular group of scattered incipient spots soon passes into a second stage, stretching out east and west with two predominant spots, — one a leader, the other a rear guard of the flock. The preceding one (in the direction of

the sun's rotation) is usually more compact and regular, though the other is sometimes the larger. The leader apparently pushes forward upon the photosphere and so increases the length of the train of spotlets between the two principals. After a time these small spots generally disappear, leaving the pair, of which the leading one usually lasts the longer and looks more like a normal

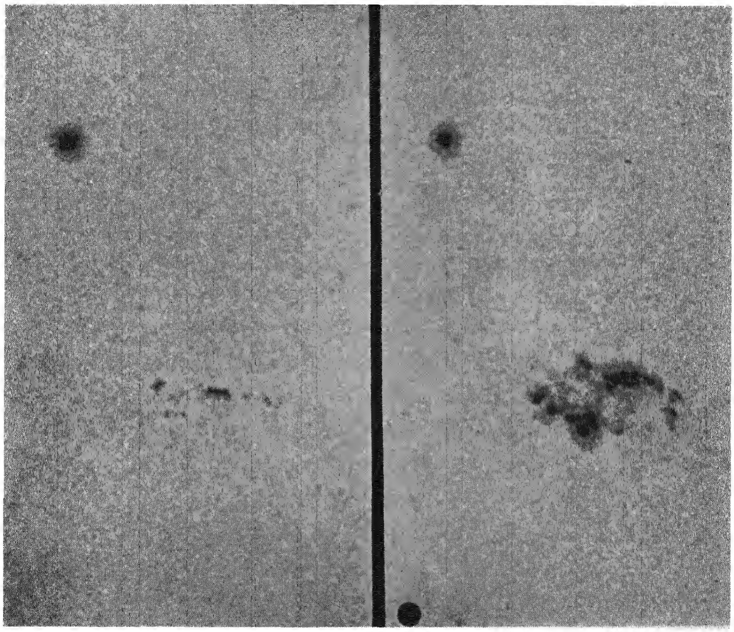


FIG. 93. Remarkable Twenty-four-Hour Development of Sun-Spot Group, August 18–19, 1916

The black disk at the bottom represents the relative size of the earth. (From photograph by Mt. Wilson Observatory)

spot. Recent studies at Mt. Wilson show that double groups are about twice as numerous as single spots. The leading member of a double group is usually the stronger (Fig. 84). Frequently a large spot divides into several, separated by brilliant bridges, and the segments fly apart with a speed of sometimes a thousand miles an hour. An active spot is an extremely interesting telescopic object; not infrequently a single day works a complete transformation (Fig. 93).

234. Proper Motions of Spots. As has already been stated, the spots are usually in motion over the sun's surface. Recent studies show that these motions take place almost at random, and that there is no definite tendency for the average drift to be in any one direction in any particular part of the sun's surface. The motions in longitude, however, average nearly twice as great as those in latitude. Akesson finds for the average daily drift 43′ of longitude and 24′ of latitude, which correspond respectively to 5400 and 3000 miles on the sun's surface. He finds, also, that the small spots move faster than the large ones.

The rate and direction of a spot's motion change from day to day, so that at the end of a solar revolution the spot (if it lasts so long) is usually not nearly so far from its original position as it would be if it had gone steadily forward in one direction. The average motion in twenty-five days, according to Dyson and Maunder, is 1°.2 in latitude and about 4° in longitude. According to the same authorities there is a definite connection between the motions of the large regular, or normal, spots and their age, which they describe as follows: "Regular spots usually form at the head of a stream, and, during the early days of the development of the stream, move rapidly forward in longitude. After the stream has attained its greatest dimensions and begun to diminish again, it is the following portion of the group that disappears first, and the regular spot which was the original leader often remains alone. In this stage of the history of the group there is a strong tendency for the regular spot to move back again toward the longitude where it was originally formed."

235. Distribution of the Spots. For the most part the spots are confined to two belts between 5° and 40° of north and south latitude (Fig. 95). A few appear near the equator at the time of the sun-spot maximum, and almost none beyond the forty-fifth parallel. A few very faint and short-lived flecks have been observed in latitudes as high as 65° and even 72°.

Generally the numbers are about equal in the two hemispheres, but sometimes there is a marked difference which persists for years. From 1672 to 1704 not a single spot was observed in the northern hemisphere, and the breaking out of a few in 1705 occasioned great surprise and was reported to

the French Academy as an anomaly. During the last fifty years the southern hemisphere has been about one fifth more spotted than the northern, but the successive revivals of activity have come a little earlier in the latter. No reason for such differences has yet been discovered.

236. Sun-Spot Periodicity. The number of spots varies greatly in different years and shows an approximately regular periodicity of about eleven years. This fact was first discovered by Schwabe, of Dessau, in 1843, as the result of his systematic watching of sun-spots for nearly twenty years, and has since been abundantly confirmed.

Wolf, of Zürich, collected all the observations available up to 1880 and summarized them in a diagram which forms part of Fig. 94. The system of sun-spot numbers introduced by Wolf is based on the number of spot-groups, and on the number of spots which can be counted in these groups, as well as singly, and also takes into account the observer and the size of his telescope. The system has been continued by later observers. The numbers are found to be closely related to the spotted area expressed as a fraction of the visible hemisphere.

During the maximum, the surface of the sun is never free from spots; sometimes a hundred are visible at once. During the minimum, weeks, and even months, pass without a single spot.

The average interval between maxima is 11.13 years according to Newcomb; but this is subject to great fluctuations, the observed intervals ranging all the way from 7.3 to 17.1 years. The rise to maximum is usually, but not always, more rapid than the fall which follows, the mean durations of the two, according to Newcomb, being 4.62 and 6.51 years. W. J. S. Lockyer has pointed out that the rise is more rapid, both in actual duration and as compared with the fall, when the maxima are highest. The whole length of the interval between minima, however, seems to bear little relation to the intensity of the maximum.

There appears also to be a decided secular variation in the intensity of the outbreaks. High and low maxima, in groups of three or four, have alternated during the interval for which reliable estimates of the activity are available; and the fragmentary records of older times indicate a similar phenomenon. Sun-spots were apparently numerous about 1610 and very rare

between 1640 and 1716. A second, less marked period of quiescence extended from 1798 to 1833, and a third has lasted from 1878 to the present time. The intervals between these times of low activity are, however, very unequal, and only the future

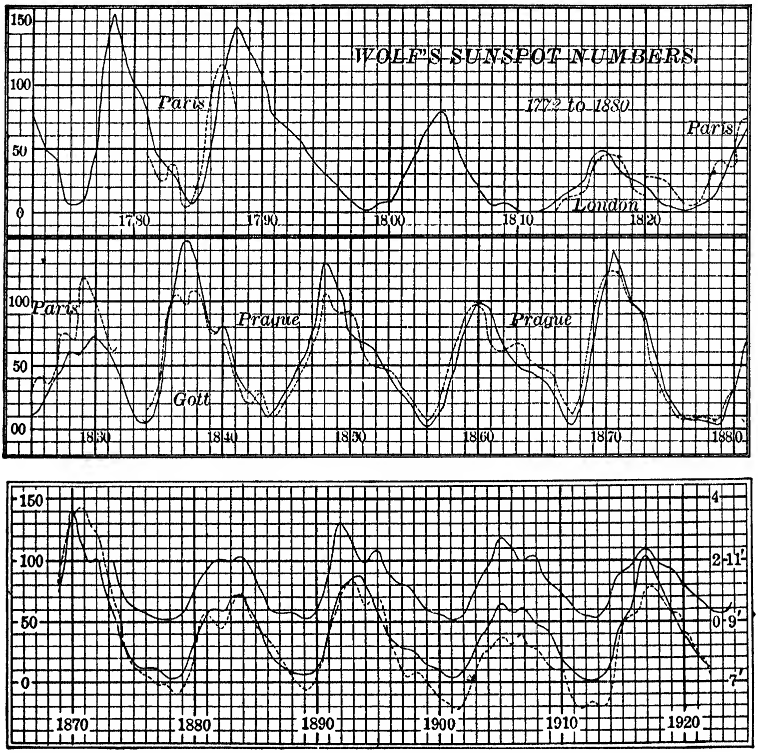


FIG. 94. Periodicity of Sun-Spot Numbers

The periodicity of the number of sun-spots is very similar to that of areas of faculæ and to that of diurnal variation of magnetic declination. Heavy curve: Wolf's sun-spot numbers, 1772–1922. Upper lighter curve: areas of faculæ (in thousandths of the visible hemisphere), Greenwich, 1873–1924. Broken line (lower part of diagram): diurnal variations of magnetic declination, Greenwich, 1869–1922

can distinguish whether these recurring changes are really periodic in their nature, or whether they are essentially irregular.

237. Spoerer's Law of Sun-Spot Latitudes. Still another fact, as yet unexplained, and probably of great theoretical importance, was first brought out by Spoerer. The disturbance which produces the spots of a given cycle first manifests itself by the

appearance of small spots 25° or 30° north and south of the sun's equator. As the solar activity increases, new spots, in increasing numbers, supplant those which disappear, and the spotted zone widens out, but almost entirely on the equatorial side, so that the mean latitude of the spots steadily decreases. Shortly before the time of maximum activity the production of spots in the highest latitudes diminishes, and ceases, and not long afterward the inner edges of the two great zones of activity approach the equator and become stationary without quite meeting. As the activity decreases, the spotted regions shrink more and more on their outer edges, until the last dying traces of the disturbance are found within a few degrees on each side of the equator, thirteen or fourteen years after its first outbreak. Two or three years before this disappearance, however, the first spots of the new cycle show themselves near latitude ± 30°, so that, at the spot minimum, there are usually four well-marked zones in which the few spots may appear, — two close to the equator, owing to the expiring disturbance, and two in high latitudes, nearly 200,000 miles away from the former, owing to the newly beginning outbreak.

Fig. 95, reproduced from a paper by Maunder, shows the distribution of the spots in heliographic latitude for each synodic rotation of the sun from 1874 to 1903, and exhibits the facts better than any verbal description could do. It is clear that if we confined our attention to a zone of definite latitude, we should always find approximately the same intervals between the times of greatest abundance of sun-spots, but that the times of maximum would differ widely from zone to zone, — maximum for the highest latitudes and minimum for the equator being almost simultaneous.

238. Irregularity of the Period of Sun-Spots. A great deal of labor has been expended in the attempt to find a mathematical formula which shall represent the changes of the observed numbers of spots in the past and may be used to predict those in the future. No formula which will stand the test of prediction has yet been obtained, and it appears very doubtful whether success is possible, for it is not unlikely that the sun-spot disturbances, like the eruptions of a geyser, are inherently only roughly periodic.

239. The cause of sun-spot periodicity is not yet known. Attempts have been made to account for it by planetary influences and meteoric swarms, but without success. There are good reasons for believing that the cause, whatever it may be, is to be found in the sun itself.

The evidence of the spectroscope and spectroheliograph (to be discussed later) shows conclusively that sun-spots are the

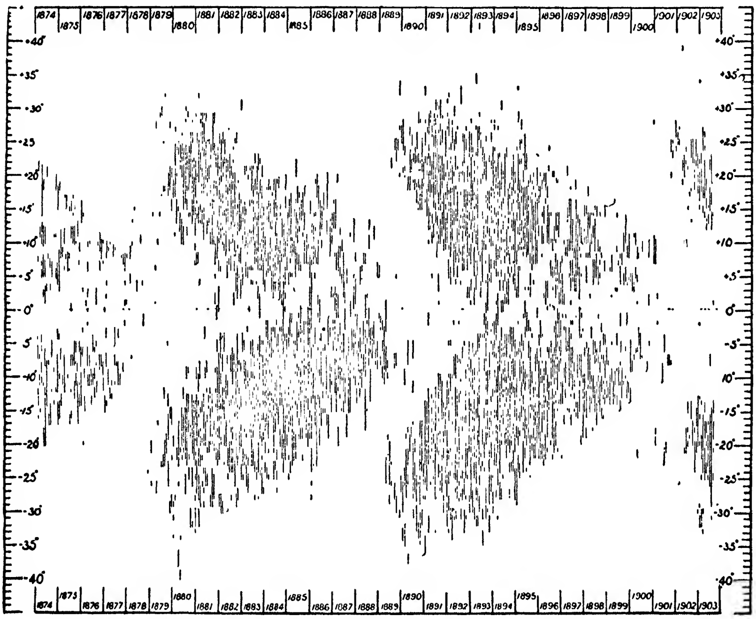


FIG. 95. "Butterfly" Sun-Spot Diagram

The lines extend over the latitudes in which sun-spots were observed during each synodic rotation of the sun. Two complete cycles of disturbances are shown, with the end of an earlier one and the start of a fourth. (After E. W. Maunder, Greenwich Observatory)

most conspicuous visible signs of a generally disturbed condition of the regions of the sun's surface in which they appear. Spoerer's law, and the associated phenomena, almost compel the belief that the individual outbreaks which produce the spot groups of the same cycle are themselves but manifestations of a single great disturbance in each hemisphere, which breaks out in high latitudes and works down toward the equator. The orderly sequence of these greater disturbances — each breaking

out as the last is dying away — points to a deep-seated cause, perhaps involving a great portion of the mass of the sun. Of the nature of this cause we can form but the vague conception that it is a gathering of deep-lying forces during an outward period of quiescence, followed by an outburst which relieves the internal strain.

On this theory the irregularity of the intervals and the intensities of the successive outbreaks are not at all surprising. It seems also natural enough that the most violent disturbances should develop the most rapidly, as is observed to be the case.

We shall see later (§ 599) that Hale, after a searching investigation of the phenomena of sun-spots, has adopted a theory which assigns their origin to disturbances in the deeper layers of the sun, well below the photosphere.

240. Many other solar phenomena show, as might be expected, periodic variations which run parallel with those of the spots. The *faculæ*, which are so closely associated with the spots, vary in almost strict proportionality to their numbers (Fig. 94); the number and distribution of the *solar prominences* and also the form and extent of the *corona* (Figs. 96, 97) also show great and regular changes with the sun-spot period; and there is evidence that the total *radiation of heat* (§ 603) from the sun increases with increasing spottedness.

241. Terrestrial Influences of Sun-Spots. There is a conspicuous relation between the abundance of sun-spots and the variations of terrestrial magnetism. The direction and intensity of the earth's magnetic field are never quite constant. In addition to the slow secular variation, which proceeds in the same direction for many years, it is subject to regular *diurnal variations*, and at irregular intervals to sudden and relatively violent disturbances, which are known as *magnetic storms*, and which have been known in extreme instances to change the direction in which the compass points by nearly three degrees in as many minutes. These magnetic storms are frequently accompanied by earth-currents of electricity, which are sometimes strong enough to interfere with the operation of telegraph lines, and by displays of the aurora borealis and aurora australis (which invariably indicate the existence of magnetic disturbance) (§ 658).

The intimate relation which exists between the number and magnitude of these magnetic disturbances and the sun-spot variations is exhibited in Fig. 94. The range of diurnal variation diminishes to about half its maximum amount at the sun-spot minima, while the number of magnetic storms (not shown

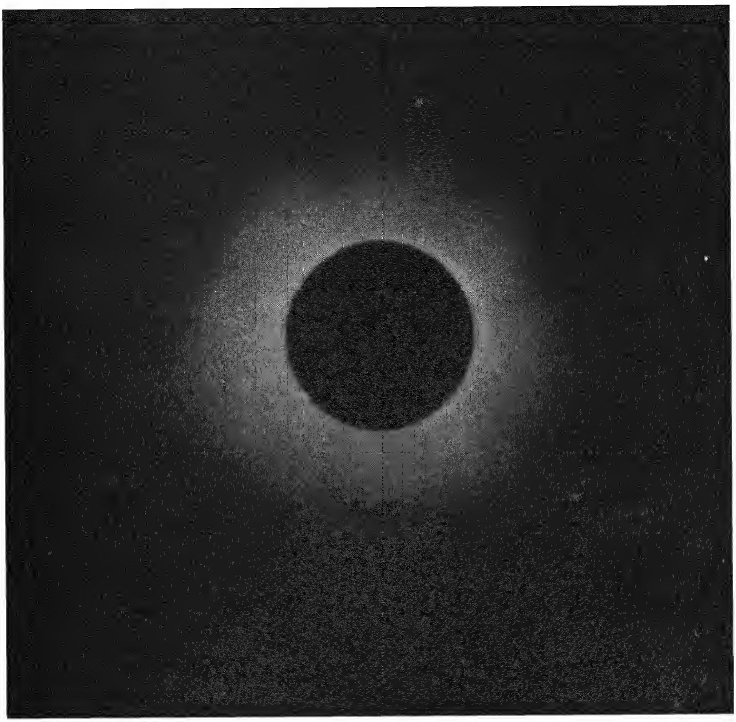


FIG. 96. Eclipse of April 16, 1893, Chile

When sun-spots are at a maximum (see Fig. 94), the corona extends out in all directions. Sometimes long streamers in the sun-spot zones give it a roughly quadrilateral form. The notches in the moon's limb shown in the photograph are not real, being due to "irradiation" of the neighboring prominences. (From photograph by Lick Observatory)

in the figure) usually drops approximately to zero. The close agreement with the curve of spottedness makes it impossible to doubt the reality of the connection. The variation in the annual number of auroræ also follows the sun-spot cycle very closely, the maxima and minima of frequency falling near those of the spots but averaging about a year later.

It is perhaps worth while to mention that magnetic storms have no connection with the electrical phenomena of thunderstorms, which show no perceptible relation to the sun-spot period.

There is often evidence connecting a magnetic storm with an individual sun-spot (Fig. 98). Maunder finds that during every

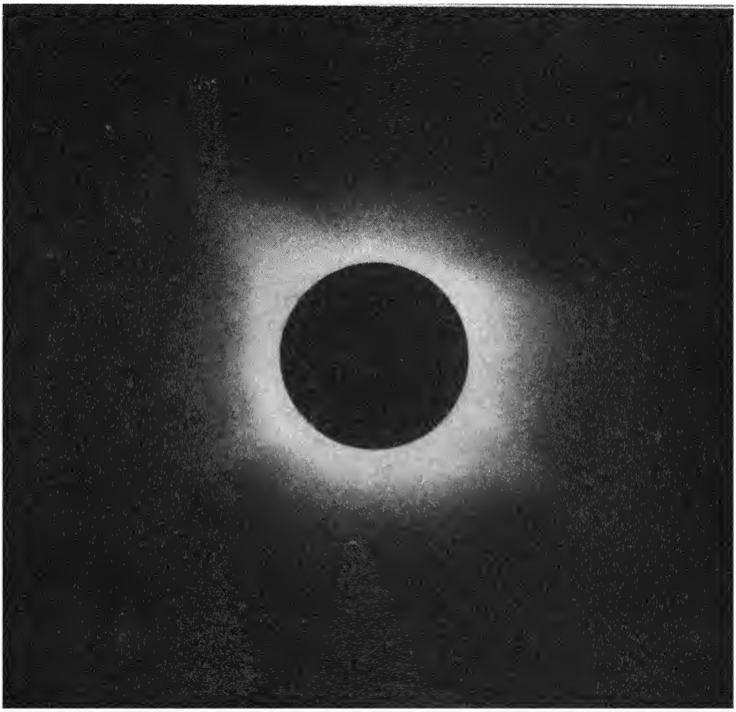


FIG. 97. Eclipse of January 22, 1898, India

At the time of sun-spot minimum the equatorial rays predominate and the short polar rays are sharply defined. Although this eclipse occurred only two years before sun-spot minimum, it still shows some of the characteristics of the maximum type. (From photograph by Lick Observatory)

one of the nineteen great magnetic storms between 1875 and 1903 a large spot, or a spot-group which had been large during a previous rotation of the sun, was on the visible side of the sun and between 20° east and 50° west of the central meridian. On the average these storms occurred twenty-five hours after the corresponding spot had crossed the central meridian. Maunder

has also shown that there are often one or more repetitions of a magnetic storm when the rotation of the sun again brings the spot region to the central meridian, and concludes that areas on the sun's surface may be magnetically active both before visible spots form and after they have disappeared.

Chree has shown that similar, although less conspicuous, relations exist in the case of the diurnal magnetic variation.

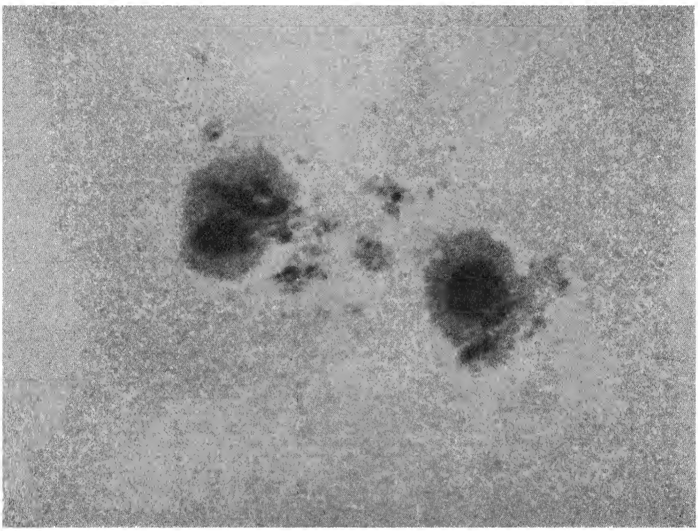


FIG. 98. Pair of Spots on the Sun

These large spots were photographed at the time of the great aurora of May 13, 1921. (From photograph by Mt. Wilson Observatory)

242. Many attempts have been made to show that greater or less disturbance of the sun's surface, as indicated by the frequency of the spots, is accompanied by effects upon the *meteorology of the earth*. Only long and homogeneous series of observations are of any value in such investigations, for an apparent relation extending over a single spot-period, or even over two, may be due only to chance.

One such relation has now been established by the concurrent results of several investigators. The mean temperature of the air at the earth's surface is *lower* when the sun-spots are most

numerous, by something between half a degree and one degree, centigrade, for every change of 100 spot-numbers.

There are indications of similar relations between sun-spot changes and the variations in rainfall and other meteorological conditions, but the effects are in all instances so small that they are very hard to separate from the much greater variations, arising from other causes, which happen from year to year.

243. It will be noticed that the accounts of solar phenomena given in this chapter have been for the most part purely descriptive, with a minimum of theoretical explanation. The reason for this is that our knowledge of solar phenomena depends to so great an extent on the information furnished by the spectroscope and other instruments of physical research that it would be premature to attempt to discuss their nature until this evidence has been presented, which will be done in Chapter XV.

EXERCISES

1. If the diameter of the sun were doubled, its density remaining unchanged, what would be the force of gravity at its surface?

2. If the sun were expanded into a homogeneous sphere, with a radius equal to the distance of the earth from the sun, its mass remaining unchanged, what would be the force of gravity at its surface?

Ans. 1/1657 of g.

3. In this case, what change, if any, would result in the orbit of the earth? *Ans.* None.

4. By how much would a change of $+ 0''.005$ in the solar parallax alter the accepted value of the astronomical unit? of the diameter of the sun?

5. What are the coördinates (celestial latitude and longitude) of the north pole of the sun?

6. How long would it take a sun-spot on the equator to gain a whole revolution upon one in latitude 30° (§ 225)?

Ans. 532 days (much longer than the life of a spot).

CHAPTER VIII

ECLIPSES

FORM AND DIMENSIONS OF SHADOWS · ECLIPSES OF THE MOON · SOLAR ECLIPSES · TOTAL, ANNULAR, AND PARTIAL ECLIPSES · ECLIPTIC LIMITS AND THE NUMBER OF ECLIPSES IN A YEAR · RECURRENCE OF ECLIPSES AND THE SAROS · OCCULTATIONS

244. The word "eclipse" is a term applied to the darkening of a heavenly body. Eclipses naturally fall into two classes, according as the eclipsed body is self-luminous or shines by reflected light.

(1) An eclipse of a *self-luminous* body can only be occasioned by the *interposition of another body*. The observer finds himself in a shadow. Thus, the sun is eclipsed when the moon intervenes and hides it from us. The components of a double star, the orbit of which lies so that it is viewed edgewise from the earth, are periodically eclipsed when one star for a brief time hides the other from us (§ 787). When the apparent size of the eclipsed body is much smaller than that of the eclipsing body, the phenomenon is known as an *occultation*. Thus, the moon in its revolution about the earth frequently occults a star, and a planet occasionally does so. Finally, a relatively very small body may *transit* the disk of some larger body, eclipsing only a very small circular area. Thus, Mercury and Venus periodically transit the sun's disk as small, round dots.

(2) A body that is *not self-luminous* suffers eclipse because the light which ordinarily makes it visible is shut off from it by some other body. In other words, it moves into a shadow. The earth, when it comes between the sun and the moon, prevents the sunlight from falling on the moon, and eclipses of several of Jupiter's satellites can be observed as they move into the shadow cast by their primary (§ 440). We shall speak here only of *solar* and *lunar* eclipses.

245. Shadows in Space. If interplanetary space were slightly dusty, we should see, accompanying the earth and moon and

each of the planets, a long, black shadow projecting behind it and traveling with it. Such a shadow is the space from which sunlight is excluded by the intervening body. As the sun, the earth, and the moon are all nearly spherical, and the sun is by far the largest, the shadows of the earth and moon are *cones* with their axes in the line joining the centers of the sun and the shadow-casting body, and with the apex always directed away from the sun. The length of the shadow varies with the distance between the sun and the earth, or the sun and the moon.

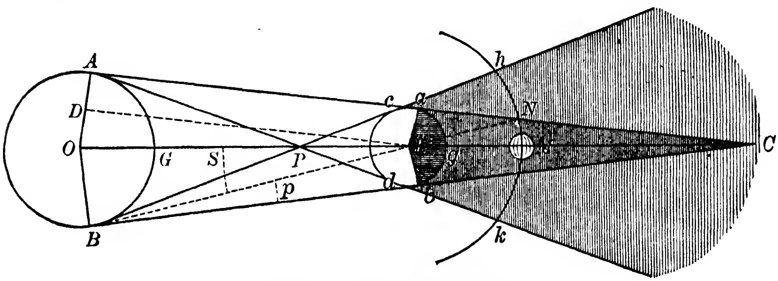


FIG. 99. The Earth's Shadow

246. Dimensions of the Earth's Shadow. The length of the earth's shadow is easily found. In Fig. 99 we have, from the similar triangles OED and ECa,

$$OD : OE = Ea : EC.$$

OD is the difference between the radii of the sun and the earth $= R - r$. $Ea = r$, and OE is the distance of the earth from the sun $= D$. Hence,

$$EC = D\left(\frac{r}{R - r}\right) = \frac{1}{108.1} D.$$

(The number 108.1 is found by simply substituting for R and r their values, R being 109.1 r.) This gives 859,000 miles for the length of the earth's shadow when D has its mean value of 92,870,000 miles. The length varies about 14,000 miles on each side of the mean, in consequence of the variation of the earth's distance from the sun at different times of the year.

From the cone aCb all sunlight is excluded, or would be if it were not for the fact that the atmosphere of the earth, by its refraction, bends some of the rays into this shadow, making it

less perfectly dark. Furthermore, the dense lower layers of atmosphere, especially if they are filled with clouds, obstruct the passage of the sun's rays and thus increase the effective diameter of the earth and its shadow by about ten miles.

247. Umbra and Penumbra. While the region from which all light is excluded (the cone defined by the lines aC and bC in Fig. 99) is usually called simply the shadow, it is more technically called the *umbra*. If we draw the lines Ba and Ab, crossing at P, between the earth and the sun, they will bound the *penumbra*, within which a part, but not the whole, of the sunlight is cut off. An observer outside of the umbra but within this cone-frustum, which tapers toward the sun, would see the earth as a black body encroaching on the sun's disk.

If the earth had no atmosphere, the boundary of the umbra would be sharp, since the contrast between darkness and even greatly diminished sunlight is notable. Since the edge of the actual shadow is cast by an atmosphere of gradually increasing density, the umbra has no sharp boundary. The outer boundary of the penumbra, while geometrically quite definite, is imperceptible, — the brightness just inside differing infinitesimally from that without.

ECLIPSES OF THE MOON

248. Total and Partial Eclipses. The axis, or central line, of the earth's shadow is always directed to a point exactly opposite the sun. If, then, at the time of the full moon, the moon happens to be near the ecliptic (that is, not far from one of the nodes of her orbit), she will pass through the shadow and be eclipsed. Since, however, the moon's orbit is inclined to the ecliptic at an average angle of $5° 8'$, lunar eclipses do not happen very frequently, — seldom more than twice a year. Ordinarily the full moon passes north or south of the shadow without touching it.

Lunar eclipses are of two kinds: partial and total, — total when the moon passes completely into the shadow; partial when she only partly enters it, going so far to the north or south of the center of the shadow that only a portion of the disk is obscured.

249. Size of the Earth's Shadow at the Point where the Moon crosses it. Since EC, in Fig. 99, is 859,000 miles, and the distance of the moon from the earth is, on the average, about 239,000 miles, CM must average 620,000 miles, so that MN, the semi-diameter of the shadow at this point, will be 620/859 of the earth's radius. This gives $MN = 2858$ miles, and makes the diameter of the shadow a little over 5700 miles, — about two and two-thirds times the diameter of the moon. It may at times be 200 miles smaller or larger.

An eclipse of the moon, when *central* (that is, when the moon crosses the center of the shadow), may continue *total* for about one hour and forty minutes, the interval from the first contact to

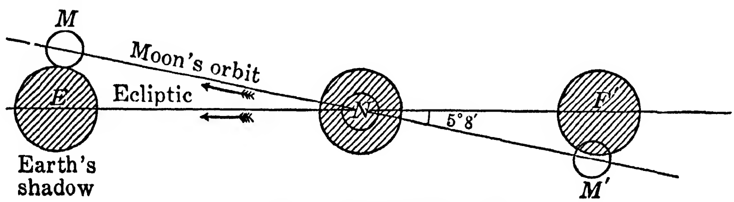


FIG. 100. Lunar Ecliptic Limit

If the earth's shadow is at the node N of the moon's orbit when the moon reaches it, there will be a central total eclipse of the moon. If the moon overtakes the shadow at E, there will be no eclipse. The distance NE is the "lunar ecliptic limit." This distance is greatest (12° 15′) when the apparent diameters of the moon and the earth's shadow have their greatest values

the last being about two hours more. This depends upon the fact that the moon's hourly motion is nearly equal to its own diameter.

The duration of a non-central eclipse varies, of course, according to the part of the shadow traversed by the moon.

250. Lunar Ecliptic Limit. The lunar *ecliptic limit* is the greatest distance of the sun from the node of the moon's orbit at which a lunar eclipse is possible. This limit depends upon the inclination of the moon's orbit, which is somewhat variable, and also upon the distance of the moon from the earth at the time of the eclipse, which is still more variable. Hence we recognize two limits, the major and the minor.

If the distance of the sun from the node, or of the shadow from the opposite node, at the time of full moon exceeds the major limit, an eclipse is impossible; if it is less than the minor, an

eclipse is inevitable. The major limit is found to be 12° 15′; the minor, 9° 30′. The shadow travels the major limit in about thirteen days. There are, therefore, two intervals during the year, of not more than twenty-six days each, when lunar eclipses are possible.

251. Phenomena of a Total Lunar Eclipse. Half an hour or so before the moon reaches the shadow its limb begins to be sensibly darkened by the penumbra, but the edge of the umbra itself, when it first encroaches on the moon, appears nearly black by contrast with the bright parts of the moon's surface. To the naked eye the outline of the shadow looks reasonably sharp, but even with a small telescope it is found to be indefinite, and with a large telescope and high magnifying power it becomes entirely indistinguishable, so that it is impossible to determine within half a minute the time when the boundary of the shadow reaches

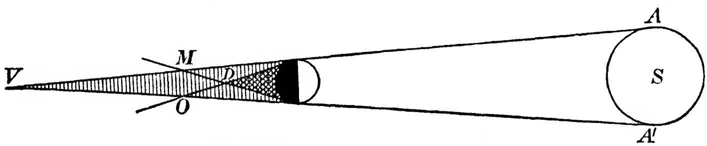


Fig. 101. Light bent into Earth's Shadow by Refraction

any particular point on the moon. After the moon has wholly entered the shadow, her disk is, as a rule, distinctly visible, illuminated with a dull, copper-colored light, which is sunlight, deflected around the earth into the shadow by the refraction of our atmosphere (Fig. 101).

Even when the moon is exactly central in the largest possible shadow, an observer on the moon would see the disk of the earth surrounded by a narrow ring of brilliant light, colored with sunset hues, — in fact, deeply colored, because the light has traversed a double thickness of our air. If the weather happens to be clear at this portion of the earth (upon its rim, as seen from the moon), the quantity of light transmitted through our atmosphere is very considerable, and the moon is strongly illuminated. If, on the other hand, the weather happens to be stormy in this region of the earth, the clouds cut off nearly all the light.

In astronomical importance a lunar eclipse is not at all to be compared to a solar eclipse. It has its uses, however. (1) It is possible to observe the moon's passage over faint stars which cannot be seen at all when near the moon except at such a time.

Observations of these star occultations made in different parts of the earth furnish data for computing the dimensions of the moon and its parallax, and for determining its position. (2) The study of the heat radiated by the moon during the different phases of an eclipse furnishes information about the absorbing power and temperature of the moon's surface.

252. Computation of a Lunar Eclipse. Since all the phases of a lunar eclipse are seen everywhere at the same absolute instant wherever the moon is above the horizon, it follows that a single computation, giving the Greenwich times of the different phenomena, is all that is needed. Such computations are published in the *Nautical Almanac.* Owing to the uncertainties mentioned above, the time is given only to the nearest tenth of a minute. Each observer has only to correct the predicted time, by simply adding or subtracting his longitude from Greenwich, in order to get the true local time. The computation of a lunar eclipse is not at all complicated.

ECLIPSES OF THE SUN

253. Dimensions of the Moon's Shadow. By the same method as that used for the shadow of the earth (§ 246) we find that the length of the *moon's* shadow at any time is very nearly 1/400 of its distance from the sun, and at new moon *averages* 232,100 miles. It varies not quite 4000 miles each way, ranging from 228,200 to 236,000 miles.

Since the *mean* length of the shadow is less than the mean distance of the moon from the earth (238,900 miles), it is evident that *on the average* the shadow (umbra) will not reach the earth.

On account of the orbital eccentricity, however, the distance of the moon is, for much of the time, considerably less than the mean. The moon may come within 221,700 miles [1] of the earth's center, or about 217,750 miles from its surface. If, at the same time, the shadow happens to have its greatest possible length (that is, if the earth is at aphelion), its point may reach nearly 18,250 miles beyond the earth's surface. In this

[1] This is the minimum distance when the moon is new and at the node. It differs from the absolute minimum distance given in section 193 (p. 165).

case the cross-section of the shadow, where the earth's surface cuts it squarely (at *o* in Fig. 102), will be about 167 *miles in diameter, which is the largest value possible.* If, however, the shadow strikes the earth's surface obliquely, the shadow spot will be oval instead of circular, and the extreme *length* of the oval may exceed the 167 miles.

Since the distance of the moon may be as great as 252,400 miles from the earth's center, or nearly 248,500 miles from its surface, while the shadow may be as short as 228,200 miles, we may have the state of things indicated by placing the earth at *B* in Fig. 102. The vertex of the shadow, *V*, will then fall 20,300 miles short of the surface, and the cross-section of the *shadow produced* will have a diameter of 192 miles at *o′*, where the earth's surface

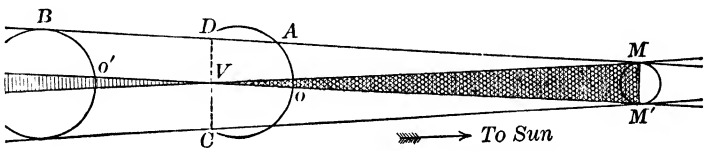


FIG. 102. Moon's Shadow on the Earth

cuts it. When the shadow falls obliquely near the limb of the earth, the breadth of this cross-section may be as great as 230 miles.

254. Total and Annular Eclipses. To an observer within the true shadow cone (that is, between *V* and the moon in Fig. 102) the sun will be *totally* eclipsed. An observer in the produced cone, beyond *V*, will see the moon smaller than the sun, leaving an uneclipsed ring around it, and will have what is called an *annular*, or ring-formed, eclipse. These annular eclipses are considerably more frequent than the total, and now and then an eclipse is annular in part of its course across the earth and total in part. In this case the point of the moon's shadow extends beyond the nearest part of the surface of the earth but does not reach as far as its center (p. 231).

255. The Penumbra and Partial Eclipses. The penumbra can easily be shown to have a diameter, on the line *CD* (Fig. 102), of a trifle more than twice the moon's diameter. An observer situated within the penumbra has a *partial* eclipse. If he is near the

cone of the shadow, the sun will be mostly covered by the moon; but if he is near the outer edge of the penumbra, the moon will encroach but slightly on the sun's disk. While, therefore, *total and annular* eclipses are visible as such only by an observer within the narrow path traversed by the shadow spot, the same eclipse will then be visible as a *partial* one everywhere within 2000 miles on each side of that path. The 2000 miles is to be reckoned perpendicularly to the axis of the shadow, and may correspond to a much greater distance on the spherical surface of the earth.

256. Velocity of the Shadow and Duration of Eclipses. If it were not for the earth's rotation the moon's shadow would pass an observer at the rate of nearly 2100 miles an hour, on the average. The earth, however, is rotating, toward the east, in the same general direction as that in which the shadow moves, and at the equator its surface moves at the rate of about 1040 miles an hour. An observer on the earth's equator, therefore, with the moon at its mean distance from the earth and near the zenith, would, on the average, be passed by the shadow with a speed of about 1060 miles an hour (2100 − 1040), or 1600 feet per second. In higher latitudes, where the surface velocity due to the earth's rotation is less, the relative speed of the shadow is higher; and where the shadow falls very obliquely, as it does when an eclipse occurs near sunrise or sunset, the advance of the shadow on the earth's surface may become very swift, — as great as 4000 or 5000 miles an hour.

A *total* eclipse of the sun observed at a station near the equator, under the most favorable conditions possible, may continue total for $7^m 40^s$. In latitude 45° the duration can barely equal $6\frac{1}{2}^m$. The greatest possible excess of the apparent semidiameter of the moon over that of the sun is only $1' 19''$.

At the equator an eclipse may continue *annular* for $12^m 24^s$,[1] the maximum width of the ring of the sun visible around the moon being $1' 35''$.

In the observation of an eclipse four contacts are recognized: the *first* when the edge of the moon first touches the edge of the sun, the *second* when

[1] This is longer in proportion to the size of the shadow than a total eclipse, because the moon is in apogee and moves more slowly.

the eclipse becomes total or annular, the *third* at the cessation of the total or annular phase, and the *fourth* when the moon finally leaves the solar disk. From the first contact to the fourth the time may be a little over four hours.

257. The Solar Ecliptic Limits. It is necessary, in order to have an eclipse of the sun, that the moon shall encroach on the cone *ACBD* (Fig. 103), which envelops the earth and sun. The diameter of this cone at the moon's distance is easily found to be a little over 10,000 miles. The solar ecliptic limits are therefore about 50 per cent greater than the lunar, the major limit being 18° 31′ and the minor one 15° 21′. Twice the latter quantity is

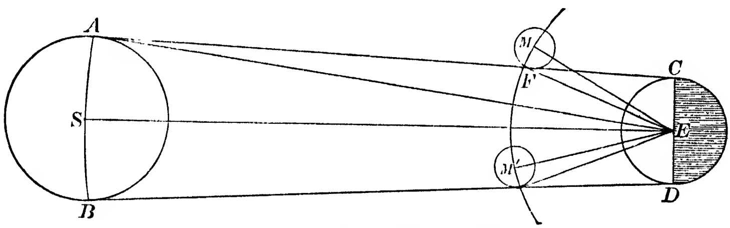


FIG. 103. Solar Ecliptic Limits

greater than the sun's motion in a synodic month, and it is therefore impossible for the sun to escape without an eclipse at an eclipse season.

In order that an eclipse may be *central* (total or annular) at any part of the earth, it is necessary that the moon shall lie *wholly inside* the cone *ACBD*, as at *M′*, and the corresponding *major central* and *minor central* ecliptic limits come out 11° 50′ and 9° 55′.

258. Phenomena of a Solar Eclipse. A partial solar eclipse is not very remarkable, though when the crescent is fairly small the loss of daylight is noticeable and the shadows cast by foliage are peculiar. The light shining through every small interstice among the leaves, instead of forming as usual a *circle* on the ground, makes a little *crescent*, — an image of the partly covered sun. A total eclipse, on the other hand, is one of the most impressive and magnificent of all natural phenomena.

About ten minutes before totality the darkness begins to be felt, and the remaining light, coming, as it does, from the *edge* of the sun alone, is much altered in quality, so that both sky

and landscape take on strange colors. Animals are perplexed, and birds go to roost. The temperature falls, and sometimes dew appears. A few minutes before the shadow reaches the observer, quivering, ripple-like *shadow bands* appear on every white surface. Just before totality, if the observer is so situated that

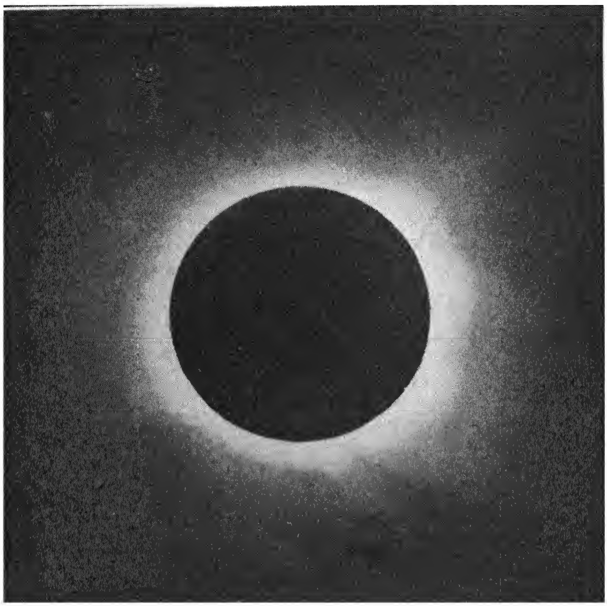


FIG. 104. The Solar Eclipse of January 24, 1925

The corona is of an intermediate type: the long equatorial and the short polar rays are characteristic of the minimum type, whereas the general extension of the corona over the polar regions is typical of the maximum type. (From photograph by Frederick Slocum, Van Vleck Observatory)

his view commands the distant horizon, the moon's shadow is sometimes seen quite distinctly, much like a heavy thunderstorm, and advancing with awe-inspiring swiftness. The last disappearing shred of the sun is often broken up, by the irregularities of the moon's limb, into specks called *Baily's beads.* With the arrival of the shadow the *corona, chromosphere,* and *prominences* become visible, together with the brighter planets and the stars of the first two or three magnitudes. The sud-

denness with which the darkness falls is startling. The sun is so brilliant that even the small portion which remains visible up to within a very few seconds of the total obscuration so dazzles the eye that it is unprepared for the sudden transition. In a few moments, however, vision adjusts itself, and it is then found that the darkness is not really very intense.

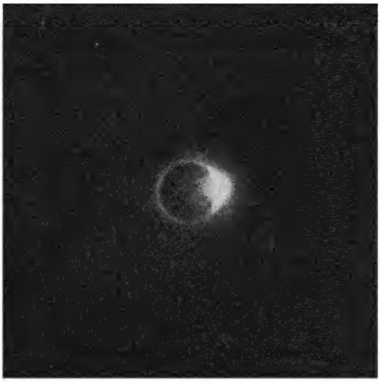


FIG. 105. The "Diamond Ring" of the Eclipse of January 24, 1925

The reappearing bit of sun is rounded out by irradiation to form the "diamond," while the still visible inner corona forms a perfect ring. Enlarged from a photograph taken with a hand camera by S. A. Korff and J. Bucher, then members of the freshman class at Princeton University

The corona and chromosphere, the lower parts of which are very brilliant, give a light about half that of the full moon. When totality is short and the shadow is of small diameter, a much larger quantity of light is sent in from the surrounding air, where, 30 or 40 miles away, the sun is still shining. In such an eclipse there is not much difficulty in reading an ordinary watch face. In an eclipse of long duration, say five or six minutes, it is much darker, and lanterns sometimes become necessary.

The darkness of totality also depends on the transparency of the atmosphere and on the presence or absence of snow on the ground. A bright glow comes from all around the horizon.

Just before the sun reappears, its lower atmosphere, shining with a steel-white light, comes into view. Then, instantaneously, the dazzling photosphere emerges, the air fills with light, and the outer corona vanishes. The inner corona remains visible for half a minute or more as a thin, yellowish ring. Owing to irradiation in the observer's eye the first speck of sun appears much larger than it really is, and strikingly resembles a jewel on a ring. The phrase "diamond ring," which is used to describe this appearance, was spontaneously coined by hundreds of people in the northeastern United States at the eclipse of January 24, 1925 (Fig. 105).

In another minute the inner corona also is drowned out by the returning light and the spectacle is over.

259. Observation of an Eclipse. The professional astronomer is too busy during the few minutes of totality to enjoy the beauties of this grandest of all spectacles. He and his assistants have spent months of preparation in constructing apparatus, in traveling perhaps to some distant and primitive region, in setting up and adjusting the instruments, and in rehearsing time and again the program which must be carried out without a hitch. During totality many plates are exposed and changed, leaving time for the astronomer to indulge in but a hurried glance at the corona.

A total eclipse of the sun offers opportunities for numerous observations of great importance which are possible at no other time. We mention:

(1) Observations for the determination of the relative positions of the sun and moon. These include the times of the four contacts (§ 256), the geographical boundaries of the shadow zone, and photographs of the partial phases.

(2) Photographic observation of the sky near the sun, including the search for possible intra-Mercurial planets (§ 423), which has practically proved their absence, and measures of the deflection of the light from stars seen near the sun in the sky, which provides one of the tests of the theory of general relativity (§ 363).

(3) Direct photography of the corona and prominences.

(4) Spectroscopic observation of reversing layer, chromosphere, and corona.

(5) Photometric measurement of the intensity of the corona and of the partial phases.

(6) Bolometric measurement of the heat radiation of the corona.

(7) Observation of the degree of polarization of the light of the corona.

(8) Miscellaneous observations upon the shadow bands, the meteorological changes during the progress of the eclipse (barometric pressure, temperature, wind, etc.), and of the effects upon the magnetic elements and upon the transmission of radio.

Most of these subjects will be discussed later. It may be said here that convincing evidence has been obtained that the shadow bands are the shadows of streaks of irregular density in the air flying with the wind. They are not usually visible because the sun's disk is so large that these shadows are completely blurred and run together, like those of the meshes of a wire screen on a surface ten feet away. But as the disappearing crescent becomes narrow, they become more and more conspicuous. Being of atmospheric origin, they may be of very different intensity and motion at points a few miles apart, as was noticed in 1925.

260. Calculation of a Solar Eclipse. The calculation of a solar eclipse cannot be dealt with in any such summary way as that of a lunar eclipse, because the times of contact and other circumstances are different at every different station. Moreover, since the phenomena of a solar eclipse admit of extremely accurate observation, it is necessary to take account of numerous little details which are of no importance in lunar eclipses. The nautical almanacs give, three years in advance, a chart of the track of every solar eclipse, and with it data for the accurate calculation of the phenomena at any given place.

261. Number of Eclipses in a Year. The least possible number is *two*, both of the sun; the largest *seven*, five solar and two lunar, or four solar and three lunar. The *most usual* number of eclipses is four.

The eclipses of a given year always take place at two opposite seasons (which may be called the *eclipse months* of the year), near the times when the sun crosses the nodes of the moon's orbit. Since the nodes move westward around the ecliptic once in about nineteen years (§ 188), the time occupied by the sun in passing from a node to the same node again is only 346.62 days, which is sometimes called the *eclipse year*.

In an *eclipse year* there can be but *two lunar* eclipses, since twice the maximum lunar ecliptic limit ($2 \times 12° 15'$) is less than $29° 6'$, the distance the sun moves along the ecliptic in a synodic month. The sun, therefore, cannot possibly be near enough to the node at *both* of two successive full moons; on the other hand, it is possible for a year to pass without any lunar eclipse, the

sun being too far from the node at all four of the full moons which occur nearest to the time of its node passage.

In a *calendar* year (of 365¼ days) it is, however, possible to have *three* lunar eclipses. If one of the moon's nodes is passed by the sun in January, it will be reached again in December, the other node having been passed in the latter part of June, and there may be a lunar eclipse at or near each of these three node passages. This actually occurred in 1898 and 1917, and will happen again in 1982.

As to solar eclipses, it is sufficient to say that the solar ecliptic limits are so much larger than the lunar that there *must be at least one solar* eclipse at each node passage of the year, at the new moon, and that there may be *two*, one before and one after, thus making four in the eclipse year. (When there are two solar eclipses at the same node, there will always be a total lunar eclipse at the full moon between them.) In the *calendar* year a fifth solar eclipse may come in if the first eclipse month falls in January. Since a year with five solar eclipses in it is sure to have two lunar eclipses in addition, they will make up seven in the calendar year. This will happen next in 1935; and in 1917 there were also seven eclipses, — four of the sun and three of the moon.

262. Relative Frequency of Solar and Lunar Eclipses. Taking the whole earth into account, the solar eclipses are the more numerous, nearly in the ratio of *three to two. It is not so, however, with those which are visible at a given place.* A solar eclipse can be seen only from a limited portion of the globe, while a lunar eclipse is visible over considerably more than half the earth (either at the beginning or the end, if not throughout its whole duration), and this more than reverses the proportion between the lunar and solar eclipses for any given station.

Solar eclipses that are *total* somewhere or other on the earth's surface are not very rare, averaging one for about every year and a half. But *at any given place* the case is very different. Since the track of a solar eclipse is a very narrow path over the earth's surface, averaging only 60 or 70 miles in width, we find that in the long run a total eclipse happens at any given station only once in about 360 years.

Most of the eclipse tracks fall mainly in the ocean; indeed, it is remarkable how a track 8000 miles long and more than a hundred miles wide may miss all land except a few small islands, as happened in 1908. Where the track crosses land, the country is often difficult of access or subject to cloudy weather, so that the number of promising observing stations is often small, and long journeys may have to be made to reach them.

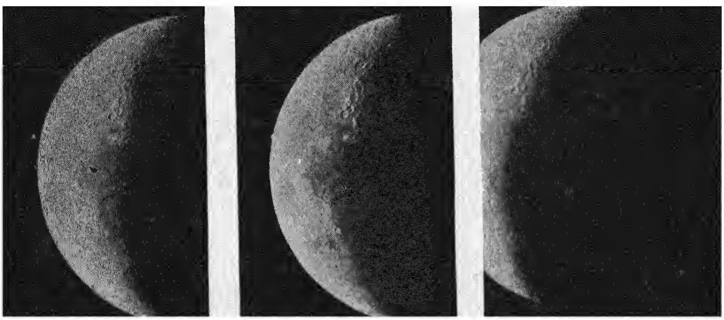
263. Recurrence of Eclipses; the Saros. It was known to the Chaldeans, even in prehistoric times, that eclipses occur at a regular interval of $18^y\ 11\frac{1}{3}^d$ ($10\frac{1}{3}$ days if there happen to be *five* leap-years, on the modern reckoning, in the interval). They named this period the *Saros*. It consists of 223 *synodic months*, containing 6585.32 days; and 19 *eclipse years* contain 6585.78. The difference is only about 11 hours; in that time the sun moves on the ecliptic, relative to the node, about 28′. The interval is also very nearly equal to 239 anomalistic months (from perigee to perigee), which amount to 6585.54 days.

If, therefore, a solar eclipse should occur today with the sun *exactly* at one of the moon's nodes, at the end of 223 months (or 18 years and 11 days) the new moon would be at almost the same distance from the earth and would find the sun again close to the node (28′ *west* of it), and a very similar eclipse would occur again; but the track of this new eclipse would lie about 8 hours of longitude farther west on the earth, because the 223 months exceed the even 6585 days by 32/100 of a day. The usual number of solar eclipses in a Saros is about seventy-one, varying two or three one way or the other. About forty-five of them are central (either total or annular) somewhere or other on the earth.

264. Occultations of Stars. In theory and computation the occultation of a star is identical with a total solar eclipse, except that the shadow of the moon is sensibly *a cylinder* instead of a cone, and has no penumbra. Since the moon always moves eastward, the star disappears at the moon's eastern limb and reappears on the western (Fig. 106). Under all ordinary circumstances both disappearance and reappearance are instantaneous, indicating not only that the moon has no sensible atmosphere, but also that the angular diameter of even a very bright star

is very small. Observations of occultations determine the place of the moon in the sky with great accuracy, and when made at a number of widely separated stations they furnish a precise determination of the moon's parallax and also of the difference of longitude between the stations.

Occasionally the star, instead of disappearing suddenly when struck by the moon's limb (faintly visible by earth-shine), appears to cling to the limb for a second or two before vanishing. In a few instances it has been reported as having reappeared and disappeared a second time, as if it had been for a moment visible through a rift in the moon's crust. In some cases



C *B* *A*

FIG. 106. Occultation of Aldebaran, March 22, 1904

A, Aldebaran is seen at the right-hand border just before disappearance at the dark limb of the moon. *B*, just emerging at the bright limb. *C*, $1^m 40^s$ after emergence. The exposure time was correct for the moon; the image of the bright star is enlarged by overexposure. (From photograph by R. J. Wallace, Yerkes Observatory)

the anomalous phenomena have been explained by the subsequent discovery that the star was double, but many of them still remain mysterious; it is quite likely that they were often illusions of the observer, due to physiological causes.

265. Recent and Coming Eclipses. Total eclipses visible in the United States occurred on May 28, 1900, from Louisiana to Virginia, duration of totality $1\frac{1}{2}$ minutes; on June 8, 1918, from the state of Washington to Florida, 2 minutes; on September 10, 1923, California, $3\frac{1}{2}$ minutes; and on January 24, 1925, from Minnesota to Connecticut, 2 minutes. The third of these was obscured by unexpected bad weather, although observations were secured in Mexico. The weather was fair for the second, and

excellent for the first and the last. In the last case the shadow passed over a densely populated region in the southern part of New York and New England, with limits extending from New York City to Providence, Rhode Island, and the eclipse was seen by millions of people. Reports from hundreds of amateur

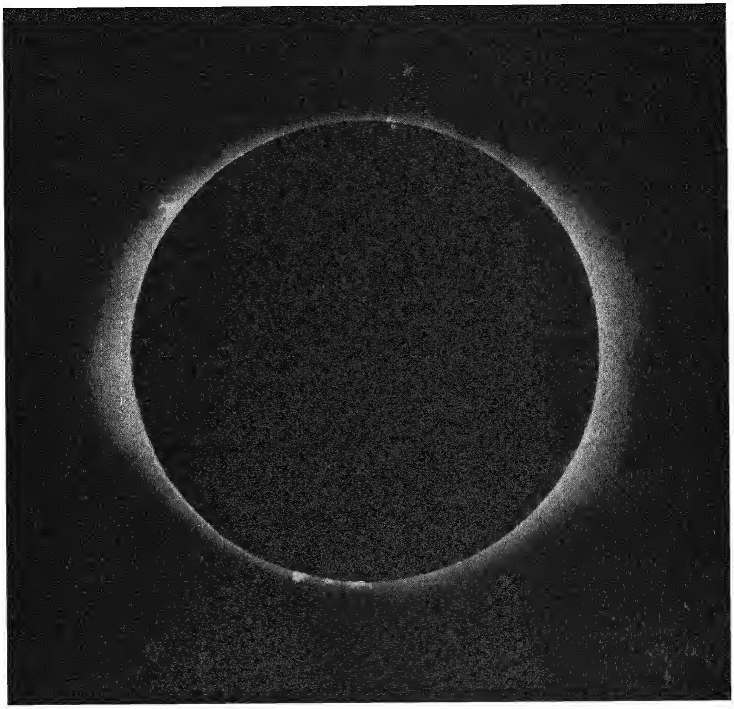


FIG. 107 *A*. The Eclipse of June 8, 1918, Goldendale, Washington

This photograph was taken soon after second contact. The chromosphere on the eastern (left) limb of the sun is not yet covered by the moon. The prominences are unusually large and spectacular. (From photograph by Lick Observatory)

observers regarding the limits of the shadow and the duration of totality promise to be of considerable scientific value. Other noteworthy recent total eclipses are those of May 18, 1901, Sumatra, 6 minutes; August 30, 1905, Labrador, Spain, and Egypt, $3\frac{1}{2}$ minutes; January 3, 1908, Pacific Ocean, 4 minutes; May 29, 1919, Brazil and West Africa, 6 minutes; September 21, 1922, Australia, 5 minutes; January 14, 1926, Sumatra, $3\frac{1}{2}$ minutes.

Eclipses will occur on June 29, 1927, England and Norway, 30 seconds; on May 9, 1929, Sumatra, 5 minutes; on August 31, 1932, Vermont to Maine, $1\frac{1}{2}$ minutes; on June 8, 1937, Pacific Ocean, 7 minutes. It will be noticed that the three eclipses of longest duration are repetitions at intervals of the Saros.

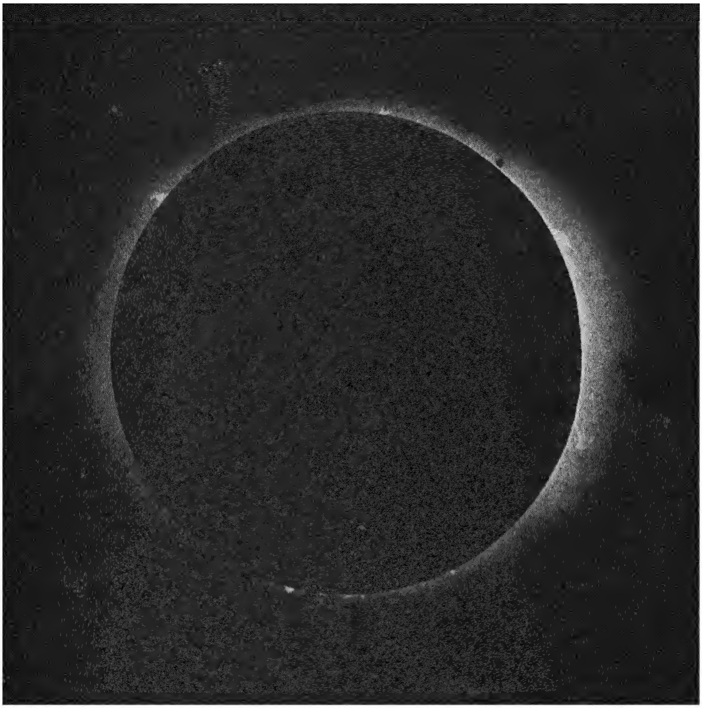


FIG. 107 *B*. The Eclipse of June 8, 1918, Goldendale, Washington

This photograph was taken a few seconds before third contact. Comparison with Fig. 107 *A* shows the motion of the moon. On the western limb the chromosphere is appearing and the huge prominences are entirely uncovered. Note the coronal arches over one of these prominences. (From photograph by Lick Observatory)

Computations made in the office of the *American Ephemeris*, and communicated before publication by courtesy of the superintendent of the Naval Observatory and the director of the *Nautical Almanac*, show that the solar eclipse of April 28, 1930, will be just total. The central line runs from California through Nevada to Montana, and the maximum duration of

totality is only one and a half seconds, since the tip of the umbra barely grazes the earth's surface. Farther west and east the eclipse is annular.

EXERCISES

1. Why cannot there be an annular eclipse of the moon?

2. Which are more frequent in New York, solar eclipses or lunar?

3. If a lunar eclipse has occurred this year in August, can there be one in June of next year? in October? If not, why not?

4. Can an occultation of Venus occur during an eclipse of the moon? Is one of Jupiter possible?

5. In a solar eclipse, which side of the sun's disk is first touched by the moon, the east or the west?

6. Can a total solar eclipse be seen at midnight?

Ans. Yes, in the polar regions. (Describe the circumstances in detail.)

7. Can a total lunar eclipse be seen at local noon?

8. What would be the appearance of the earth, as seen from the moon, at the time of a total solar eclipse? of a total lunar eclipse?

9. Under what circumstances can an eclipse give us information regarding the weather at and near the north pole?

10. The battle of Princeton was fought on January 3, 1777. There was a solar eclipse on January 9. At what hour (approximately) did the moon rise on the night before the battle?

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S. A. Mitchell, *Eclipses of the Sun* (Columbia University Press), a complete account of what has been learned about the sun from observation during eclipse.

T. Oppolzer, *Canon der Finsternisse* (Canon of Eclipses), a remarkable book containing the elements of all eclipses (8000 solar and 5200 lunar) occurring between 1207 B.C. and A.D. 2162, with maps showing the approximate tracks of the moon's shadow on the earth.

William F. Rigge, *The Graphic Construction of Eclipses and Occultations* (Loyola University Press, 1924), gives the methods by which the circumstances of an eclipse are predicted graphically.

CHAPTER IX

THE PLANETS IN GENERAL

NAMES, DISTANCES, AND PERIODS · APPARENT MOTIONS · THE ELEMENTS OF A PLANET'S ORBIT · THE DETERMINATION OF ORBITS · STUDY OF THE PLANETS THEMSELVES: DIAMETER, MASS, SURFACE PECULIARITIES, ROTATION · SATELLITE SYSTEMS · CLASSIFICATION OF THE PLANETS

266. The *stars* preserve their relative configurations, however much they may alter their positions in the sky from hour to hour. The Dipper always remains a dipper in every part of the diurnal circuit.

But certain of the heavenly bodies, and the most conspicuous of them, behave differently. The sun and the moon move always steadily eastward through the constellations; and a few others, which look like brilliant stars, but are not stars at all, creep back and forth among the star groups in a less simple manner.

These moving bodies were called by the Greeks *planets*, that is, wanderers. The steadiness of their light, that is, the absence of twinkling (§ 117), unless they are low down near the horizon, also distinguishes them from the stars. The Greeks enumerated seven, — Mercury, Venus, Mars, Jupiter, and Saturn, and, in addition, the Sun and Moon.

267. List of Planets. At present the sun and moon are not reckoned as planets, but the earth is, and the number known to the ancients has been increased by two new worlds (Uranus and Neptune, of great magnitude though inconspicuous on account of their distance), besides a host of little bodies called asteroids.

The list of the principal planets in their order of distance from the sun stands thus at present: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune.

Between Mars and Jupiter, where there is a wide gap in which another planet might be expected, there have been discovered more than a thousand asteroids, which may represent a single planet that was somehow "spoiled in the making," so to speak.

One of this family, Eros, discovered in 1898, crosses the inner boundary mentioned (the orbit of Mars), and at times comes nearer to the earth than any other heavenly body except the moon.

The planets are *non-luminous* bodies which shine only by *reflected sunlight*; they are globes which, like the earth, *revolve around the sun* in orbits nearly circular, moving all of them in the same direction and (with exceptions among the asteroids) nearly in the common plane of the ecliptic. How much of the illuminated hemisphere can be seen from the earth depends upon the angle at the planet between lines to the earth and to the sun. Where this *phase angle* is always very small, as it is for the most distant planets, there is never any perceptible deviation from a circular disk. The planets Mercury and Venus, whose orbits are smaller than that of the earth, go through all *phases*, like the moon, from new to full.

All but the inner two planets and the asteroids are attended by satellites. Of these the Earth has one (the moon), Mars two, Jupiter nine, Saturn nine, Uranus four, and Neptune one. Four of these satellites have been discovered since 1900, and other faint ones may await discovery.

268. Names, Distances, and Periods. The distances of these planets from the sun, and their periods of revolution about it, have been very accurately determined. Their values are given in Table I.

TABLE I. NAMES, DISTANCES, AND PERIODS OF THE PLANETS

NAME	SYMBOL	DISTANCE		SIDEREAL PERIOD		SYNODIC PERIOD
		Astronomical Units	Millions of Miles	Sidereal Years	Mean Solar Days	Days [1]
Mercury	☿	0.387	36.0	0.2408	88.0	116
Venus .	♀	0.723	67.2	0.6152	224.7	584
Earth . .	⊕	1.000	92.9	1.0000	365.3	
Mars . .	♂	1.524	141.5	1.8808	687.0	780
Asteroids		1.46 to 5.71	136 to 530	1.76 to 13.7		394 to 845
Jupiter .	♃	5.203	483	11.862		399
Saturn .	♄	9.539	886	29.457		378
Uranus .	⛢ or ♅	19.191	1782	84.013		370
Neptune	♆	30.071	2793	164.783		367½

[1] Unless distinctly stated otherwise, "days" refers to mean solar days (§ 36).

The *sidereal period* of a planet is the time of its revolution around the sun, from a *star* to the same star again, *as seen from the sun*. The *synodic period* is the interval between two successive times when the planet occupies the same position in relation to the sun *as seen from the earth.*

Let us imagine ourselves on the sun, watching the planets traveling around their orbits at different speeds. The earth and another planet start together. One gains a lap on the other and completes the synodic period of the planet. Mercury travels so fast that it catches up with the earth again before the earth has gone more than one third of the way around. Neptune moves only about two degrees while the earth is making a complete circuit, and is overtaken in 367½ days. The earth and Mars are so evenly matched that it takes the earth over two years to overtake Mars.

The sidereal and synodic periods are connected by the same relation as the sidereal and synodic months (§ 186), namely, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$, in which E, P, and S are, respectively, the sidereal periods of the earth and the planet, and the planet's synodic period; and the numerical difference between $\frac{1}{P}$ and $\frac{1}{E}$ is to be taken *without regard to sign*; that is, for an *inferior* planet, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$; for a *superior* one, $\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$.

269. The Harmonic Law. There is evidently a relation between the distance of a planet and its period; the more distant planets have the longer periods. The exact nature of this relation was discovered by Kepler early in the seventeenth century, and constitutes his famous harmonic law, *the squares of the periods are proportional to the cubes of the mean distances from the sun*, which may readily be verified from the table. This relation was shown by Newton to be a consequence of the law of gravitation, and will be discussed in Chapter X.

There is a curious approximate relation between the distances of the planets from the sun, usually known as *Bode's law* because first brought prominently into notice by Bode in 1772, though it appears to have been discovered by Titius of Wittenberg some years earlier.

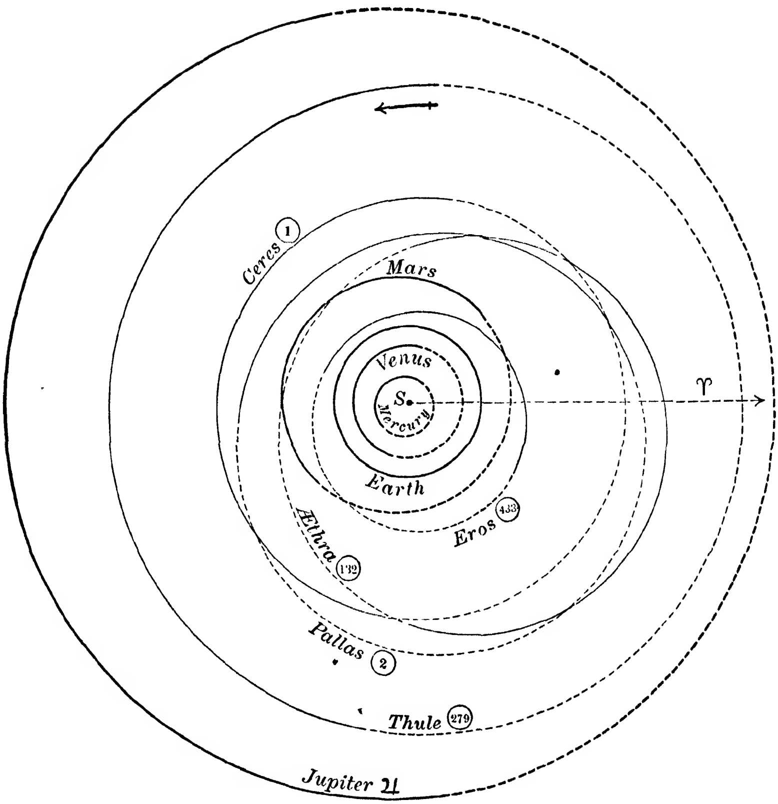

FIG. 108. The Planetary Orbits

The radius of the earth's orbit is taken as one centimeter, and the other orbits (as far out as Jupiter) are drawn to scale, as seen from the northern side. The orbits of Mercury, Mars, Jupiter, and of several of the asteroids are quite distinctly eccentric

It is this: Write a series of 4's. To the second 4 add 3; to the third add 3 × 2, or 6; to the fourth, 6 × 2, or 12; and so on, doubling the added number each time, as in the following scheme:

	4	4	4	4	4	4	4	4	4
		3	6	12	24	48	96	192	384
Sum	4	7	10	16	28	52	100	196	388
Distance	3.9	7.2	10.0	15.2	26.5	52.0	95.4	191.9	300.7

The resulting numbers are approximately equal to the mean distances of the planets from the sun, if the earth's distance is taken as 10. It is not known whether Bode's law is a mere coincidence or whether it has some

physical explanation which may be understood in the future. For Neptune the law breaks down, and it really does so for Mercury, for which the law should give the distance 5.5 (adding half of 3, instead of 0, to 4).

270. Planetary Configurations. Fig. 109 illustrates the meaning of the terms used in describing the position of a planet with respect to the sun. E is the position of the earth, the inner circle is the orbit of an *inferior* planet (Mercury or Venus), and the outer circle is that of a *superior* planet, — Mars, for instance.

The *elongation* of a planet is the angle between lines drawn from the observer to the planet and to the sun, that is, the apparent angular distance of the planet from the sun; for a planet at P it is the angle SEP.

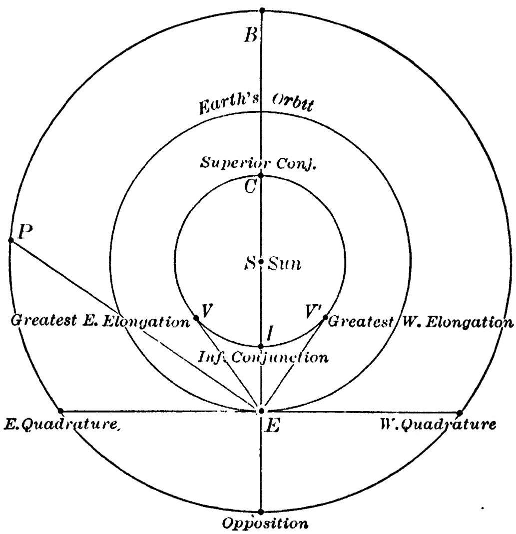

FIG. 109. Planetary Configurations

For a *superior* planet the elongation can have any value from 0° to 180°. For an *inferior* planet there is a certain maximum value, called the *greatest elongation*, which must be less than 90°. This greatest elongation is the angle between a line drawn from the earth to the sun and another line drawn tangent to the planet's orbit, — the angle SEV in the figure.

Absolute conjunction occurs when the elongation of the planet is zero; *superior conjunction*, when the planet is in line with and beyond the sun; *inferior conjunction*, when the planet is between the earth and the sun, — a position which is impossible for a superior planet. *Conjunction in longitude* occurs when the planet's longitude is the same as the sun's; *conjunction in right ascension*, when it has the same right ascension as the sun.

Opposition in longitude occurs when the difference in longitude of sun and planet is 180°; *quadrature* in longitude, when this difference is 90°. A planet when in opposition is on the meridian about midnight; when in quadrature, about 6 A.M. or 6 P.M. No inferior planet can come to either opposition or quadrature.

These planetary positions were included among the "aspects" once recognized by astrologers.

271. Apparent Motions of the Planets. If we imagine ourselves looking down upon the orbits perpendicularly from their northern side, so as to see them *in plan*, they appear as shown in Fig. 108, and the planets travel regularly forward (counter-clockwise) with a steady, almost uniform, motion. Viewed from the earth, however, the orbits appear nearly edgewise, and the apparent motions are complicated, being made up of each planet's own motion around the sun, combined with an apparent motion due to the movement of the earth.

Their apparent motion as seen by us may be considered under three different aspects:

(1) The motion *in space relative to the earth.*

(2) The motion *on the celestial sphere relative to the constellations*, that is, change of right ascension and declination or of celestial latitude and longitude.

(3) *Changes in the apparent angular distance from the sun*, that is, motion *in elongation.*

272. Motion in Space Relative to the Earth. The fundamental principle of relative motion is that if we look at a body at rest while we ourselves are moving, its *relative motion*, that is, the *change in its distance and direction from us*, will be the same as if we were at rest and it possessed our motion *reversed.* If we look at a body while we move to the *south*, it appears to move toward the *north.* If we *approach* it, the effect is the same as if it *were coming toward us*, and so on.

If the body has a motion of its own, then the total apparent, or relative, motion will be the *resultant* of its own motion combined with our reversed motion, according to the law of composition of motions.

A planet at rest with respect to the sun, therefore, would appear to move in an orbit precisely like that of the earth in

form and size, and in the same plane, always keeping its motion opposed to our own though going around this apparent orbit *in the same direction* as the earth (just as any two opposite points on the circumference of a revolving wheel are always moving in opposite directions though going the same way around the axis). And since the planets are really revolving around the sun, it follows that their apparent, or *geocentric*, motion is a combination of two motions, — that of a body moving once a year around the circumference of a circle[1] equal to the earth's orbit, while at the same time the center of that circle is carried around the sun in the real orbit of the planet, and in the planet's period. Jupiter, for instance, as seen from the earth, appears to move as in Fig. 110.

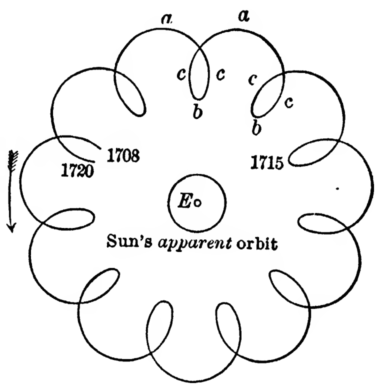

FIG. 110. Geocentric Motion of Jupiter from 1708 to 1720

After Cassini

This is the orbit that we should find if we were to attempt to map it out by the method used for determining the form of the orbit of the earth around the sun (§ 158), that is, by observing the *direction* of the planet from the earth and at the same time measuring its *angular diameter* in order to get its relative distances at different times. Practically, however, this method would not here succeed very well, since the planet's apparent diameter is too small to permit the necessary precision in determining the variations of distance.

A motion of the kind represented in the figure is loosely called epicycloidal, — not quite accurately, because the orbits concerned are not true circles, so that the loops are of varying size.

The Ptolemaic theory of the solar system was fundamentally an acceptance of this apparent motion of the planets, relative to the earth, as real, though the theory involved in addition certain serious errors of arrangement and proportion (§ 277).

[1] The "circles" spoken of here are, strictly, ellipses of small eccentricity.

273. Motion of a Planet on the Celestial Sphere, that is, in Right Ascension and Declination, or in Longitude and Latitude. Looking at Fig. 110, we see that, viewed from the earth, the planet moves most of the time "direct," that is, *eastward* in the direction of the arrow, as at the points *aa*; but while rounding the loops at *bb*, where it comes nearest the earth, its apparent motion is reversed and "retrograde," and at certain points *cc* on each side of the loop the planet is "stationary" in the sky, its motion at the time being directly toward or from the earth.

Starting from the time of superior conjunction, when the planet is at *a*, it moves eastward, or direct, among the stars, always increasing its right ascension or longitude, but at a rate continually slackening, until at last the planet becomes stationary at an elongation from the sun which depends upon the size of the orbit and its distance from the earth.

From the stationary point it reverses its course and moves *westward* around the loop until it comes to the second stationary point on the other side of the loop. There it resumes its eastward motion and continues it until it reaches the next superior conjunction, at the end of a synodic period.

The middle of the *arc of regression* is always very near the point where the planet comes nearest the earth, that is, at opposition for a superior planet, and at inferior conjunction for an inferior planet. In time, as well as in the number of degrees passed over, the direct motion always exceeds the retrograde in each synodic period of the planet.

As observed with a transit instrument, all planets, when moving eastward (direct), come *later* to the meridian each night *by the sidereal clock*, and vice versa when retrograding.

274. Motions in Latitude. If the orbits of the planets all lay precisely in the same plane with the earth's orbit, their apparent orbits relative to the earth would do so as well, and their apparent motions on the celestial sphere would be simply *forward and backward along the ecliptic*.

But while the orbits of the planets are only slightly inclined to the ecliptic, so that they never go very far from it, they do, in fact, deviate a few degrees on one side or the other, so that their paths in the heavens form more or less complicated loops and kinks.

Fig. 111 shows the loops made by Jupiter and Saturn in 1901, when they were very near each other and, for a short time, near the rapidly moving Venus.

Venus may attain a latitude of almost 9°, Mars nearly 7°, and Mercury almost 5°. None of the other planets can reach 3°.

Certain of the asteroids have orbits greatly inclined to the ecliptic and very eccentric. The description of apparent motions as given above would therefore require very serious modification in their case. Eros is sometimes found in circumpolar regions more than 40° north of the ecliptic; sometimes its nearest approach to the earth does not coincide with the time of its opposition within several weeks; and sometimes at the time of its opposition its motion is more nearly from *north to south* than from east to west. Fig. 112 shows the track which Eros will follow through opposition in 1931.

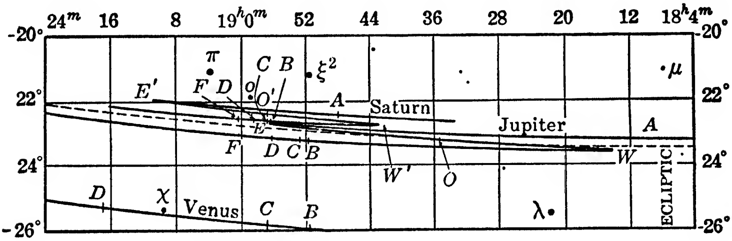

FIG. 111. Apparent Motions of Saturn, Jupiter, and Venus in 1901

Saturn and Jupiter were describing their loops through opposition, while Venus passed close by to the south. The opposition points, at the middle of the retrograde arcs, are marked O; the stationary points, E and W (east and west). Corresponding points A, B, C, D, and F are reached by two or more planets on the same date. Venus and Jupiter are in conjunction at B, Venus and Saturn at C, Jupiter and Saturn at F. The dates are as follows: Saturn, E', April 25; O', July 5; W', September 14. Jupiter, E, April 29; O, June 30; W, August 30. A, January 30.0; B, November 17.7; C, November 18.7; D November 23.0; F, November 27.7. Saturn, being more distant, describes a shorter loop than Jupiter and moves more slowly. Jupiter passed the descending node near the time of opposition, and Saturn was approaching the corresponding point in its orbit

275. Motion of the Planets in Elongation, that is, with Respect to the Sun's Place in the Sky. The visibility of a planet depends mainly on its *elongation*, because when near the sun the planet will be above the horizon only by day. Considered from this point of view, there is a marked difference between the inferior planets and the superior planets.

(1) *The superior planets always drop steadily westward with respect to the sun's place in the heavens*, continually increasing their western elongation or decreasing their eastern elongation.

As observed by an ordinary timepiece (keeping *mean solar time*), they therefore invariably rise earlier and come *earlier to the meridian* on each successive night, never moving eastward among the stars as rapidly as the sun, even when their direct motion is most rapid. This relative motion westward with respect to the sun is not,

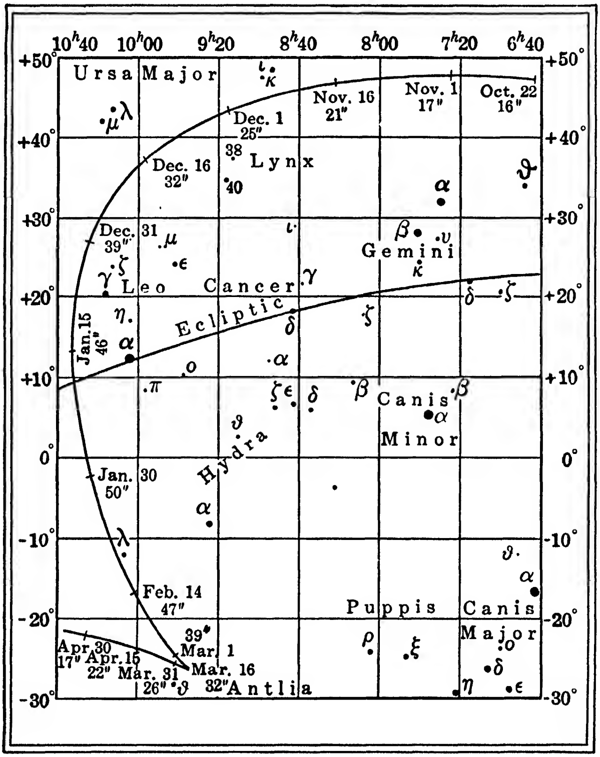

FIG. 112. Path of Eros through Opposition, in 1931

Positions of the planet two weeks apart are marked on the path. Below the date is given the parallax. In four and one-half months Eros travels from declination 48° N to 26° S. Opposition occurs on February 17, but the parallax is not greatest on that date

however, uniform. It is slowest near superior conjunction, when the planet is moving eastward among the stars (that is, in the same direction as the sun is apparently moving), and most rapid at opposition, when the planet is retrograding.

Starting at conjunction, the planet is then behind the sun, at its greatest distance from the earth, and invisible. Soon, how-

ever, it reappears in the morning, rising before the sun as a morning star, and passes on to western quadrature, when it rises near midnight. Thence it moves on to opposition, when it is nearest and brightest, and rises at sunset. Still dropping westward, and now receding, it passes to eastern quadrature and is on the meridian at sunset. Thence it still crawls sluggishly westward as an evening star until it is lost in the twilight and completes its synodic period by again reaching conjunction.

276. (2) The *inferior planets*, on the other hand, apparently *oscillate* across the sun, moving out equal or nearly equal distances on each side of it, but making the westward swing, between us and the sun, much more quickly than the eastward swing, through superior conjunction.

At superior conjunction an inferior planet is moving eastward *faster* than the sun. Accordingly it creeps out into the twilight as an evening star and continues to increase its apparent distance from the sun until it reaches its *greatest eastern elongation* (47° for Venus; from 18° to 28° for Mercury). Then the sun begins to gain upon it, and as the planet itself soon begins to retrograde, the elongation diminishes rapidly and the planet hurries back to *inferior conjunction*, passes it, and then, as a morning star, moves swiftly out to its western elongation. There it turns and climbs slowly back to superior conjunction again.

277. The Ptolemaic System. Assuming the fixity and central position of the earth and the actual revolution of the heavens, Ptolemy (who flourished at Alexandria about A.D. 140) worked out the system which bears his name.

In his great work, the *Almagest* (from Arabic *al*, "the," and the Greek μεγίστη meaning "greatest"), which for fourteen centuries was the authoritative "scripture of astronomy," he showed that all the apparent motions of the planets (including the sun and moon) so far as then observed, could be accounted for by supposing each planet to move around the circumference of a circle called the epicycle, while the center of this circle, sometimes called the fictitious planet, itself moved ***around the earth*** on the circumference of another and larger circle called the deferent (see Fig. 113).

It was as if the real planet were carried on the end of an arm which turned around the fictitious planet as a center in such a way as to point toward or from the earth at times when the planet was in line with the sun.

Fig. 113 represents this Ptolemaic system, except that no attention is paid to dimensions, the deferents being spaced at equal distances.

It will be noticed that the epicycle radii which carry at their extremities the planets Mars, Jupiter, and Saturn are always parallel to the line which joins the earth and the sun.

In the case of Venus and Mercury this is not so. Ptolemy supposed that for these planets the *deferent* circles lay *between* the earth and the sun, and that the fictitious planet in both cases revolved in its deferent once a year, always keeping exactly between the earth and the sun; the motion in the *epicycle* in this case was completed in the time of the planet's period.

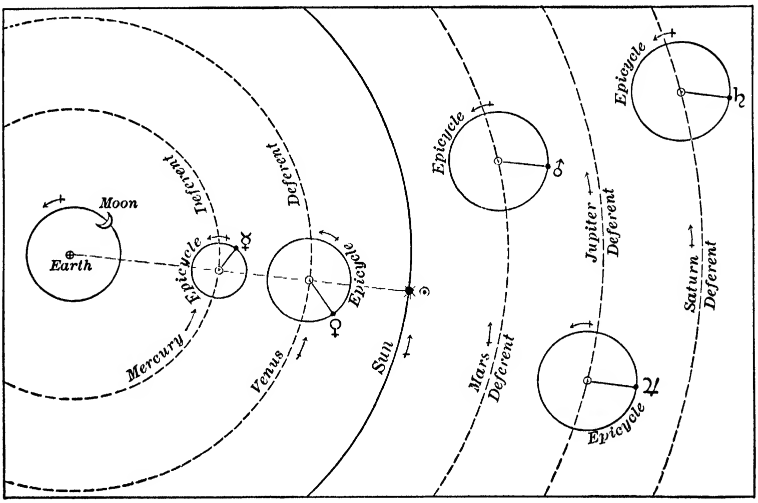

FIG. 113. The Ptolemaic System

He did not recognize that for these two planets there should be only one deferent, namely, the orbit of the sun itself, as the ancient Egyptians are said to have understood.

To account for some of the irregularities of the planets' motions it was necessary to suppose that both the deferent and the epicycle, though circular, are eccentric, the earth not being exactly in the center of the deferent, nor the fictitious planet in the exact center of the epicycle. In later times, when the knowledge of the planetary motions had become more accurate, the Arabian astronomers added epicycle upon epicycle until the system became very complicated.

278. The Copernican System. Copernicus (1473–1543) asserted the diurnal rotation of the earth on its axis, which was rejected by Ptolemy, and showed that it would fully account for the appar-

ent diurnal revolution of the stars. He also showed that nearly all the known motions of the planets could be accounted for by supposing them to revolve around the sun, with the earth as one of them, in orbits *circular* but slightly *out of center*. His system, as he left it, was nearly that which is accepted today. He was, however, obliged to retain a few small epicycles to account for certain of the irregularities.

Up to this time no one dared to doubt the exact circularity of celestial orbits. It was considered metaphysically improper that heavenly bodies should move in any but *perfect* curves, and the circle was regarded as the only perfect one. It was left for Kepler, some sixty years later than Copernicus, to show that the planetary orbits are *elliptical*, and to bring the system into substantially the form in which we know it now.

It was nearly a century before the Copernican system, with the improvements of Kepler, finally replaced the Ptolemaic. In our oldest American universities, Harvard and Yale, the Ptolemaic was for a considerable time taught in connection with the Copernican.

279. Tychonic System. Tycho Brahe, who came between Copernicus and Kepler, found himself unable to accept the Copernican system for two reasons. One was that it was unfavorably regarded by the Church, and he was a good churchman; the other was the really scientific objection that if the earth moved around the sun, the fixed stars all ought to appear to move in a corresponding manner, each star describing annually an ellipse in the heavens of the same apparent dimensions as the earth's orbit seen from the star. Technically speaking, each star ought to have an *annual parallax.*

His instruments were by far the most accurate that had ever been made, and he could detect no such parallax (although it really existed and can now be observed); hence he concluded, not illogically though incorrectly, that the earth must be at rest.

He rejected the Copernican system, placed the earth at the center of the universe, according to the then received interpretation of Scripture, made the sun revolve around the earth once a year, and then (this was the peculiarity of his system) made the apparent orbit of the sun the *common deferent* for the epicycles of all the other planets, making them revolve around the sun.

This theory just as fully accounts for all the motions of the planets as the Copernican or Ptolemaic, but, like the Ptolemaic, breaks down absolutely when it encounters the *aberration of light* and the *annual parallax of the stars*,

now observable with modern instruments, though not with Tycho's. The Tychonic system was never generally accepted, and the Copernican was soon firmly established by Kepler and Newton.

280. Elements of a Planet's Orbit. These are a set of numerical quantities, seven in number, which describe the orbit with precision and furnish the means of finding the planet's place in the orbit at any given time, whether past or future, so far as that place depends upon the attraction of the sun alone. They are as follows:

(1) The semi-major axis, a.

(2) The eccentricity, e.

(3) The inclination to the ecliptic, i.

(4) The longitude of the ascending node, ☊.

(5) The angle from the ascending node to the perihelion point, ω.

(6) The period, P, or else the daily motion, μ.

(7) The epoch, E, or the time of perihelion passage, T.

Of these the first five pertain to the orbit itself, regarded as an ellipse lying in space with one focus at the sun, while two are necessary to determine the planet's place in the orbit.

281. The *semi-major axis*, a (CA in Fig. 114), defines the *size* of the orbit and is usually expressed in astronomical units. (It will be remembered that the earth's mean distance from the sun is the "astronomical unit.")

The *eccentricity*, e, defines the orbit's *form*. It is the fraction c/a, obtained by dividing the distance between the sun and the center of the orbit by the semi-major axis. In some computations it is convenient to use, instead of the decimal fraction itself, the angle ϕ which has e for its sine, so that $e = \sin\phi$.

The third element, i, the *inclination*, is the angle between the plane of the planet's orbit and that of the earth's. In the figure it is the angle KNO, the plane of the ecliptic being lettered $EKLP$ and that of the orbit $ORBT$.

The fourth element, ☊ (*the longitude of the ascending node*), defines what has been called the aspect of the orbit plane, that is, the direction in which it faces. The line ot nodes is the line NN' in the figure (the intersection of the two planes of the orbit and ecliptic), and the angle ♈ SN is the longitude of the ascending

node. This angle lies in the plane of the ecliptic and is measured from the vernal equinox in the direction of the earth's motion. The planet passes from the lower, or southern, side of the plane of the ecliptic to the northern at the point n in its orbit, so that N is the ascending and N' the descending node.

The fifth and last of the elements which belong strictly to the orbit itself is ω, which defines the direction in which the major axis of the ellipse (the line pA) lies in the plane $ORBT$. It is

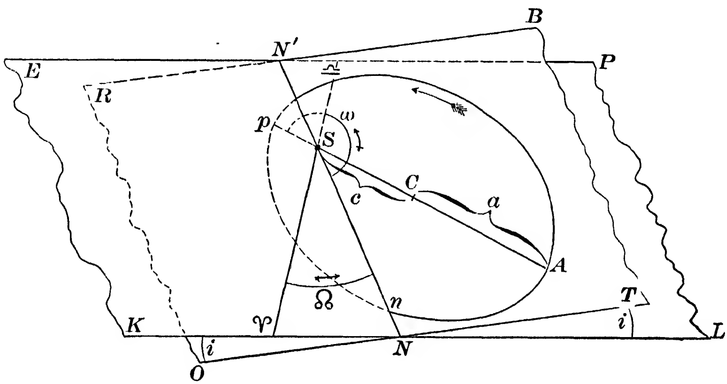

FIG. 114. The Elements of a Planet's Orbit

measured in the plane of the planet's orbit and in the direction of the planet's motion. The "longitude of the perihelion," π, is given by the equation $\pi = ☊ + \omega$.

When the motion of the body in its orbit is retrograde (that is, in the opposite sense from that of the earth), the inclination is regarded as greater than 90°. Thus, a comet moving backward in a plane making an angle of 10° with the plane of the ecliptic, is said to have an inclination of 170°. The definitions of ☊ and ω then apply without alteration.

If we regard the orbit as an elliptical wire hoop suspended in space, these five elements completely define its *position*, *form*, and *size*. The *plane* of the orbit is fixed by the two elements i and ☊; the *position* of the orbit in this plane, by ω; the *form* of the orbit, by e; and, finally, its *size*, by a.

To determine where the planet will be at any date we need two more elements:

Sixth, *the periodic time.* We must have the sidereal period, P, or else the mean daily motion, μ, which is simply 360° divided by the number of days in P.

Seventh, and finally, we must have a starting-point, the *epoch*, so called, — that is, the mean longitude of the planet as seen from the sun at some given date, as January 1, 1900 or 1925, or else the precise date at which the planet passed the perihelion.

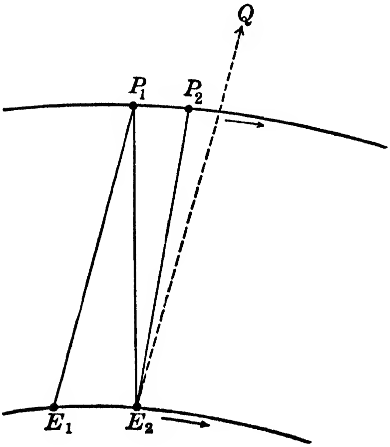

FIG. 115. Antedating an Observation of a Planet

Light leaves a planet at the time $T - t$, when it is at P_1, and reaches the earth at time T, when the earth is at E_2. Owing to aberration the planet appears to be in the direction E_2Q, which is not the same as its geometrical direction E_2P_2 at that instant. The ordinary aberration is P_1E_2Q; the planetary, $P_1E_2P_2$. Since the earth moves from E_1 to E_2 and light from P_1 to E_2 in the same interval, it is easy to show that the angle $E_1P_1E_2$ is equal to the aberration P_1E_2Q, and hence that E_1P_1 is parallel to E_2Q (§ 162). Hence the apparent direction of the planet at time T is the same as its true direction at time $T - t$

282. Ephemeris. If it were not for the perturbations (§ 328) caused by the mutual attractions of the planets these elements would never change and could be used for computing the planet's place at any date in the past or future; but in any accurate work, especially if the calculations are to extend over several revolutions of the planet, these perturbations must be calculated and allowed for. It may also be noted that if Kepler's harmonic law (§ 269) were strictly true, there would be no necessity of knowing both a and P, as either could be found from the other; but, owing to the mutual attractions of the planets, this law does not precisely hold.

When the elements of a planet's orbit are known, its direction and distance at any instant from the sun or the earth may be calculated. A series of such positions for equidistant dates constitutes an *ephemeris*. Accurate ephemerides of the sun, moon, and all the larger planets (taking full account of perturbations) are published annually in all the nautical almanacs.

We must distinguish here between the *true* direction of a planet from the earth at any moment and the *apparent* direction. An ephemeris of true directions is computed immediately from the elements of the planet's orbit. The apparent direction at the moment T differs from the true direction, both because of the aberrational displacement resulting from the earth's motion (§ 162) and because of the so-called *planetary aberration.* This is the angular distance through which the planet moves relatively to the earth in the time, t, that it takes light to travel from the planet to the earth. At the moment T the observer sees the planet where it was at the time $T - t$, when the light, by which it is observed, left it. Both aberrations may be allowed for, either by applying a correction to the true position at the time T (the procedure followed in the nautical almanacs) or (as readily can be proved) by antedating the observation by the light-time t before comparing with the ephemeris (Fig. 115). In dealing with observations of asteroids or comets great care must be taken to find out whether this correction has been applied, and if so, how.

THE DETERMINATION OF ORBITS

283. The Modern Method. By utilizing to the full the knowledge that can be obtained from the theory of gravitation it is possible to calculate all the elements of a planet's orbit from three accurate observations of its right ascension and declination, separated by a few weeks (in a few special cases a fourth observation is necessary).

The observations may be made with the meridian circle, with a filar micrometer, or by photography.

The theory is intricate and the calculation long and complicated (§ 319), but a skilled computer can carry the work through in a day or two. This method is habitually employed by astronomers when a new asteroid or comet is discovered. This problem was solved in 1801 by Gauss, then a young man of twenty-three, in connection with the discovery of Ceres, the first of the asteroids, which, after its discovery by Piazzi, was lost to observation by passing into conjunction with the sun.

284. Older Geometrical Methods. In earlier times, however, when gravitational theory was unknown, the orbits of the planets

were worked out by relatively simple geometrical methods. These demand, however, that the computer have available not three observations, but a large number, distributed over many years. Since the principal planets are bright and can hardly be confused with one another, such observations have been available for centuries past. From such a series the position of the planet at any particular moment, whether an observation was made at that time or not, can be determined by interpolation.

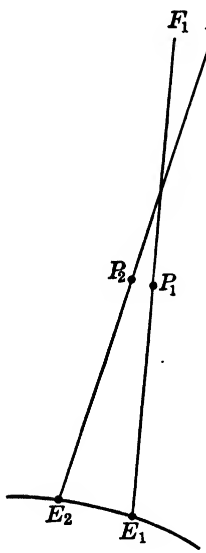

FIG. 116. Direction of a Planet

Observation of a planet gives its direction from the earth, but not its distance

This can be done *graphically* by plotting the observations on squared paper with a scale of times as abscissas, the observed data being plotted as ordinates, and then drawing a curve through the points determined by observation, as in so many operations of the physical laboratory. Whatever can be done *graphically* can, of course, be worked out still more accurately by *calculation*. The principle is of very extensive application.

The chief difficulty of the problem is that while our observations tell us the direction of the planet from the earth at a given instant, they tell us nothing directly about its distance. We know that the planet, at a certain instant, was on a certain line (E_1F_1 in Fig. 116) passing through the earth, but we do not know *at what point* on this line it was. If we observe again, a few days later, the earth will have moved to E_2, but the planet will also have moved to P_2 and will not be at the intersection of the lines E_2F_2 and E_1F_1. This difficulty can be overcome by taking advantage of the fact that the planet moves in a definite orbit and returns to the same position at regular intervals, equal to its sidereal period.

First, therefore, we must find this period.

285. Determination of the Sidereal Period of a Planet. The sidereal period cannot be directly determined from our observa-

tions, but the synodic period can, for it is the interval between successive conjunctions or oppositions.

The exact instant of opposition is found from a series of right ascensions and declinations observed about the proper date. By comparing the deduced longitudes of the planet with the corresponding longitudes of the sun we easily find the precise moment when the difference was 180°. When the synodic period is found, the sidereal is at once given by the equations in section 268 (p. 235), namely, $\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$ for an inferior planet, and $\frac{1}{S}=\frac{1}{E}-\frac{1}{P}$ for a superior one. In the first case $P=\frac{S\times E}{S+E}$.

It will not answer for this purpose to deduce the synodic period from two *successive* oppositions, because, on account of the eccentricity of the orbits, both of the planet and of the earth, the synodic periods are notably variable. The observations must be sufficiently separated in time to give a good determination of the *mean* synodic period.

In the case of all the older planets we have observations running back nearly two thousand years, so that no difficulty arises on this score.

286. The position of a planet (for example, Mars) in its orbit may then be found as follows: Let A (Fig. 117) be the position of the earth at any date when Mars was observed and found to lie on the line AM. After one sidereal period (686.95 days) the earth will be at C. Even if no observation was obtained on this date, the direction of the line CM, on which the planet then lay, can be found by interpolation among neighboring observations. The intersection M of these two lines is the planet's position at both dates. What could thus be done graphically could be done more accurately by trigonometric calculation.

Two observations of Venus, V, separated by a period of 225 days, will similarly mark out a point on its orbit.

In practice the lines AM and CM will usually not be in the plane of the ecliptic, but by considering the planet's longitudes alone the construction just given will fix the projection of the planet on the plane of the ecliptic. From the observed latitudes the distance above or below this plane may then be found.

From a sufficient number of such pairs of observations distributed around an orbit it is evidently possible to work out completely its magnitude and form; and it was by similar methods that Kepler, utilizing the rich mine of data contained in Tycho's long series of observations, proved that the orbit of Mars is an *ellipse*, and later those of the other planets also, and deduced

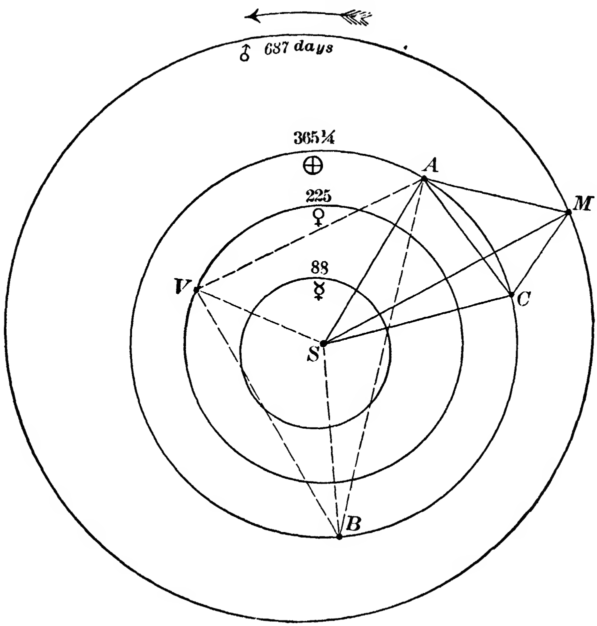

FIG. 117. Distance of a Planet

If a planet is observed at any date, and again after one sidereal period, its position in the orbit, which is the same on both dates, can be found, and its distance from the sun in astronomical units can be determined

their distances from the sun as compared with that of the earth. His harmonic law was then discovered simply by comparing the periods with the distances. Now that we have the harmonic law, a planet's *approximate* mean distance can of course, after its period is known, be much more easily found by applying that law than by the geometrical method just explained.

287. Solar Parallax from Distance of a Planet. By such methods it is possible to find with great accuracy the mean distance of a planet from the sun, at any time, in terms of the astronomi-

cal unit, and a map of the solar system can be formed, correct in all its proportions but without a scale of miles. The measurement by trigonometrical parallax of any one planetary distance in the solar system will then suffice to express them all in miles. The parallax of the sun cannot be measured directly with accuracy; the sun is too bright and too large, and its parallax is small. The asteroid Eros has proved to be the most favorable subject for parallax measurements. It can be as accurately observed as a star, and at opposition may come within little more than thirteen million miles of the earth.

STUDY OF THE PLANETS THEMSELVES

In discussing the individual characteristics of the planets we have to consider a variety of different data, obtained by telescopic observation, — micrometric, spectroscopic, photometric, and radiometric measurements, — and then to study their *diameters*, their *masses* and *densities*, their *satellite systems*, their *axial rotation*, their *surface markings*, their *light* and *temperature*, their *atmospheres* if present, and the *physical conditions* which prevail on their surfaces.

288. Determination of Size: Diameter, Surface, and Volume. The size of a planet is found by measuring its *apparent* diameter in seconds of arc with some form of micrometer (§ 82) attached to a powerful telescope. Since from the elements of the orbit of a planet and of the earth we can find the distance of the planet from the earth at any time in astronomical units, we can at once deduce the real linear diameter from the apparent diameter D'', by the equation given in section 109 (p. 96), namely,

$$\text{linear diameter} = \Delta \sin D'', \text{ or } \frac{\Delta D''}{206{,}265},$$

Δ being the distance of the planet from the earth. This will give the linear diameter as a fraction of the astronomical unit and can be converted into miles by simply multiplying it by 92,870,000, the number of miles in the unit, or into kilometers by multiplying by 1.4945×10^8.

For many purposes it is convenient to express the planet's radius in terms of the earth's radius by dividing half the diam-

eter in *miles* by 3959 (the number of miles in the mean radius of the earth), designating this *relative radius* by r.

The *surface area* of the planet in terms of the earth's surface is then r^2, and the *volume*, or *bulk*, of the planet is r^3 in terms of the earth's volume; for example, if, as is nearly true in the case of Jupiter, $r = 11$, then the surface of the planet is 121 times that of the earth, and its bulk 1331 times that of the earth.

The nearer the planet, other things being equal, the more accurately r and the quantities derived from it can be determined. An error of $0''.1$ in measuring the apparent diameter of Venus when nearest counts for less than 13 miles, but in the case of Neptune it would correspond to more than 1300 miles.

Irradiation is the greatest obstacle to accurate measurement. It can be partially eliminated by comparing observations made by day and by night, or when the planet is at very different distances, or, for Mercury or Venus, when it is in transit across the sun.

289. Mass. If the planet has a satellite, its *mass* compared with the sun is very easily and accurately found.

If S is the sun's mass, E, the earth's, and P, the planet's; A, the earth's mean distance and T its period; a, the satellite's mean distance and t its period, we have, by equation (1), section 220, $\frac{S}{E} = \frac{4\pi^2 A^3}{gr^2T^2}$ and, similarly, for the planet, $\frac{P}{E} = \frac{4\pi^2 a^3}{gr^2t^2}$. Whence

$$P : S = \frac{a^3}{t^2} : \frac{A^3}{T^2}.$$

Strictly speaking, what we find here is the ratio of the sums of the masses of the two bodies concerned (Sun + Earth : Planet + Satellite), but in practice this correction need not be considered, except when the perturbations by other planets or satellites are taken into account.

The observations upon which this method of determining a planet's mass depend are those of the satellite's *period* and its *greatest elongation*, the measures of *distance* being especially important, since the distance enters into the formula by its cube.

When a planet has no satellite, as is the case with Mercury and Venus, its mass can be determined only by means of the *perturbations which it produces* in the motions of other planets, or of comets that happen to come near it, and the calculations are intricate.

In the case of Mercury the mass is still very uncertain. Venus, however, disturbs the earth and Mars sufficiently to give a good determination of her mass.

290. Surface Gravity and Density. When the mass has been determined, the surface gravity and density follow at once. The surface gravity is P/r^2 times that on the earth. The *density*, compared with the earth, is simply P/r^3; if we want the density as compared with water, we must multiply the result by 5.52, the density of the earth. Any error in the measured diameter, of course, very seriously affects the computed density and gravity.

291. Rotation Period and Data connected with it. The length of the planet's "day," when it can be determined at all, is usually ascertained by observing some well-marked spot on its disk and noting the times of its successive returns. An approximate value of the rotation period is obtained from the observation of such returns during a few days or weeks, and this is afterward corrected by data furnished from observations extending over the longest interval obtainable, a century or more if possible.

Mars, however, is the only planet of which the rotation period is known with great accuracy; the others either show no well-defined markings or show only such markings as seem to be more or less movable on the planet's surface, like spots on the sun.

In reducing the observations, account has to be taken of the continual change in the direction of the planet from the earth and also of the variations of its distance, which alter the time taken by light to reach us.

Even when the planet has no distinct surface markings, methods are available which may yield at least an approximate value for the period of rotation. (1) The speed of approach and recession of opposite limbs may be measured with the spectroscope. (2) Periodic variation of brightness sometimes arises from the presentation, by rotation, of areas of various reflecting power. (3) The oblateness of the planetary disk may, with certain assumptions concerning the internal distribution of the planet's mass, give a measure of the centrifugal force and hence of the speed of rotation.

The *inclination* of the planet's equator to the plane of its orbit, and the positions of its poles and equinoxes, are deduced

from the observations of the *paths* of the spots as they cross the disk. Such data, however, are available only in the cases of Mars, Jupiter, and Saturn. When this method fails, the direction of the axis may be indicated by polar flattening, or the plane of the equator may be found from the perturbations of the satellites (§ 342).

The *oblateness*, or polar compression, of the planet, due to its rotation, is found simply by measuring the difference between the polar and equatorial diameters; but the difference is always very small, so that the percentage of its probable error is rather large.

The oblateness can also be determined from observation of the perturbations of the planet's satellites.

292. Data relating to the Light and Heat of a Planet. The *brightness* of the planet and the reflecting power of its surface, or *albedo*, are determined by observations with the photometer (§ 568).

Since the illumination of the planet's surface by the sun varies inversely as the square of its distance r, while the apparent diameter of the disk varies inversely as the distance Δ from the earth, and the apparent area of the disk as $1/\Delta^2$, the brightness of the planet, as seen from the earth, varies as $1/r^2\Delta^2$ — to which must be added the effects of phase (compare § 508).

The *spectroscopic peculiarities* of the planet's light are of course studied with a spectroscope, and usually by spectroscopic photography. A planet always shows, so far as its brightness permits, the lines of the solar spectrum and, in some cases, additional lines or bands of its own, which give information as to the constitution of its atmosphere (§ 619).

Suitable *radiometric* measurements show how much *heat* is radiated by the planet, and furnish information regarding the rate of absorption and radiation of heat and the probable temperature (§ 618).

293. The Planet's Surface Markings and Topography. These are studied with the telescope by making careful notes and drawings of the appearances and markings seen at different times. If the planet has any well-defined and characteristic features by which its rotation can be determined, it is soon pos-

sible to identify such as are permanent and to chart them more or less perfectly. Photography is proving an increasingly valuable aid but cannot compete with visual observation in the study of the finest details.

Just as the study of the surface of the earth is known as geography, so that of the surface of the moon is called selenography, and that of the surface of Mars, areography.

294. The Satellite Systems. The principal data to be determined in respect to these systems are the distances and periods of the satellites. These are found, along with the eccentricities, inclinations, etc., by *micrometric* measures of the apparent distance and direction of each satellite from the planet, or from other satellites. The latter is now the usual method, since the distance and direction between two satellites (which appear as mere points of light) can be measured much more precisely than between a satellite and the center of the large disk of a planet. The reduction of the observations in this case is, however, very complicated.

The diameters of some of the larger satellites are measurable. Those of the smaller ones can be only roughly guessed at, on the basis of their photometrically observed brightness. In several instances satellites show periodic variations in brightness, which indicate that they make an axial rotation in the time of one revolution around the primary, just as our moon does.

Where there are a number of satellites attending a planet, their mutual perturbations furnish a very interesting subject of study and make it possible to determine their *masses* relative to that of the planet.

All those satellites of the planetary system which lie relatively near their primary (at a distance less than ten or twelve times its diameter) move in very nearly circular orbits, whose planes are nearly coincident with that of the equator of the primary. The remoter satellites, among which the moon is to be counted, usually have orbits of considerable eccentricity and inclination.

295. Classification of Planets. Humboldt has classified the planets into two groups, — the terrestrial planets, as he calls them, and the major planets. The terrestrial group contains the four planets nearest the sun, — Mercury, Venus, Earth, and

Mars. They are all of similar magnitude, ranging from 3000 to 8000 miles in diameter, and are solid bodies, not very different in density and perhaps roughly alike in composition; but they are very different with respect to the physical conditions of their surfaces, — temperature, force of gravity, presence and amount of air and water, and so on.

The four major planets, Jupiter, Saturn, Uranus, and Neptune, are much larger bodies, ranging from 32,000 to 90,000 miles in diameter, and are, on the average, only about one fourth as

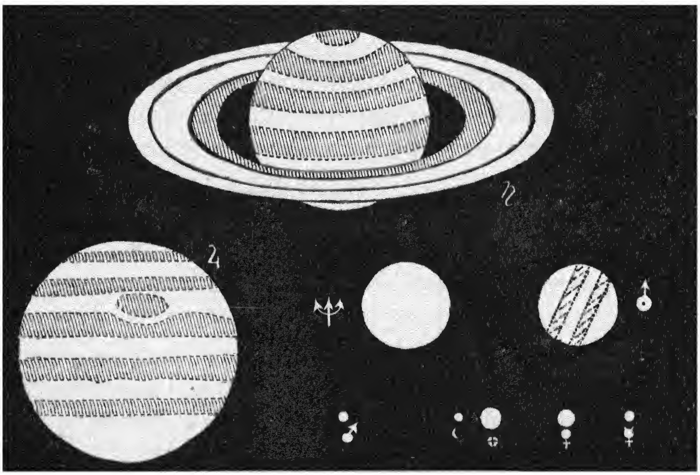

FIG. 118. Relative Sizes of the Planets

dense as the terrestrial planets. So far as we can make out, they present only a surface of cloud. The inner portions of these planets are probably very hot, but they are enveloped in relatively cool superficial layers and are not self-luminous.

As to the *asteroids*, the probability is that they represent a fifth planet of the terrestrial group, which, as has already been intimated (§ 267), failed somehow in its evolution. All of those so far discovered, if united, would not make a planet of more than one tenth of the earth's diameter, or one thousandth of its mass.

296. Tables of Planetary Data. In the Appendix are presented tables of the different numerical data of the solar system,

derived from the best authorities and calculated for a solar parallax of 8″.803, the sun's mean distance being therefore taken as 92,870,000 miles. These tabulated numbers differ widely in accuracy. The *periods* of the planets and their *distances in astronomical units* are very precisely known; probably the last decimal place in the table may be trusted. The remaining *orbital elements* — inclinations, eccentricities, etc. — are also very accurately known. Next, but with less percentage of accuracy, come the *masses* of such planets as have satellites, expressed in terms of the sun's mass. The masses of Venus and especially of Mercury are much more uncertain. The distances of the planets in miles, their masses in terms of the earth's mass, and their diameters in miles, all involve the solar parallax and are affected by the slight uncertainty in its amount. For the remoter planets, moreover, *diameters*, *volumes*, and *densities* are subject to a very considerable percentage of error, as explained above (§§ 288, 290). The student need not be surprised, therefore, at finding serious discrepancies between the values given in these tables and those given elsewhere, amounting in some cases to 10 or 20 per cent or even more. Such differences indicate the actual uncertainties of our knowledge.

EXERCISES

1. What is the mean daily gain of the earth on Mars as seen from the sun, that is, the synodic motion of Mars, assuming their sidereal periods as 365.25 days for the earth, and 687 days for Mars?

2. Find the synodic period of Venus, the sidereal period being 225 days.

3. Given the synodic period of a planet as 3 years, what is its sidereal period?

Ans. $\begin{cases} \frac{3}{4} \text{ of a year, or} \\ 1\frac{1}{2} \text{ years.} \end{cases}$

4. Given a synodic period of 4 years, find the sidereal period.

5. What would be the sidereal period of a planet which had its synodic period equal to the sidereal? *Ans.* 2 years.

6. Within what limits of distance from the sun must all planets lie which have synodic periods longer than 2 years? (Apply Kepler's harmonic law after finding the sidereal periods that would give a synodic period of 2 years.)

Ans. $\begin{cases} 0.763 \text{ astron. units, or } 70{,}860{,}000 \text{ miles, and} \\ 1.588 \text{ astron. units, or } 147{,}480{,}000 \text{ miles.} \end{cases}$

CHAPTER X

CELESTIAL MECHANICS

NEWTON'S LAWS OF MOTION · MOTION UNDER CENTRAL FORCE · KEPLER'S LAWS · LAW OF GRAVITATION · THE CONICS · PROBLEM OF TWO BODIES · DETERMINATION OF AN ORBIT · EFFECTS OF RADIATION PRESSURE · EXERCISES · PROBLEM OF THREE BODIES · PERTURBATIONS · DEFINITIVE ORBITS · THEORY OF MOON'S MOTION · ELLIPTICITY AND INTERNAL CONSTITUTION OF PLANETS · PHENOMENA OF THE TIDES · ANALYSIS AND PREDICTION OF TIDES · DIRECT MEASUREMENT OF TIDE-RAISING FORCE · TIDAL FRICTION AND ITS EFFECTS · THE THEORY OF RELATIVITY

297. Practical Importance. No better example of the practical value of an investigation which must have appeared at the start to be of purely intellectual interest can be found than Newton's study of the motions of the planets and the moon.

For fifty years after Kepler discovered the laws which define the planetary orbits they remained an unexplained mystery; then Newton showed that they were all consequences of the single law of gravitation, and that many other previously inexplicable things were accounted for by the same law. But he did far more. His study, following Galileo's observations of falling bodies, led to the first comprehension of the manner in which bodies move under the influence of forces, and so to the science of mechanics; and the mathematical difficulties of the problem stimulated the invention of the calculus.

Modern engineering, as well as modern physics, rests on these two foundations, for without mechanics and the calculus we could neither recognize the forces which are in play around us nor work out their effects. The mechanical basis of civilization, therefore, involves principles and methods which were discovered as a rather direct result of astronomical research.

Only a general account of the theory of the motion of the heavenly bodies can be given in this chapter. A complete discussion would involve difficult mathematical treatment.

GENERAL PRINCIPLES

298. The laws of motion, formulated by Newton (though some of the principles involved had been recognized earlier by Galileo), form the basis of this theory. According to the first law *a moving body on which no force is acting travels in a straight line with uniform speed.* Failure to realize this held back the advance of mechanical science for nearly two thousand years after the time of Aristotle, while it was supposed that rest was more natural to a body than motion. This is not true; mere motion implies no acting force. According to Newton, *change of motion* only, either in speed or in direction, implies such action. The second law states that *change of motion is proportional to the force which acts on a body (the impressed force), and takes place in the direction of this force.*

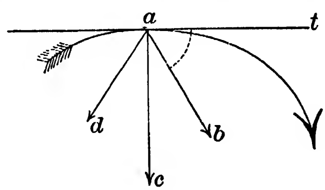

FIG. 119. Curvature of an Orbit

Thus, if the force is directed forward, the body will move faster and faster, traveling in the same straight line; if it is directed backward, the body will be retarded, still moving in the same line; if it is directed laterally, the path will curve toward the side to which the force urges it. Conversely, if the speed of a body moving in a curve (Fig. 119) increases, we know that the acting force pulls not only crosswise but forward, as *ab*; if it decreases, the force is directed backward, like *ad*; if the speed is constant, the force is exactly crosswise, along *ac*. This force may be, and often is, the resultant of several forces, but they act as one.

By "motion" in this law Newton meant what is now called *momentum*, the product of the velocity of the moving particle by its mass; and by "change" he meant the *rate of change* per unit of time. The rate of change of velocity alone, whether in amount or in direction, is called *acceleration*. If m is the mass, a the acceleration, and f the force, then

$$f = ma.$$

The third law states that *action and reaction are equal and opposite.* If, for example, the earth attracts the moon, the moon

attracts the earth with a force which is exactly equal in amount and oppositely directed.

299. Application. By means of these laws the force which acts on any body of known mass may be determined if the resulting acceleration is known; or, conversely, if the force is given, the way in which the body's velocity changes from moment to moment can be found, and thus the whole circumstances of the motion. In the simplest cases the problems can be solved by geometrical devices, but the more complicated ones can be handled only by the methods of the calculus, which afford also the shortest and most elegant solutions of the simple cases.

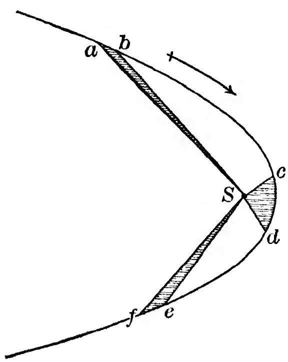

FIG. 120. The Law of Equal Areas

The curve represents part of a comet's orbit around the sun. If the arcs *ab*, *cd*, and *ef* are described in equal intervals of time, the shaded areas are all equal

These mathematical discussions are usually omitted here, as beyond the province of the present work, and many important results are merely stated, the proofs being left to more technical treatises.

300. Motion under Central Force: Law of Areas. If a particle moves under the action of a force directed always toward a fixed center, it follows from Newton's laws that its path will be a *curve, concave toward the center of force, and lying in one plane* which includes the center (Fig. 120). It is easy to prove, further, that it will move in such a way that the line joining it to the center (the radius vector) *will sweep out equal areas in equal times* (though even this simple demonstration will be omitted here). Conversely, if this law of areas holds good, the force determining the orbit must be central.

When the force is directed ever so little ahead of the radius vector, the rate of description of area increases; when the force is directed behind, the rate decreases. With a repulsive force, directed away from the center, the orbit is convex; but the law of areas still holds true, and would do so even if the force should suddenly change from attraction to repulsion.

301. Linear, Angular, and Areal Velocities. The *linear velocity* of a particle is the number of linear units (centimeters, feet, miles) that it moves over in a unit of time, say a second. Its symbol is usually V. The *angular velocity* is the number of units of angle (radians or degrees) swept over by the radius vector in a unit of time. The usual symbol for this is ω. The *areal velocity* is the area swept out by the radius vector in unit time (square miles per second) and is constant if the force is central.

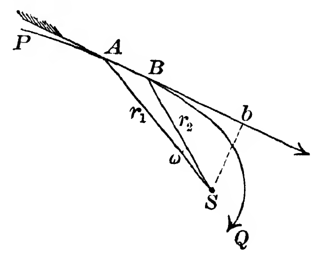

FIG. 121. Linear and Angular Velocities

In Fig. 121, if AB is the length of the path described in unit time, AB is the *linear velocity* V, the angle ASB is the *angular velocity* ω, and the area ASB is the *areal velocity*, which is constant. Calling this A and regarding the sector as a triangle (which it is nearly enough), we have $A = \frac{1}{2} V \times x$, x being the line Sb drawn from the center of force perpendicular to the line of motion; so that if we regard AB as the base of the triangle, x is its altitude. Hence we have the equation

$$V = \frac{2\,A}{x}. \tag{1}$$

Also, $A = \frac{1}{2} r_1 r_2 \sin ASB$. Since in a second of time the angle ASB, or ω, is so small that it may be taken equal to its sine, and $r_1 r_2$ equals (sensibly) r^2, we have

$$\omega = \frac{2\,A}{r^2}. \tag{2}$$

In every case of motion under a central force, therefore, (1) the *areal velocity is constant* in all parts of the orbit; (2) the *linear velocity varies inversely as the perpendicular drawn from the center to the line of motion*; and (3) the *angular velocity varies inversely as the square of the radius vector.*

These three statements are not independent laws, but simply different geometrical equivalents for one law. They hold good regardless of the nature of the force, requiring only that it act *directly toward, or from, the center*, along the line of the radius vector.

302. Angular Momentum. The areal velocity is closely connected with a very important quantity known as the *angular momentum*, which may be defined as the product of the mass of the moving particle into the area swept out in unit time by the line joining it to a given point. In the present case the angular momentum about the center of force is constant. This is a simple example of the general principle of the *conservation of angular momentum*.

In a system of several particles moving in one plane the total angular momentum relative to a given point is found by multiplying the mass of each by its areal velocity about the point and adding the results (counting backward motions as negative).

If no external forces act on the system, and if the internal forces are central (that is, directed along the lines joining the various particles) *the total angular momentum of the system remains unchanged* (is conserved), whatever may be the details of the motion.

This principle is of great generality and finds important astronomical applications (§§ 321, 331, 358, 540). For an extended body the angular momentum of each separate part must be taken, and added together, and for bodies in space the momentum must be taken about an axis instead of a point; but the principle still holds good. It is an immediate consequence of the third law of motion. If the action of one body on a second increases the angular momentum of the latter, the reaction of the second body on the first decreases its angular momentum by exactly the same amount, and the sum is unaltered.

303. Circular Motion. In the special case when the path of a body is a circle described under the action of a force directed to its center, both the linear and angular velocities are constant, as is also the force, which is given by the familiar formula, already several times used,

$$f = \frac{mV^2}{r}, \tag{3}$$

or

$$f = 4\,\pi^2 m \frac{r}{t^2}, \tag{4}$$

obtained by substituting for V in equation (3) its value $2\,\pi r$ (the circumference of the circle) divided by t (the time of revolution).

As the orbits of the principal planets are all nearly circular, these formulæ will find frequent application.

304. Kepler's Laws. Early in the seventeenth century Kepler discovered, as unexplained facts, three laws which describe the motions of the planets, — laws which still bear his name. He worked them out from a study of the observations which Tycho Brahe had made during many preceding years upon the planets, especially Mars. They are as follows:

(1) The orbit of each planet *is an ellipse with the sun in one of its foci* (§ 311).

(2) *The radius vector of each planet describes equal areas in equal times.*

(3) *The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun; that is,* $t_1^2 : t_2^2 :: a_1^3 : a_2^3$. This is the so-called harmonic law.

To make sure that the student apprehends the meaning and scope of this third law we add a few simple examples of its application.

1. What would be the period of a planet having a mean distance from the sun of one hundred astronomical units, that is, a distance a hundred times that of the earth?

$$1^3 : 100^3 = 1^2 \text{ (year)} : X^2,$$

whence $$X \text{ (in years)} = \sqrt{100^3} = 1000 \text{ years.}$$

2. What would be the distance from the sun of a planet having a period of 125 years?

$$1^2\text{(year)} : 125^2 = 1^3 : X^3,$$

whence $$X = \sqrt[3]{125^2} = 25 \text{ astronomical units.}$$

3. What would be the period of a satellite revolving close to the earth's surface?

$$\text{(moon's distance)}^3 : \text{(distance of satellite)}^3 = (27.3 \text{ days})^2 : X^2,$$

or, $$60^3 : 1^3 = 27.3^2 : X^2,$$

whence $$X = \frac{27.3}{\sqrt{60^3}} = 1^{\text{h}}\,24^{\text{m}}.$$

305. Inferences from Kepler's Laws. From what has already been said (§ 300), Kepler's second law indicates that *the force which determines the orbits of the planet is directed toward the sun.*

This, by itself, tells us nothing about the magnitude of the force; but from the fact that the orbit is an ellipse, with the sun at its focus, Newton proved (by a demonstration a little

beyond the scope of this book) that *the force acting on a given planet at different points on its orbit varies inversely as the square of the radius vector.*

Finally, from the third law, he proved that *the forces acting on all the different planets are inversely proportional to the squares of their distances and directly proportional to their masses.* It makes no difference whether a planet is hot or cold, made of hydrogen or of iron; if the mass and distance are the same, so is the force.

306. The proof for circular orbits is very simple. From equation (4) we have, for the first of two planets,

$$f_1 = 4\,\pi^2 m_1 \frac{r_1}{t_1^2},$$

in which f_1 is the central force, r_1 the planet's distance from the sun, and t_1 its periodic time. For a second planet

$$f_2 = 4\,\pi^2 m_2 \frac{r_2}{t_2^2}.$$

Hence $$\frac{f_1}{f_2} = \frac{m_1 r_1 t_2^2}{m_2 r_2 t_1^2}.$$

But, by Kepler's third law, $$\frac{t_2^2}{t_1^2} = \frac{r_2^3}{r_1^3}.$$

Substituting in the preceding equation, we have

$$\frac{f_1}{f_2} = \frac{m_1 r_2^2}{m_2 r_1^2}, \quad \text{or} \quad f_1 : f_2 = \frac{m_1}{r_1^2} : \frac{m_2}{r_2^2}.$$

In the case of elliptical orbits the proposition is equally true if for r we substitute a, the semi-major axis of the orbit; but the proof is much less simple.

307. The Law of Gravitation. One more step remains. Since the sun attracts the planets, Newton's third law demands that the planets must attract the sun with equal force; and since the attraction which the sun exerts on any planet is proportional to the mass of that planet, the attraction of a planet on the sun must be proportional to the mass of the sun. This led Newton to the general statement of the law of gravitation. *Every particle of matter attracts every other with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them.* This may be expressed by the equation

$$f = G\frac{Mm}{r^2}, \tag{5}$$

which has already been used (§ 146).

308. The constant of gravitation (G in the above equation) is believed to be an absolute constant of nature, like the velocity of light, — the same throughout the material universe, although the numerical value which is assigned to it depends upon the units used in measuring mass, length, and time. In the c.g.s. (centimeter-gram-second) system its value is 6.673×10^{-8}. Two masses of one gram, one centimeter apart, therefore attract one another with a force of 6.67×10^{-8} dynes, or about one fifteen-millionth of a dyne.

A dyne (the c.g.s. unit of force) is that force which, acting for one second on a mass of one gram, imparts to it a velocity of one centimeter per second; it is about equal to the weight of one milligram. Gravitational attraction between bodies of ordinary dimensions is thus exceedingly small, and very delicate apparatus must be employed to measure it (§ 148). Nevertheless, the constant G has been determined to about one part in a thousand. It is only because of the huge masses of the sun and the planets that their gravitational attraction becomes important.

309. Newton's Test of his Theory of Gravitation by the Motion of the Moon. When, in 1665, Newton first conceived the idea of universal gravitation, he saw at once that the moon's motion around the earth ought to furnish a test. Since the moon's distance (as was known even then) is about sixty times the radius of the earth, the distance it should fall toward the earth in a second ought to be, if his idea of gravitation was correct, 1/3600 of 193 inches (the distance which a body falls in a second at the earth's surface. This assumes, however, that the earth attracts as if its mass were all collected at its center — a theorem Newton had much trouble in proving, as it involved the use of his new mathematical invention of "fluxions").

Now 1/3600 of 193 inches is 0.0535 inches. Does the moon fall toward the earth, that is, deflect from a straight line, by this amount each second?

According to the law of central forces, considering the moon's orbit as circular, its acceleration is

$$a = 4\,\pi^2 \times \frac{r}{t^2},$$

and the *deflection* is one half of this, namely, $2\pi^2\frac{r}{t^2}$. If we compute the result, making $r = 238{,}840$ miles reduced to inches, and t the number of seconds in a sidereal month, the deflection comes out 0.0534 inch, a difference of only 1/10,000 of an inch, — practically an exact accordance.

Unfortunately, when Newton first made this test, the distance of the moon in miles was not known, because the size of the earth had not then been determined with any accuracy. The length of a degree was supposed to be about 60 miles instead of 69, as it really is. Newton computed the radius of the earth on this erroneous basis, and, multiplying it by 60, obtained for r, the distance of the moon, a quantity about 16 per cent too small; from this he calculated a corresponding deflection of only about 0.044 inch. The discordance between this and 0.0535 was too great, and he loyally abandoned the theory as contradicted by facts.

Six years later, in 1671, Picard's measurement of an arc of a meridian in France corrected the error in the size of the earth, and Newton, on hearing of it, at once repeated his calculation, or tried to, for the story goes that he was too excited to finish it, and a friend completed it for him. The accordance was now satisfactory, and he resumed the subject with zeal and soon established the correctness of his theory.

At the present time the acceleration of gravity at the earth's surface and the length of the month are known from observation with greater percentage precision than the distance of the moon. The distance at which a body of the moon's mass would revolve about the earth in its actual period can be computed, making full allowance for perturbations, thus obtaining a more accurate value of the distance than observation is yet able to secure.

310. The Inverse Problem. Newton did not rest with merely showing that the motion of the planets and of the moon could be explained by the law of gravitation; he also investigated and solved the more general *inverse* problem and determined what kind of motion is necessary according to that law. He found that the orbit of a body moving around a central mass under the law of gravitation need not be a circle, or even an ellipse of slight eccentricity like the planetary orbits; but it must be a *conic.* Whether it will be a circle, an ellipse, a parabola, or a hyperbola depends on circumstances.

311. The ellipse, parabola, and hyperbola, which are the principal types of conics, are illustrated in Fig. 122. The ellipse is a

closed curve, returning into itself, and such that the *sum* of the distances of any point N of the curve from the two foci equals the major axis, that is, $FN + F'N = PA$.

The hyperbola does not return into itself. The portions PN'' and Pn'' of the curve go off indefinitely, becoming ultimately nearly straight, and diverging from each other at a definite angle. In the hyperbola the *difference* of the distances of a point

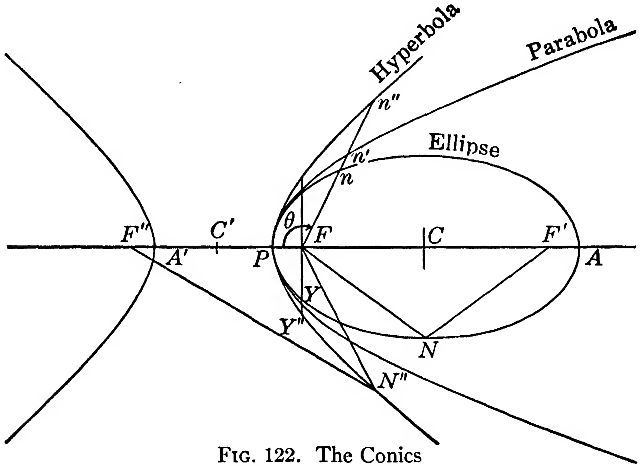

FIG. 122. The Conics

on the curve from the two foci equals the major axis, that is, $F''N'' - FN'' = PA'$. A second branch of the curve surrounds the "empty focus," F''.

The parabola, like the hyperbola, fails to return into itself; but its two portions, though still gradually separating, become more and more nearly parallel. It has but one accessible focus, and may be regarded either as an ellipse, with its second focus, F', removed to an infinite distance, and therefore having an infinite major axis, or, equally well, as a hyperbola of which the second focus F'' is pushed infinitely far in the opposite direction, so that it has an infinite (negative) major axis.

In the ellipse the *eccentricity* (FC/PC) is *less than unity*; in the hyperbola it is *greater than unity* (FC'/PC'). In the parabola it is *exactly unity*; in the circle it is *zero*.

The eccentricity of a conic determines its form. All parabolas are of the same form, differing only in size, as do circles. Ellipses

and hyperbolas have an infinite variety of forms, — some very narrow and some broad.

The distance from the focus to any point n on a conic is given by

$$r = \frac{p}{1 + e \cos \theta}, \tag{6}$$

where θ is the angle PFn (Fig. 122), often called the *true anomaly*, e is the eccentricity, and p is the distance FY, — half the *parameter* of the conic (the chord through the focus perpendicular to the major axis). The perihelion distance FP is $p/(1 + e)$, and the semi-major axis a is $p/(1 - e^2)$. For an ellipse a is positive (PC in the figure), but for a hyperbola it is negative (PC').

312. The Problem of Two Bodies. This problem, proposed and solved by Newton, may be stated as follows:

Given the masses of two spheres and their positions and motions at any instant; given also the law of gravitation: required the motion of the bodies ever afterwards and the data necessary to compute their place at any future time.

The mathematical methods by which the problem is solved require the use of the calculus, but the general results are easily understood.

In the first place, the motion of the *center of gravity* of the two bodies is not in the least affected by their mutual attraction.

In the next place, the two bodies will describe as orbits around their common center of gravity two curves precisely alike in form, but of size inversely proportional to their masses, the form and dimensions of the two orbits being determined by the masses and velocities of the two bodies.

It is often convenient to ignore the center of gravity entirely and to consider simply the *relative* motion of the smaller body around the center of the other. It will move with reference to that point precisely as if its own mass m had been added to the principal mass M while it had become itself a mere particle. This *relative orbit* will be precisely like the orbit which m actually describes around the center of gravity, except that it will be magnified in the ratio of $(M + m)$ to M; that is, if the mass of the smaller body is 1/100 of the larger one, its *relative* orbit around M will be just 1 per cent larger than its actual orbit around the common center of gravity of the two.

313. Equation for the Orbital Velocity. The velocity of one body relative to the other, at the distance r, is given by the equation

$$V^2 = G(M+m)\left(\frac{2}{r}-\frac{1}{a}\right), \tag{7}$$

where a is the semi-major axis of the orbit and G the constant of gravitation. This equation is often called the *equation of energy*, since it expresses the conservation of energy.

The velocities of m and M, relative to the center of gravity, are respectively $V_1 = MV/(M+m)$ and $V_2 = mV/(M+m)$. The kinetic energy of the two bodies is $\frac{1}{2}(mV_1{}^2 + MV_2{}^2)$, which is readily found to be $\frac{1}{2}\frac{mM}{m+M}V^2$. Equation (7) may now be written

$$\tfrac{1}{2}\frac{mM}{m+M}V^2 - \frac{GMm}{r} = -\frac{GMm}{2a}.$$

The first term is the total kinetic energy of the orbital motion, the second the potential energy of gravitation, and the sum of these is constant.

314. The Parabolic Velocity. For a parabola a is infinite, and if the velocity in this special case is called U, equation (7) of the last section becomes

$$U^2 = 2G\left(\frac{M+m}{r}\right). \tag{8}$$

A particle projected with this velocity U at the distance r will move in a parabola, whatever be the direction of projection. This is therefore called the *parabolic velocity.*

If the velocity V exceeds the parabolic velocity U, a comes out negative and the orbit is a hyperbola; but if V is less than U, a is positive and the orbit is an ellipse. In the former case the particle never comes back, but in the latter it returns at regular intervals. The parabolic velocity is therefore often called the *velocity of escape.*

The velocity of escape at the surface of any body may be computed from (8), neglecting the mass m of the small body which is supposed to be in motion. Thus, in the case of the earth $M = 5.97 \times 10^{27}$ g., $r = 6.37 \times 10^8$ cm., and $G = 6.67 \times 10^{-8}$, whence $U = 1.13 \times 10^6$ cm./sec., or 11.3 km./sec. For the sun, for which M is 332,000 times as great and r 109 times, U is $\sqrt{332000/109}$, or 55.2 times as great, that is, 622 km./sec. For the moon it comes out 2.4 km., or not quite 8000 ft./sec. A body

projected from the moon with a speed greater than this would never return, — which probably explains why the moon has no atmosphere (§ 200).

It may be noticed that in the above calculation all quantities are expressed directly in standard units, — centimeters, grams, and seconds. When very large or very small numbers have to be handled, this procedure has decided advantages, and, once the student is familiar with it, the chances of error are lessened, whether the calculation deals with stars or with atoms.

315. Equation for the Major Axis of an Orbit. From the equations of sections 313 and 314, eliminating $G(M+m)$,

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right). \tag{9}$$

If V is less than the parabolic velocity U, a is positive and the orbit is an ellipse; while if $V > U$, a is negative and the orbit is a hyperbola. If V is very nearly equal to U, a is very great and the orbit is nearly parabolic in form.

For a *circular* orbit $a = r$, which demands that $V^2 = \frac{1}{2} U^2$ (and also, of course, that the velocity be at right angles to the radius vector). The velocity of a body moving in a circular orbit is therefore equal to the parabolic velocity multiplied by $\sqrt{\frac{1}{2}}$. At the earth's distance from the sun this "circular velocity" is 29.76 km./sec. (18.47 mi./sec.). The parabolic velocity is $\sqrt{2}$ times this, or 42.09 km./sec.

316. The expression for a planet's period is now easily found. At the extremity B of the minor axis of the orbit (Fig. 123) $r = a$, and by equation (7) $V = \sqrt{\dfrac{G(M+m)}{a}}$. For the areal velocity A we have, by section 301, $A = \frac{1}{2} Vx$, where x denotes the perpendicular FB' from F on the tangent at B. But this is evidently equal to the minor axis b of the ellipse. Hence,

$$A = \tfrac{1}{2}\, b \sqrt{\frac{G(M+m)}{a}}.$$

Now the whole area of the ellipse is πab, and since it is swept out by the radius vector at the rate A, the period will be

$$t = \frac{\pi ab}{A} = 2\,\pi a^{\frac{3}{2}}/\sqrt{G(M+m)}.$$

This embodies Kepler's third law, and shows that all planets moving in ellipses which have the same major axis will have the same period, whatever the eccentricities of their orbits.

Strictly speaking, the values of a^3/t^2 for the different planets should not be equal, but proportional to $M + m$, where M is the mass of the sun and m that of the planet in question; but corrections due to perturbations by the other planets (§ 325) may be larger than this.

317. Expression for the Areal Velocity. By the geometry of the ellipse $b = a\sqrt{1 - e^2}$, and also, if p is the semi-parameter of the ellipse (FY', Fig. 123), $p = a(1 - e^2) = b^2/a$. The expression for the areal velocity may then be written

$$A = \tfrac{1}{2}\, p^{\frac{1}{2}} \sqrt{G(M + m)};$$

that is, the *areal velocity is proportional to the square root of the semi-parameter.*

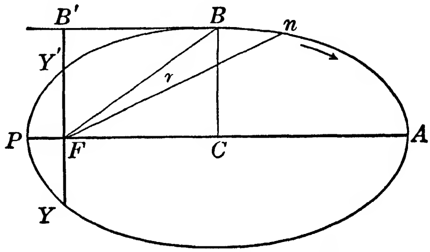

FIG. 123. An Elliptical Orbit

The sun is at the focus F; $FB = PC = a$, the semi-major axis; and $FB' = CB = b$, the semi-minor axis. The semi-parameter, p, is $FY' = FY$. The eccentricity, e, is FC/PC, which is less than unity. The radius vector r is measured from F to any point n on the orbit; and the angle PFn is θ, the true anomaly. The motion is in the direction of the arrow

All orbits for which the parameter YY' is the same will therefore be described with the same areal velocity. This is true not only of elliptic but of parabolic and hyperbolic orbits, and makes it possible to compute the motion in these, although such motion is not periodic and Kepler's third law is therefore inapplicable.

The principles summarized above have many important applications. We will mention two.

318. Calculation of the Position of a Body at Any Time: Ephemerides. If we know the orbit, the areal velocity A, and the time T at which the body passed perihelion, all that is necessary in order to find its position in its orbit at any other time t is to know how to draw a radius vector Fn (Fig. 124) such that the area between this line, the orbit, and the radius FP to the perihelion is equal to $A(t - T)$. This is known as "Kepler's problem." For a parabolic orbit it leads to a cubic equation;

for an ellipse or a hyperbola, to a transcendental equation, — which, however, may readily be solved. An approximate solution may be found graphically or with a slide rule, and the exact values are then easy to determine.

Knowing the position of the body on its orbit, and being similarly able to calculate the earth's position in its orbit, it is a mere matter of geometry to find the distance and direction of one from the other, that is, the *right ascension and declination of the body as seen from the earth.*

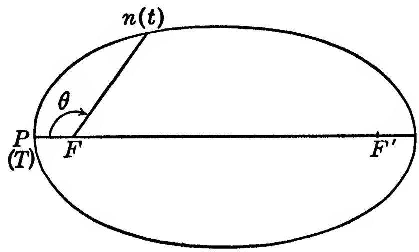

FIG. 124. Kepler's Problem

It is in this way that an *ephemeris* of any heavenly body is calculated, so that we know in advance where to look for it. In the ephemerides of the sun and major planets, which are given in the *Nautical Almanac*, the perturbations (see below) are carefully computed and allowed for. For a newly discovered comet or asteroid this is not necessary.

319. Determination of the Orbit of a Newly Discovered Planet. If the planetary orbits were circular, this would be a very simple matter. Suppose that we have two observations of the right ascension and declination of the planet, made a few days apart. The positions E_1 and E_2 of the earth at the times of observation (Fig. 125) are known, and the observations fix two lines, E_1L_1 and E_2L_2, on which the planet lay at these dates.

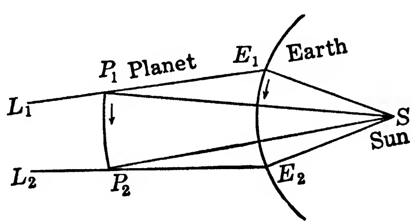

FIG. 125. Determination of a Circular Orbit

We may start by guessing at the distance E_1P_1 of the planet from the earth at the first date. On the second date it must have been on the line E_2L_2 and at the same distance as before from S, which fixes its position P_2. The angle P_2SP_1 is thus determined. But the period of a body moving at the distance SP is given by Kepler's third law, and P_2SP_1 should be the same fraction of

360° as the time interval $t_2 - t_1$ is of the period. If this relation is satisfied, our guess at the distance is right; if not, it is wrong, and we proceed to further trials, which soon lead to the solution.

Since the actual orbit is not a circle but an ellipse, this process will give only approximate results, but it is often actually used to predict the motion of a newly discovered asteroid for a few weeks and to facilitate further observations.

The determination of an elliptic orbit requires *three* observations and the determination of *two* geocentric distances E_1P_1 and E_2P_2 (preferably for the first and last observations). If these distances are given, and the points P_1 and P_2 thus fixed, the whole orbit of the planet around the sun can be determined, and the direction of the planet from the earth at the time of the middle observation can be calculated. The two conditions that the calculated right ascension and declination shall agree with the observed values suffice to determine the two geocentric distances. Equations can be set up which, though complicated, make the solution much shorter than it would be by trial and error. In some cases (for example, when the planet is in the plane of the ecliptic) a fourth observation is required for a solution.

If the orbit is a parabola or a hyperbola, this general solution will show it and will determine the eccentricity. Comets usually move in almost parabolic orbits. Therefore, when computing the orbit of a new comet, it is assumed that it is a parabola, which considerably simplifies the calculations. If the orbit is really an ellipse, the calculated parabolic orbit cannot be made to represent both the right ascension and the declination for the middle observation, and the general method must be employed.

The calculation of an orbit from three observations takes a skilled computer two days or sometimes less. The novice may take as many weeks, most of his time being occupied in finding and correcting the numerical mistakes which are only too easy to make.

320. Radiation Pressure and its Effects. For very small bodies the effects of the sun's attraction are modified by those of another force, radiation pressure (§ 553). A beam of light falling on any body exerts a force in the direction of the incident light. This force is proportional to the intensity of the light and to the cross-section A of the beam which is intercepted by the body.

All bodies which obstruct the outward passage of the sun's light are therefore slightly repelled by the sun. At the earth's distance the repulsive force (in dynes) amounts to $4.5 \times 10^{-5} A$ (if A is measured in square centimeters). The gravitational attraction at the same distance is $0.59\ m$ dynes (if m is the body's mass in grams). Both repulsion and attraction vary inversely as the square of the sun's distance, so that their ratio for the same body is a fixed quantity, being $7.6 \times 10^{-5} A/m$. For a sphere of radius r cm. (large compared with a wave-length) and density ρ g./cm.^{3},

$$A = \pi r^2, \quad m = \tfrac{4}{3}\,\pi \rho r^3, \quad \text{and} \quad A/m = \frac{3}{4\,\rho r}.$$

For even the smallest asteroids this quantity is very small and radiation pressure is negligible, but for minute particles (since their mass diminishes more rapidly than the area) the repulsion due to radiation pressure may equal or even exceed the gravitational attraction. Very fine dust is therefore actually repelled by the sun.

The orbit of such a particle is a hyperbola *convex* toward the sun, — the other branch of the curve shown in Fig. 122. The laws of areas and of energy still apply, but the constant G must be replaced by one representing the combined influence of the two forces, which is negative and different for particles of different sizes. Motions of this sort are actually met with in the tails of comets (§ 516).

EXERCISES

1. Given a comet moving in an ellipse with the eccentricity 0.5. Compare the velocities, both linear and angular, at the perihelion and aphelion.

Ans. Linear velocity at perihelion is *three* times that at aphelion. Angular velocity at perihelion is *nine* times that at aphelion.

2. What would be the result if the eccentricity were 1/3? What if it were 3/4?

3. What would be the periodic time of a small body revolving in a circle around the sun close to its surface? (Apply Kepler's harmonic law.)

Ans. $2^h\,47^m.4$.

4. What would be its velocity? *Ans.* 435.8 km./sec.

5. If the earth had a satellite with a period of 8 months, what would its distance be? *Ans.* Four times that of the moon.

6. If Jupiter were reduced to a particle, how much would its period be lengthened? (Consider its mass to be 1/1048 of the sun's, and see § 316.)

Solution. Let x be the new period; then

$$x^2 : t^2 \frac{1049}{1048} = r^3 : r^3 = 1 : 1,$$

since r is not changed; whence

$$x = t\left(\frac{1049}{1048}\right)^{\frac{1}{2}} = t(1 + \tfrac{1}{2} \times 1/1048 + \text{etc.}) = t(1 + 1/2096) \text{ very nearly.}$$

But $\quad t = 4332.6$ days, and $(x - t) = \dfrac{4332.6}{2096} = 2.067$ days. *Ans.*

7. How much longer would the earth's period be if it were a particle?

Ans. 1/660,000 of a year, or 47.8 sec.

8. If the sun's mass were a hundred times greater, what would be the parabolic velocity at the earth's distance from it (§ 314)?

Ans. Ten times its present value; that is, 420.9 km./sec.

9. If the sun's mass were reduced 50 per cent, what would be the parabolic velocity at the distance of the earth? *Ans.* 29.76 km./sec.

10. If the sun's mass were to be suddenly reduced by 50 per cent or more, what would be the effect upon the now practically circular orbits of the planets? (See § 315.)

11. What would be the effect upon the orbit of the earth if the sun's mass were suddenly doubled? *Ans.* It would immediately become an eccentric ellipse, with its aphelion near the point where the earth was when the change occurred.

12. How long would a comet, which is moving in a parabolic orbit with a perihelion distance of one astronomical unit, take to move through 90° from perihelion (from P to Q in Fig. 126)?

Ans. The velocity of the comet at perihelion is $\sqrt{2}$ times that of the earth in its orbit, and its areal velocity is also $\sqrt{2}$ times that of the earth. The latter is π "square astronomical units" per year (since this is the area of the earth's orbit), so that for the comet $A = \pi\sqrt{2}$. By well-known properties of the parabola $SQ = 2\,PS$, and the area of the sector SPQ is 2/3 that of the rectangle $PRQS$, or 4/3 of a square astronomical unit. The time in question is therefore $4/(3\,\pi\sqrt{2})$ years, or very nearly 3/10 of a year (109.61 days).

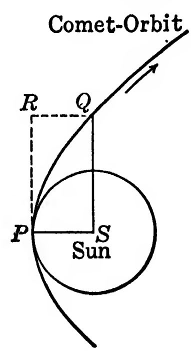

FIG. 126. Perihelion Passage of a Comet

13. Show that a comet moving in a parabola with perihelion distance q will take $109.61 \times q^{\frac{3}{2}}$ days to move through 90° from perihelion.

DISTURBED MOTION

321. The Problem of Three Bodies. As has been said, the problem of two bodies is completely solved, and (with the aid of the calculus) the full solution can be given in a page or two of print. If, however, instead of *two* spheres attracting one another, there are *three* or more, the general problem of determining their motions and predicting their positions becomes extraordinarily difficult. In spite of the labors of the greatest mathematicians for more than two centuries, no general solution was reached until recently, when K. Sundman, of Helsingfors, obtained one, and this solution is so complicated that it is valueless for purposes of practical calculation.

The problem of three bodies is in itself as determinate and capable of solution as that of two. Given the initial data, that is, *the masses, positions, and motions of the three bodies at a given instant*, and assuming the law of gravitation, their motions for all the future, and the positions which they will occupy at any given date, are exactly predetermined.

All the *general* relations met with in the problem of two bodies still hold true. The center of gravity of the whole system moves uniformly in a straight line, and the principles of the conservation of angular momentum and the conservation of energy are true, whatever the number of bodies. The harmonic law, also, holds in this generalized form: given any exact solution of the problem, for any number of bodies, an infinite number of other solutions may be obtained by multiplying all the linear dimensions by any factor A, and all the time-intervals by $A^{\frac{3}{2}}$.

These relations, however, are insufficient to define the motion. The difficulty of obtaining a general solution lies mainly in the complexity of the problem.

322. Why the Problem is so Complex. The character of the difficulties is well illustrated by a simple example.

Let the three bodies be the sun (S), Jupiter (J), and a meteorite (M) weighing but a few grams, so that its attraction does not alter the motion of Jupiter about the sun to any perceptible degree. The orbit of Jupiter will then be a fixed Keplerian ellipse. When M is not near Jupiter, the planet's attraction will not

greatly affect its motion, and its path will be nearly, though not quite, an ellipse such as *A* (Fig. 127). Suppose, however, that *M* makes a close approach to Jupiter, as at *J* in the figure. The planet will then attract it strongly and may greatly change its velocity relative to the sun, so that after leaving Jupiter's vicinity it will pursue quite a different orbit *B*. Moving in this orbit, *M* will return at regular intervals to the point where it met Jupiter, but unless its new period happens to be the same as Jupiter's, at first it will not find the planet close by. Sooner or later, however, Jupiter and the meteorite will return to the point of encounter at very nearly the same time. A second close approach will take place, and the orbit will again be greatly changed.

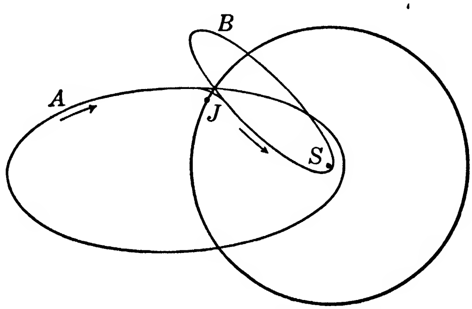

FIG. 127. Problem of Three Bodies

If the meteorite's velocity is this time sufficiently increased, it may be sent off in a parabola or hyperbola, never to return; if not, it will be diverted into still another elliptic orbit and will pursue this until another encounter with Jupiter takes place. A very small difference in the circumstances of the first encounter (perhaps a few miles of difference in the minimum distance) will lead to greater differences in the size and period of the second orbit. After the dozen revolutions or more which may elapse before the second encounter, this will result in still greater changes in the third orbit, and so on.

Any general formula that would be capable of taking into account all the possible sequences of successive encounters would have to be of almost inconceivable complexity.

Two facts — that the attraction of Jupiter would be continually modifying the meteorite's orbit, to a less degree, between the close encounters, and that the orbits are, in general, not in one plane — add further complexity to the problem. If *M* were another planet, of mass comparable to Jupiter's, the changes in Jupiter's orbit would also have to be taken into account, and

things would be still worse. The difficulty of the problem of three bodies, therefore, appears to lie mainly in the inherent complexity of the possible motions, and not in the limitations of mathematical analysis.

323. Solution by Quadratures. In a practical sense the problem of three or more bodies is nevertheless soluble. In any individual case, when the initial data are known with sufficient precision, it is always possible to work out the motions, past and future, with any desired degree of precision, by a fairly simple process of computation *step by step*, — determining, for example, the influence of the forces upon the motions of the bodies in a single day, then calculating anew the forces corresponding to their new positions at the end of the day, and advancing another day, and so on.

This method will handle any particular problem, but it is very laborious and never leads to any general formulæ that can be applied in the case of other bodies or for the same body at remote times. It is therefore used only when all others fail, as in the case of the eighth satellite of Jupiter. A special use is in calculating the trajectories of artillery projectiles as affected by the resistance of the air.

324. Solution by Successive Approximations. In the more important cases which occur in the solar system it fortunately happens that the attraction of some one body is dominant. Thus, the attraction of the sun upon a planet greatly exceeds that of all the other planets combined, and the earth has a similar preponderant influence on the moon. Under these conditions a first approximation to the solution may be made by neglecting the smaller attractions altogether (as has been done in the first part of this chapter). If this approximation were exact, each planet would pursue an unaltered Keplerian ellipse around the sun, and the moon a similar orbit around the earth.

This approximation may be used to obtain nearly, though not quite, correct values of the positions of the planets at any time, and hence of their attractions on one another. The influence of these attractions in changing the motions can then be computed, and a second approximation obtained, from which much more accurate values of the positions of the planets follow. These lead

to improved values of the forces, from which a third approximation to the motions and positions may be derived, and so on.

In practice this process is rapidly convergent. The second approximation is very nearly sufficient for the smaller planets, and the third leaves little more to do even in the case of Jupiter and Saturn.

325. Perturbations. A planet may therefore be regarded as moving in an ellipse about the sun but having this motion modified by the attraction of the other planets. The resulting changes in its position and orbit are called *perturbations*. Though the motions of the planets are thus technically said to be "disturbed," they are actually just as natural and regular (controlled by law) as in the simpler case of undisturbed motion, only they are much more complicated.

For bodies which, like the planets and their satellites, move in nearly circular orbits it is possible to obtain analytical expressions for these perturbations, from which their values at any desired time may be computed. Such expressions are called *general perturbations*. They appear in the form of infinite series, and a great number of terms have usually to be included to obtain accurate results. The calculation of these perturbations is highly intricate.

When, as in the case of comets, the orbit is very eccentric, the perturbations must be computed by quadratures, step by step, for so long an interval as is necessary; such values are called *special perturbations*.

326. Perturbations of the Planets. The actual motions of the planets may best be described as follows:

(1) The orbits of the planets are not fixed but gradually change in eccentricity, inclination, etc., — so slowly, however, that during any one revolution the orbit suffers very little alteration.

(2) A planet, in its motion, does not exactly follow Kepler's laws, but, if compared with an ideal planet which strictly obeys these laws, is continually being shifted, forward or backward, toward or from the sun, above or below the orbit plane, though never to any great distance.

Changes of the first type are called *secular perturbations*. They depend on the relative positions of the planetary orbits. Those

of the second sort are called *periodic perturbations*. They depend on the positions of the planets in their orbits and can be represented as a sum of periodic terms, most of them of short period.

327. The periodic perturbations of the four inner planets are small, being at maximum about 15″ of longitude for Mercury (as seen from the sun), 30″ for Venus, 1′ for the earth, and 2′ for Mars. Those of the outer planets are much greater and may reach about 30′ for Jupiter, 70′ for Saturn, 60′ for Uranus, and 35′ for Neptune. The largest terms are of relatively long period, — 913 years in the case of Jupiter and Saturn.

The magnitude of these periodic perturbations is proportional to the mass of the disturbing planet. If the former can be found by observation, the latter can be determined. This affords the only way of finding the masses of Mercury and Venus, and a good way of finding those of several of the other planets.

It gives, of course, the sum of the masses of the planet and all its satellites. The distinction is important only in the case of the earth and the moon. When the mass of the earth-moon system has been found in this way, the solar parallax may be determined (§ 220).

328. The secular perturbations of the orbits, which are so named because they run on "from age to age," may themselves be expressed by sums of periodic terms of very long period (from fifty thousand to nearly two million years). About a century ago Laplace and Lagrange showed that these changes, though considerable, will not alter the general plan of the solar system.

The major axes of the orbits, and the periods, suffer no secular changes. The eccentricities oscillate irregularly but can never become great, while the perihelia, on the whole, advance, with occasional intervals of regression. In a similar fashion the nodes usually, though not always, regress, while the inclinations oscillate but never become very large.

The orbits of the planets are always so nearly circular that we can form a good idea of their secular changes by regarding them as circles, of fixed radius, whose centers shift slowly about in complicated curves but always keep near the sun, while the planes in which they lie tilt and swing a few degrees in one direction or another but always keep near a mean position.

Another effect of perturbations is that the relation between the period and the mean distance of a planet is not exactly that given by Kepler's third law, even when the mass of the planet itself is taken into account. On the average, planets inside the given planet's orbit increase the central attraction; those outside diminish it, increasing the period. Thus, the period of Jupiter

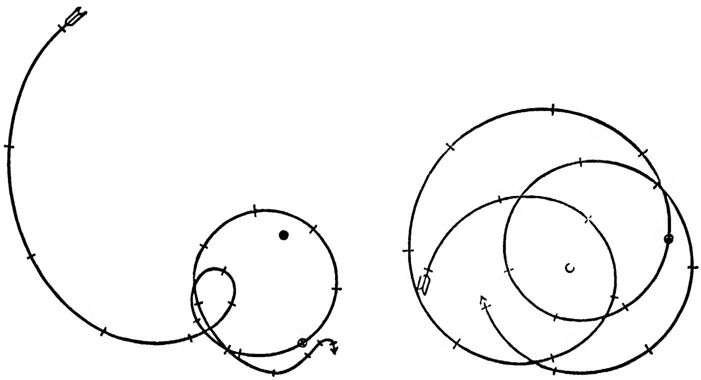

FIG. 128. Secular Perturbations of the Earth's Orbit

The calculated motions, for 200,000 years, of the center of the earth's orbit, in its own plane, are shown in *A*; and those of the pole of the ecliptic, on the celestial sphere, in *B*. The positions at intervals of 10,000 years are marked by short lines, and those at the present time by small circles. The black dot in *A* represents the sun, and the isolated circle in *B* represents the pole of the invariable plane. The earth's orbit itself, on the scale of *A*, would be about 40 inches in diameter. The eccentricity of the orbit is now 0.016 but is diminishing and will reach a minimum value of 0.003 about 24,000 years hence. The inclination to the invariable plane is now 1° 35′ and is also diminishing. It will reach a minimum value of 47′ in about 20,000 years

is one and one-half hours longer than it would be if no other planets existed, while that of Saturn is nearly a week shorter.

329. Perturbations of Asteroids. Many asteroids have orbits of high inclination and eccentricity, which pass relatively near to Jupiter. Their perturbations are correspondingly great, and the theory of these is very complicated.

If the period is very nearly an exact sub-multiple ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, etc.) of that of Jupiter, the perturbations may gradually increase to very large values, as they reënforce each other in successive revolutions. It is significant that there are gaps in the distribution of the asteroids for just these values of the period.

It is possible, however, as Lagrange showed, for an asteroid to move permanently in an orbit with a period just equal to Jupiter's, provided it moves in the plane of Jupiter's orbit and in such a way that its distances from the sun and Jupiter are always equal to the distance between the two, so that the three bodies form an equilateral triangle. Moreover, this motion is *stable*; an asteroid that is originally near one of the "triangular points" and moving at nearly the same rate will always remain near it, circulating about it in a complex curve. In the course of ages, however, the attraction of Saturn and the other planets may upset the stability. Several such asteroids are known, forming the "Trojan group" (§ 419).

Eros and one or two of the other asteroids with small perihelion distances are considerably perturbed by the earth, and may be used to find its mass.

330. Stability of the Planetary System. According to the propositions stated above, the mutual perturbations of the planets alter only the details of their orbits, not the general arrangement of the system. No planet is ever in danger of colliding with another, or of falling into the sun, or of being sent off in a hyperbolic orbit, though such accidents might perhaps happen to an asteroid. But the equations on which these propositions are based represent only the leading terms of an infinite series, and Poincaré has shown that these series, instead of being convergent, as was supposed, may become divergent when a very long interval of time is taken into account. The stability of the planetary system, therefore, has not been *proved*, although its age is so great as to make it very probable that it is actually stable.

331. The Invariable Plane. One property of the system, however, is really stable. A certain plane passing through the center of gravity of the whole system maintains its direction in space exactly invariable (except for the infinitesimal effects of the attraction of the stars). This plane is defined by the condition that the total angular momentum of the system about an axis perpendicular to this plane is a maximum, while it is zero about any axis lying in this plane. Since no actions within the system can alter the total angular momentum, this plane must be invariable.

The invariable plane, and not that of the earth's orbit, is the really fundamental plane of reference for the solar system. It is inclined 1° 35′ 8″ to the ecliptic of 1850, and has its ascending node in longitude 186° 9′. It is intermediate between the planes of the orbits of Jupiter and Saturn, and nearer the former.

332. Definitive Orbits. In making a precise determination of the orbit of a planet or a comet all reliable observations are first collected, corrected, so far as possible, for the systematic errors of the various observers, and combined into "normal," or mean, places, giving the more accurate observations higher weights. These positions are compared with those calculated from a preliminary orbit. The outstanding deviations arise partly from perturbations, partly from errors in the elements of the preliminary orbit, and partly from errors of observation. The perturbations are computed and allowed for, and corrections to the orbital elements are then determined by the method of least squares (§ 124), so as to leave the smallest possible discrepancies to be attributed to errors of observation. The process is very laborious.

The calculations may be designed to find the *osculating orbit*, that is, the orbit which the body would pursue if, at a specified instant, all the planets were annihilated and it moved thereafter under the attraction of the sun alone. This is usually done for comets and often for asteroids. If the general perturbations have been computed, the *mean orbit* may be found, that is, the orbit which the body would follow if its periodic perturbations were abolished. This calculation is still more laborious but has been made for all the major planets and, with less precision, for many asteroids.

333. Planetary Tables. The positions of the planets given in the *Nautical Almanac* are derived from *tables* in which all known perturbations are taken into account. The design and calculation of such tables demands great mathematical knowledge and skill, but their use, with the aid of the "precepts" given in connection with them, is fairly simple.

Certain quantities called arguments, which depend on the positions of the planets in their orbits, are first calculated, and with these arguments the tables are entered which give the perturbations. By adding these to the values given by Kepler's laws (which are taken from other tables) the planet's longitude, latitude, and distance (all referred to the sun) are found. The tables of Jupiter, for example, cover ninety-one quarto pages. The planet's longitude is found by adding thirty quantities taken from them.

334. Perturbations of the Moon: the Disturbing Force. The changes in the moon's orbit are due almost entirely to the attraction of the sun. At new moon (Fig. 129) the sun attracts the earth and the moon in the same direction, but the moon more strongly, since it is nearer, and hence tends to pull the moon away from the earth. This is illustrated by the arrows attached to E and M_1 in the figure, which represent the acceleration of the earth and moon caused by the sun's attraction. The net effect is equivalent to that of a disturbing force acting on the moon and directed away from the earth. At the first quarter (M_2) the accelerations of E and M are equal but are along converging lines, so that the earth and the moon tend to approach one another, and the disturbing force is directed toward the earth. At full moon (M_3) the earth is pulled away from the moon, and the disturbing force is directed away from the earth again.

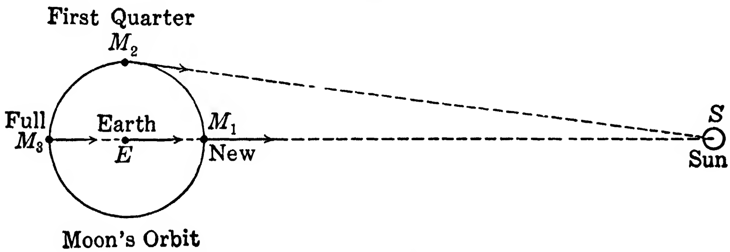

FIG. 129. The Sun's Disturbing Force on the Moon

It should be noticed that this disturbing force represents only a small fraction of the whole attraction of the sun on the moon. The latter is more than twice the earth's attraction, so that if the earth could be held fixed by some supernatural power, the sun would pull the moon right away from it. But since the earth and moon are both free to move, only the difference in their respective accelerations enters into their relative motions, and this is never as much as 1/80 the acceleration of the moon by the earth's attraction.

It should be noticed, also, that the *accelerations*, and not the *forces*, are what really count. The force of the sun's attraction on the earth is more than eighty times as great as that on the moon, owing to the difference of mass; but the accelerations at the same distance are equal. This use of the term "force" instead of "acceleration" is common in celestial mechanics.

It is noteworthy that exact calculations can be made of the motion of a body (for example, a comet) when we have no idea what its mass is and consequently do not know the force of attraction ($f = GmM/r^2$); for the acceleration ($f/m = GM/r^2$) is accurately known.

The effects on the moon's orbit are very similar in kind to the perturbations of the planets, but greater in degree.

335. Motions of the Perigee and Node: Effect on the Length of the Month. The eccentricity of the moon's orbit, on the average, remains the same, but the *line of apsides advances*, though not continuously; so that, on the average, it completes a revolution in 8.8503 years (according to Brown). The inclination too is subject only to periodic changes, but the *nodes regress* at an average rate of a revolution in 18.5995 years.

The disturbing force, on the average, is away from the earth, so that the earth's effective attraction is diminished, and *the month is longer*, by one part in 734, or about 53 minutes, than it would be if the moon's motion were undisturbed.

These effects are clearly analogous to the secular perturbations of the planets.

336. The periodic perturbations are large and extremely numerous. According to Professor Brown's calculations (the latest and most exhaustive) there are 155 periodic terms in the expression for the moon's longitude with coefficients exceeding $0''.1$, and more than 500 smaller ones, which, though individually insignificant, may at times add up to a sum which is not negligible; so that they must all be considered if we wish to compute the longitude to $0''.1$. Each of these terms has its own period. The number of sensible terms in the latitude is about half as great; and almost 150 have to be taken into account in computing the parallax.

337. The Principal Terms. The nature of the results may best be understood by examining the first few terms in the expressions for the moon's longitude and parallax. These may be written

$$\lambda = L + 377' \sin l + 13' \sin 2l + 76' \sin(2D - l) + 40' \sin 2D - 11' \sin l' + \cdots,$$

$$\pi = 3424'' + 187'' \cos l + 10'' \cos 2l + 34'' \cos(2D - l) + 28'' \cos 2D + \cdots.$$

Here L is the moon's mean longitude, or the angular distance of an imaginary uniformly moving mean moon from the vernal equinox; l, the distance of the mean moon from the mean perigee; D, its distance from the mean sun; and l', the distance of the latter from its perigee (which corresponds to the earth's perihelion).

The periodic terms in the longitude have names. Those in l and $2\,l$ are called the *elliptic terms*. They represent motion in a Keplerian ellipse and are not perturbations. The term in $(2\,D - l)$ is the *evection*. It has a period of 31.8 days and gets into step with the principal elliptic term whenever the sun crosses the line of apsides of the moon's orbit. At this time the eccentricity of the orbit is increased by about 20 per cent. When the sun is 90° from the perigee, the eccentricity is correspondingly diminished. This perturbation was discovered by Hipparchus about 150 B.C. The corresponding term in the parallax acts similarly. There is a somewhat similar *evection in latitude*, which makes the inclination of the moon's orbit 9′ less than its average when the sun is at the node, and 9′ greater when it is 90° from it.

The term in $2\,D$ is the *variation*. It sets the moon ahead between new moon and first quarter and between full moon and last quarter, and behind in the other two quarters of the month, while the corresponding terms in the parallax make its distance smaller at new and full and larger at the quarters. The term in l' is the *annual equation* and arises from the changes in the sun's disturbing force which are due to its varying distance.

Of the multitude of smaller periodic terms the most important is the *parallactic inequality*, which, in the complete expression for the longitude, appears as $-\,125''\sin D$, and which sets the moon back at the first quarter and ahead at the third. The coefficient of this term is proportional to the solar parallax (whence its name). The ratio of the two (14.214) can be accurately found from theory. Jones, from 1438 occultations observed at the Cape of Good Hope, finds the coefficient to be $125''.15 \pm 0''.06$, from which follows a solar parallax of $8''.805 \pm 0''.004$, — one of the best determinations.

338. Planetary Perturbations of the Moon. The planets, both by their direct action on the moon and by their indirect action in altering the earth's motion around the sun, give rise to a multitude of periodic perturbations. One of these, due to Venus, has a coefficient of about 14″ and a period of 271 years.

The action of the planets has also, indirectly, an important, though small, influence on the length of the month. The eccentricity of the earth's orbit is slowly decreasing, owing to secular perturbations. Since the major axis of the orbit is unaffected, this implies a slight increase of the minor axis and therefore of the average distance of the earth from the sun during a year. This increased average distance results in a decreased lengthening of the month by the sun's disturbing action. The sidereal month is therefore slowly becoming shorter, and will continue to do so as long as the eccentricity of the earth's orbit continues to

diminish. After about 24,000 years (Fig. 128) the effect will diminish to zero and then become reversed.

As a result of this the moon slowly forges ahead of the position computed with a uniform period, to an extent that increases as the *square* of the time, since in a longer interval there are more months and the average length of each is less. The theoretical change amounts to only 6″.01 at the end of a century (according to Brown), but in 2000 years this becomes 40′, a quantity great enough to be detected even from the rough records of ancient observations.

More than two hundred years ago Halley found this secular acceleration of the moon's motion by comparing ancient with modern eclipses. Fotheringham (1915), from a thorough discussion of all the records of ancient observations, finds an acceleration of 10″.3 per century. The difference between this and the theoretical value is undoubtedly real, and must arise from a gradual slowing of the earth's rotation and lengthening of the day (§ 355). This explanation is confirmed by the fact that the ancient eclipses indicate an apparent secular acceleration of the sun at the rate of 1″.5 per century, which can be explained only by an increase in the length of the day.

339. The Lunar Theory. The precise calculation of the motion of the moon is one of the most difficult and laborious tasks in the whole realm of applied mathematics. With respect to the sun's action, the agreement of the results of Hansen, Delaunay, and Brown (who worked by altogether different methods, each spending between ten and twenty years on the problem) is conclusive evidence that the calculations are correct and complete.

The even more complicated planetary action has now been fully computed by investigations whose complexity Professor Brown compares to "playing chess in three dimensions blindfolded," and in which several thousand terms were examined to see whether they were of sensible magnitude. Brown's new *Tables of the Moon* fill three quarto volumes, totaling more than 360 pages.

340. Outstanding Irregularities in the Moon's Motion. After the influence of the attraction of all known bodies has been allowed for, there remain unexplained and very remarkable discordances between the computed and observed longitudes of the

moon, which sometimes exceed 10″ and may change by several seconds in a few years.

The greater part of these appear to be of a roughly periodic nature. There is evidence of a fluctuation, of period about 260 years, with a range of some 14″ on each side of the mean, and of another of period 60 or 70 years, ranging about 3″ each way. But it would require the addition of several more such empirical terms to represent the observations within reasonable limits of error, and Newcomb and Brown agree that the law of these fluctuations is still unknown, and that their future course cannot be predicted with certainty.

Their cause is also unknown. The observed positions of the sun and the inner planets also show deviations from theory which, though smaller, are remarkably similar. All these deviations would be fairly well explained by supposing that the earth, compared with a perfect clock, gets sometimes fast and sometimes slow, by as much as eight or ten seconds, at rather irregular intervals of a few decades; but no known forces could influence its rotation to anything like this extent.[1]

341. Ellipticity and Internal Constitution of the Planets. The principles of celestial mechanics give a full explanation of the polar flattening of the earth and other planets. This arises from the centrifugal force due to the rotation (§ 150), and the oblateness of a planet (other things being equal) will depend on the ratio ϕ of centrifugal force to gravity at the planet's equator.

If M is the planet's mass, r its radius, and P its rotation period, the centrifugal force at the equator on unit mass is $4\pi^2 r/P^2$, and the force of gravity GM/r^2; whence $\phi = 4\pi^2 r^3/GMP^2$. But if ρ is the planet's mean density, $M = \frac{4}{3}\pi\rho r^3$, whence

$$\phi = \frac{3\pi}{G\rho P^2}.$$

This ratio depends, therefore, only on the density and rotation period of the planet.

[1] Professor Brown has suggested (1926) that alternating expansion and contraction of the earth, by a few feet, may be involved; and thus that the problem is of a geological rather than of an astronomical nature. (Compare § 136.)

The oblateness ϵ depends also on the planet's internal constitution. For a homogeneous planet it may be proved that $\epsilon = \frac{5}{4}\phi$. For a planet in which almost the whole mass is concentrated into a small lump at the center, $\epsilon = \frac{1}{2}\phi$. For several of the planets ϵ may be found by direct measurement, and ϕ may be computed by the equation given above. The ratio ϵ/ϕ is found to be 1.14 for Mars, 0.97 for the earth, 0.76 for Jupiter, and 0.62 for Saturn. It is evident, therefore, that Mars must be of nearly uniform internal density, while Jupiter, and especially Saturn, must be very much denser at the center than near the surface.

The force of gravity at the surface depends on both the ellipticity and the internal constitution. If W is the fraction by which gravity at the pole exceeds that at the equator, then

$$W = \tfrac{5}{2}\phi - \epsilon, \tag{10}$$

as was proved by Clairaut. This equation is used in finding the earth's ellipticity from measures of gravity (§ 138).

342. Perturbations of Satellites. For a close satellite of an oblate planet the principal disturbing force arises from the attraction of the equatorial protuberance, or bulge. If no other disturbing forces are at work, the satellite revolves with a fixed mean distance and period, and the eccentricity of the orbit, and its inclination to the plane of the planet's equator, remain practically constant. The line of apsides advances uniformly, and the nodes on the equatorial plane regress at the same rate, completing a revolution in the same period.

The period of this revolution is

$$\frac{a^2}{r^2} \times \frac{T}{\epsilon - \frac{1}{2}\phi},$$

where a is the satellite's mean distance and T its period, while r, ϵ, and ϕ have the meanings defined above. The ellipticity of a planet may thus be found from observations of its satellites.

For those satellites upon which the action of the sun is sensible the nodes regress upon a "proper plane" which lies between the planes of the planet's equator and orbit and is a sort of compromise between them. The inclination of the satellite's orbit to this plane varies but little. For remote satellites, like the moon, this proper plane almost coincides with that of the planet's orbit, and the principal perturbations are due to the sun.

The ellipticity of the earth increases the motions of the perigee and node of the moon's orbit by about $6''$ per year, and causes a few small periodic perturbations, of which the largest has a coefficient of $8''$. From these Brown deduces an ellipticity of 1/293.7.

In the systems of Jupiter and Saturn the mutual attractions of the satellites produce considerable perturbations of much theoretical interest. By the study of these the masses of a number of the satellites have been determined (§§ 436, 464).

THE TIDES

343. By far the most important (for practical affairs) of the effects of disturbing forces are the *tides* which the attractions of the sun and moon produce in the ocean. These consist in a regular rise and fall of water, generally twice a day and with a range of several feet. The resulting changes of depth and currents are of the utmost importance to navigators, and in many places, such as shallow harbors, the tides still control daily life more closely than any other natural phenomena except the succession of day and night and of the seasons.

344. Phenomena of the Tides. When the water is rising, it is *flood tide*; when it is falling, it is *ebb tide. High water* occurs at the moment when the water level is highest, and *low water* when it is lowest. Successive high waters come about $12\frac{1}{2}$ hours apart; more precisely, the average interval between corresponding high waters on successive days is $24^h\ 51^m$. This is the same as the average interval between two successive passages of the moon across the meridian; and the coincidence, maintained indefinitely, makes it certain that there must be some close connection between the moon and the tides. As someone has said, the odd fifty-one minutes are the "moon's earmark."

When the moon is new or full, the range of the tides is considerably greater than the average, high water rising higher and low water falling lower, without affecting mean sea-level. These are called *spring tides*. The *neap tides*, at the first and last quarters of the moon, have the smallest range, — usually rather less than half that of the spring tides.

Further evidence that the moon is largely responsible for the tides is found in the fact that when the moon is in perigee (nearest the earth) their range is nearly 20 per cent greater than when it is in apogee. The greatest range of all happens when the moon is new or full at the time when it is in perigee.

The "establishment of a port" is the mean interval between the time of high water at that port and the next preceding passage of the moon across the meridian. The establishment of New York, for instance, is $8^h\ 13^m$, — on the average, high water occurs there $8^h\ 13^m$ after the moon has crossed the meridian; but the actual interval varies fully half an hour on each side of this.

In North Atlantic waters the morning and afternoon high tides are about equally high, and the low tides equally low; but in many seas, as in the Gulf of Mexico and the North Pacific, there is a marked *diurnal inequality* in the heights, coupled with considerable irregularity in the times of high and low water. Indeed, in extreme cases there is but one high water and one low water a day. This inequality occurs only when the moon is far north or south of the equator. When the declination is zero, there are two equal tides daily.

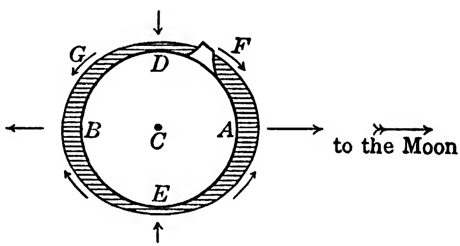

FIG. 130. The Tide-Raising Force

All these complicated phenomena are readily explicable by the effects of the disturbing forces, due to the sun and moon, on the water of the oceans.

345. The tide-raising force is very similar in nature to the disturbing force of the sun upon the moon, which has already been discussed (§ 334). The moon, for example, attracts the ocean at *A* (Fig. 130), directly beneath it and nearer to it, more powerfully than it attracts the solid mass of the earth, and so tends to pull the two apart. The ocean at *B*, on the far side, is less strongly attracted than the mass of the earth, and so the disturbing force again tends to separate the two. At *D* and *E* the moon's attrac-

tion pulls the earth and the ocean along converging lines, and draws them together. At intermediate points, such as F, G, a combination of the two effects takes place, and the resultant forces are indicated by the arrows.

The magnitude of the tide-raising force is very small. If M is the moon's mass (in terms of the earth's), R its distance in radii of the earth, and g the acceleration of gravity at the earth's surface, then the acceleration due to the moon's attraction at C will be gM/R^2, and that at A, $gM/(R-1)^2$. The tide-raising force (on unit mass) f is the difference of these two; that is,

$$f = \frac{gM(2R-1)}{R^2(R-1)^2}.$$

Since R is a large number, we may set R in place of $R-1$, and $2R$ instead of $2R-1$, without serious error. This gives $\frac{f}{g} = \frac{2M}{R^3}$; that is, the tide-raising force is proportional to the mass of the attracting body, and inversely proportional to the cube of its distance.

For the moon $M = 1/81$ and $R = 60$. Hence $f/g = 1/8{,}800{,}000$. This is the maximum tide-raising force. The force at D is half as great, and at F about three fourths, or $1/11{,}700{,}000$ of g.

For the sun, $M = 332{,}000$ and $R = 23{,}400$, so that $f/g = 1/19{,}300{,}000$. The sun's tide-raising force is thus nearly $5/11$ of the moon's.

346. Tides on a Slowly Rotating Earth. If the earth were covered by a deep ocean, and if it kept always the same face toward the moon, the tide-raising forces would produce a permanent distortion of the ocean surface, raising it at the points A and B (Fig. 130) under the moon and opposite to it, and lowering it all around the earth on the circle 90° from these points, as at D and E. The difference in level between A and D would be about two feet (1/11,700,000 of the earth's radius).

Suppose now that the earth were set in *slow* rotation, so that the ocean currents had plenty of time to bring the surface into equilibrium under the tide-raising forces. The lunar tide would then move around the earth like a long, low wave in the ocean, with high water keeping at the points where the moon was at the zenith or nadir. Near an isolated island (F in Fig. 130) the sea would rise whenever the rotation carried it to A or B, and would fall when it was at D or E.

The sun's tidal force would raise a similar but lower tide-wave with its crests under the sun and opposite to it, which would be superposed on the lunar wave and modify its effects.

347. Explanation of the Principal Tidal Phenomena. At new and full moon the lunar and solar high tides would come together, and the range of level would be unusually great. At the quarters the solar high tide would coincide with the lunar low tide, and would partly fill up the depression, producing a decreased range. This explains the spring and neap tides. The greatest and least ranges should be in the ratio of $(11 + 5)$ to $(11 - 5)$, or as 8 to 3. The observed ratio is usually not quite so great.

When the moon is in perigee, it is fully 10 per cent nearer than at apogee, and its tidal force is about 30 per cent greater, which immediately explains the increased range of the tides at perigee.

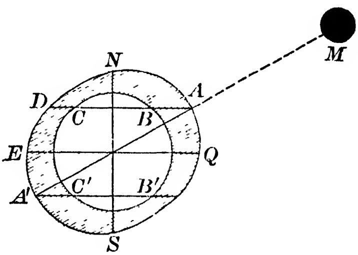

FIG. 131. The Diurnal Inequality

When the moon is farthest north of the celestial equator (*EQ*, prolonged, Fig. 131), the tidal summits will be at *A* and *A'*; the tide which occurs at *B* when the moon is overhead will be great, while the tide in the corresponding southern latitude at *B'* will be small. Twelve hours later, when the earth's rotation has carried the observers to *C* and *C'*, the tide at the northern station will be small and the one at the southern station will be large. The existence of a diurnal inequality in the tides is thus explained. For points much nearer the pole than *B* there will be but one high tide and one low tide a day, while on the equator, at *E* or *Q*, the two tides should be equal.

When the moon is on the celestial equator, *A* and *A'* are on the earth's equator, and everywhere there will be two equally high tides a day, once more accounting for an observed fact.

348. Complexity of the Actual Problem. This simple theory (worked out by Newton) accounts for the general behavior of the tides very well, but it breaks down in details. It does not explain why high tide, at different points, comes at all sorts of different intervals after the moon's transit, or why the range of the tides is

many times greater in some places than in others, or why there is a marked diurnal inequality of the tides in some places near the equator (like Manila) and very little at high latitudes in the North Atlantic.

There are two reasons for these discrepancies. First, the earth rotates rapidly, so that the tide-raising forces in a given region of the ocean change too fast for the water-level to adjust itself fully to them; second, the ocean basins are very irregular in form and depth. The first of these facts makes it a very difficult problem to calculate the tides theoretically even in an ocean of uniform depth, bounded by parallels of latitude and longitude; the second destroys all hope of general theoretical calculation.

349. Tides in Lakes. For a lake which covers so small a part of the earth's surface that the tide-raising forces at all points of its area are practically equal and parallel, the problem is simple. When the moon is on the horizon, the tidal force is vertical and the surface of the lake will be undisturbed. With the moon above the horizon the horizontal component of the tidal force urges the water toward the side of the lake nearest the moon (as at F in Fig. 130), so that the high water will swing around the shore from the east side to the south and west (in the northern hemisphere), while the low water will move along the opposite shore of the lake. With the moon below the horizon the high water will be on the side opposite to it (G in Fig. 130).

In even the largest lakes the tides are very small, — much less than the variations in level due to wind and weather, — and they can only be detected from the average of long series of observations. At Chicago the tide in Lake Michigan has a range of about $1\frac{3}{4}$ inches. The establishment of the port is about 30 minutes (as would be expected from the position of Chicago just west of the southern end of the lake). The tides in the Mediterranean are substantially of this type, and average about one foot in range, reaching three feet at a few points, including Venice and Tunis.

350. Oceanic Tides. For bodies of water as large, as irregular, and as much cut up by land barriers as the oceans are, theoretical calculation is hopeless, except in one very important particular, — it enables exact prediction of the *periods* of the actual tidal oscillations.

Consider, for example, the sun's influence at the time when it is on the celestial equator. At any given point the tide-raising force varies in amount and direction from hour to hour, but returns to the same value (as may easily be proved) at intervals of twelve

hours. After such a force has acted long enough for a "steady state" to be established, the surface of the ocean will be thrown into oscillations. The character of these oscillations — whether they are simple motions, like those in a small lake, or complicated ones, with several regions in which the water rises at the same time, separated by others in which it falls — will depend on the size, shape, and depth of the sea and on the range of level. The time of high water will differ from place to place, but the period will be exactly that of the impressed force.

Similarly, the lunar tide-raising force, if acting alone, would set up a series of oscillations at intervals of $12^h 25^m.2$ with a greater range and a somewhat different "pattern" of regions of high and low water. These two oscillations are called the solar and lunar semi-diurnal tides. Under the combined action of the moon and sun they are superposed without sensibly modifying one another, and their combination produces the spring and neap tides.

The other variations in the tide-raising force may be taken into account by introducing additional periodic terms. For example, the main part of the changes due to the moon's varying distance may be allowed for by superposing on the lunar semi-diurnal tide, the period of which is $12^h 25^m 14^s$, another tide of about 1/5 the range, and of period $12^h 39^m 30^s$. This falls into step with the first whenever the moon is in perigee, and increases the range, while at apogee it is out of step and diminishes it. The lunar diurnal inequalities may be represented by a pair of oscillations, of period nearly one day, which annul each other when the moon is on the equator, but reënforce each other when it is farthest north or south.

351. Analysis and Prediction of the Tides. When a record of the actual fluctuations of the water-level at a given port for a year or more has been obtained (which may readily be done by a self-registering tide-gauge), the ranges and phases of the separate periodic changes of level are not hard to find. For example, taking readings made at noon on every day of the year, the lunar tide will be sometimes high and sometimes low, and its effects will practically disappear in the average, while the solar tide will always be the same. Taking averages of the readings at 1 P.M., 2 P.M., etc., the course of the solar tides can be found. Tides of other periods can be treated similarly.

Having thus found the various component tides at the given port (of which from twenty to thirty are usually large enough to take into account), the level of the water at any time may be predicted by adding together their effects. This may be done mechanically, with the aid of a very ingenious machine which, when once set with the tidal constants for a given port, will automatically draw a curve on a long roll of paper, showing the oscillations of the tide-level for a whole year. The United States Coast Survey uses such a machine to prepare the *Tide Tables* which give the predicted times and heights of every high and low water for about seventy of the principal ports of the world.

352. Height of the Tides. In the open ocean the range of the tides (as observed on isolated islands) averages about two and a half feet, but is very different on different islands, owing to the complicated mode of oscillation of the ocean waters. On the coast the range is usually greater, for the oscillations tend to grow higher as they run into shallow water. In shallow water the wave moves slowly, so that the time of high water differs widely at stations only a few score miles apart.

Where the configuration of the coast forces the tide into a corner, or where the natural period of oscillation of the water in a bay or gulf is nearly the same as that of the tides, the range becomes very great. At the head of the Bay of Fundy the mean range of spring tides is fifty feet, and on the eastern coast of Patagonia it is almost as great. It is also very great in the Bristol Channel, on the coast of Normandy, and in Hudson Strait. In some tidal estuaries in these regions the flood tide comes in as a breaking wave several feet high, called the bore. The tide wave runs up great rivers, such as the Amazon, far into fresh water and many feet above sea-level.

353. Effect of Wind and Barometric Pressure. In quiet weather the predictions of the tide tables are usually very accurate, but a strong wind may drive in the water before it, raising the level sometimes by several feet. A northeast wind, for example, raises the level at the western end of Long Island Sound. In such a case the tide runs in longer than usual, and high water comes late. A wind in the opposite direction produces the reverse effect.

When the barometer is lower than usual, the level of the water is usually higher than it would otherwise be, at the rate of about a foot for each inch of barometric height. In the center of a tropical hurricane, when the barometer is very low and the wind drives the water in spirally toward the center, the tide may rise very high. This effect was largely responsible for the great disaster at Galveston (1900).

354. Direct Measurement of the Tide-Raising Force; Rigidity of the Earth. The tidal force, even when the moon and sun act together, is so small that it is very difficult to detect by direct observation. The problem was solved by Michelson in 1913, by remarkably simple means. A carefully leveled line of iron pipe 500 feet long was buried in a trench; the pipe was half filled with water and sealed air-tight. At each end was a window, through which the water-level could be observed by a microscope. The attraction of the sun and moon produced tides within this pipe, the water rising at one end and falling at the other. The height of these tides amounted at most to less than 1/1000 of an inch, but this could be measured to about 1 per cent. All the phenomena of the tides — springs and neaps, perigee tides, and diurnal inequality — were beautifully exhibited. Two pipe lines, one running north and south and the other east and west, permitted a study of the tidal force in both directions.

Discussion of such measures showed that the times of high water, for the various component periodic oscillations, and their relative heights, agreed admirably with theory; but the heights were in all cases only 69 per cent of the predicted heights.

The reason for this is that the solid earth is elastic and is deformed by the tidal forces, so that its surface rises, falls, and tilts as that of a liquid sphere would do, but to only 30 per cent as great an extent. From this it can be shown that the rigidity of the earth, as a whole, must be about equal to that of steel. It appears, also, that the earth is almost perfectly elastic; there is no evidence of any lag in the action, as there would be if there were any slow permanent yielding, such as would occur in a semi-solid or viscous mass. The actual rise and fall of the earth's surface amounts to about nine inches at the time of spring tides.

355. Tidal Friction: Effects on the Earth's Rotation. In the deep sea the tidal motions of the water are slow and there is little friction, but near the land great masses of water flow in upon the shallows and out again to sea, with a large amount of fluid friction, and this involves the expenditure of a considerable amount of energy, which is converted into heat.

This energy must be derived mainly from the earth's energy of rotation, and the necessary effect is to lessen the rate of rotation

and to lengthen the day. Compared with the earth's whole stock of rotational energy the loss by tidal friction is small and the lengthening of the day is very gradual, but it has been detected with certainty by the study of ancient observations.

Fotheringham's values (§ 338) show that at the end of a century the moon is ahead of the position computed from gravitational theory by $4''.3 \pm 0''.7$, and the sun by $1''.5 \pm 0''.3$. A lengthening of the day at the rate of 1/1000 of a second per century would set the moon ahead by about $5''.8$, and the sun by $0''.75$; these figures agree with the observed values well enough in view of the uncertainty of the data.

356. Rate of Dissipation of Energy. Although this change in the earth's rotation is very slow, it involves a great expenditure of energy. A lengthening of the day by 1/1000 of a second per century requires the continuous dissipation of energy at the rate of *2100 million horsepower*, as is shown by a simple calculation. If the observed accelerations of the moon and sun are really due to tidal friction, the power expended in driving the tidal currents must add up to this gigantic total.

If the depth of water and the velocity and direction of the currents are accurately known, the dissipation of energy by tidal friction in any given area of the sea may be calculated. Jeffreys, and G. I. Taylor, have done this for all the regions where the tides are strong. They find that, on the average, the total rate of dissipation is 1500 million horsepower, and that about two thirds of all the friction occurs in Bering Sea, where the water is shallow and the currents are strong. The numerical data on which the calculations are based are rather rough, and it is quite within the bounds of probability that more precise information would lead to a closer agreement with the value of 2100 million horsepower.

If the earth's interior were somewhat viscous, the deformation of its whole mass by the tidal forces would be attended with serious friction; but evidence of various kinds shows that it is almost perfectly elastic, and that such friction is small compared with that in the sea.

357. Tidal Power. A century and more ago the tides were used to drive small water-mills (tide-mills) all along the New England

coast, but such small units are now unprofitable. There are few places where the height of the tides and the volume of the flow would justify a modern hydro-electric development, and the fact that high water comes at a different time each day complicates the economic side of the problem, so that no such plants are in operation, though some are projected.

358. Effects of Tidal Friction on the Moon's Orbit. Tidal friction has also important effects on the motion of the moon, less obvious than those on the earth's rotation. As the earth's rotation grows slower the angular momentum associated with it diminishes. But angular momentum is indestructible (§ 302); it may be transferred from one body to another in a system, but cannot disappear. If the earth and the moon were wholly isolated, all the angular momentum lost by the earth would be gained by the moon and would alter its orbital motion. Now the angular momentum for two bodies moving in a Keplerian ellipse is proportional to the square root of their distance at a point 90° from the perigee, that is, of the semi-parameter p of the orbit (§ 317). Hence, as the moon gains angular momentum its orbit must expand, it must recede from the earth, and the month must grow longer.

At the present time the angular momentum of the orbital motions of the earth and moon about their center of gravity is 4.82 times that of the earth's rotation. Taking the latter as a unit, the whole momentum is 5.82. If the length of the day should change from its present value d_0 to d, the rotational momentum would change from 1 to d_0/d, since it is proportional to the rate of rotation; and if the moon's distance, measured as defined above, should change from p_0 to p, the orbital momentum would become $4.82\sqrt{p/p_0}$. Hence, if it were not for the solar tides, we should have

$$\frac{d_0}{d} + 4.82\sqrt{\frac{p}{p_0}} = 5.82$$

throughout the whole course of the changes, and this equation would give the moon's distance corresponding to any given length of the day.

The solar tides also slow the earth's rotation, and transfer angular momentum to the earth's orbital motion around the

sun (the solar and lunar tides acting independently, as is easily proved). This causes the earth to recede from the sun, and lengthens the year; but the present orbital angular momentum is almost a million times as great as in the case of the moon, and the changes in the earth's orbit produced by tidal action are therefore practically insensible.

359. "Tidal Evolution." These influences, acting through the course of ages, are capable of producing profound alterations in the system of the earth and moon. Looking backward in time, it appears that the earth's axial rotation must have been faster in remote times, the moon nearer, and the month shorter. The limiting state can be shown to be that in which the day was not quite five hours long, the moon a little more than nine thousand miles from the earth, and the month but a little longer than the day.

The "tidal evolution" of the system from this initial condition to its present state has been studied in detail by Sir George Darwin (son of the great naturalist). The present inclination of the moon's orbit, and the obliquity of the earth's equator to the ecliptic, may also have resulted from tidal action, starting with an initial inclination of about 12° for the system. These changes must have taken a very long time. Jeffreys estimates roughly that they may have required four thousand million years.

In the future the day will grow longer, and the moon's distance will increase, until at last the earth keeps the same face toward the moon, and the sidereal day and month are equal to forty-seven of our present days. Before this happens an interval many times as long as that of the past history of the system must elapse. If the moon once rotated rapidly, its rotation would have been far more quickly slowed by the great tides raised in it by the earth's attraction. Thus the moon, as regards its axial rotation, appears already to have reached the state which the earth will not attain for ages to come.

This state, however, will not be final. The solar tides will still be at work (provided the oceans have not frozen up), and will gradually slow the earth's rotation until the day is *longer* than the month. The friction of the lunar tides will then tend to accelerate the earth's rotation, and oppose that of the solar tides. In this way the angular momentum will be very slowly

transferred to the orbital motion of the system around the sun, until at last the moon comes back close to the earth. Then (according to Jeffreys) it may be torn to pieces by the tidal forces (§ 461) and form a ring around the earth like those of Saturn. The sun itself, however, may have ceased to shine before time enough has passed for these exceedingly slow changes to be completed.

RELATIVITY

Within the last two decades many fundamental physical conceptions have been modified by the advent of the *theory of relativity*, first propounded by Albert Einstein (1905). A general discussion of this theory would lead far outside the range of this book, but certain of its applications, and some of the most important observational tests of the theory, fall within the field of astronomy and must be mentioned here.

360. The principle of relativity is the postulate that the laws of nature are such that all physical phenomena depend only on the *relative* positions and motions of the bodies concerned (including everything on which observations are made, even if it be a distant star), and are quite unaffected by any uniform rectilinear motion which may be common to them all.

With regard to the motions of material bodies this follows immediately from Newton's laws of motion and is confirmed by everyday experience. Thus, the earth's orbital motion (which is nearly enough rectilinear to serve as an illustration) has no influence on the relative positions or motions of objects on its surface, nor does the rapid motion of the solar system through interstellar space (§ 740) have the least influence on the paths which the planets pursue relatively to the sun. (The displacement of Aristotle's views by Newton's was therefore the first and greatest triumph of relativity.)

The modern question was, whether experiments on *light* (or electrical experiments of certain types) could detect uniform rectilinear motion, — if the observer and all his apparatus, for example, were inclosed in a great box, moving with him, which shut out all influences from the universe outside. Though some experimental difficulties and disagreements remain, the general outcome of the investigations of the past generation is in favor of the conclusion that here too the principle of rela-

tivity holds good, in that such motion cannot be detected by these experiments.

The consequences of this principle point to revolutionary changes in our notions of the nature of time and space, the discussion of which must be left to other works. Ordinary dynamical calculations, even in astronomy, are affected hardly at all, since the differences between the old theory (Newton's) and the new theory (Einstein's) involve the square of the ratio of the velocity of a moving body to the velocity of light, and this is an exceedingly small quantity. The principle, however, like that of the conservation of energy or of angular momentum, often greatly simplifies the consideration of a complicated problem.

Two of its consequences may be mentioned here. First, the velocity of light in empty space, as measured by any observer, must always have the same value, no matter whether the observer is at rest or in motion, or in what direction the light is moving. This value c is an absolute constant of nature.

Second, the mass of a body depends upon its content of energy, increasing slightly when this is increased (as by heating it or increasing its velocity). In other words, energy possesses mass. The mass m associated with an amount E of energy is given by the equation $m = E/c^2$. Such a change in mass is great enough to be observable only in the case of electrons moving with great speed (and kinetic energy) in vacuum tubes, but the principle is astronomically important in dealing with the history of the stars (§§ 672, 961).

361. General Relativity. The "special" theory of relativity, just described, was generalized by Einstein (1915) to take account of accelerated motion. The mathematical details of the resulting theory are of notorious difficulty, and, even with the intricate tensor analysis, mathematicians have as yet succeeded in working out only the simpler cases; but some of its underlying ideas, and the observational evidence by which it has been tested, are not hard to understand.

Consider, for example, two observers, each inclosed, with his instruments, in a great box, such as was previously imagined. Suppose that the first observer and his box are at rest in space (or in uniform motion), and that the second — box, apparatus,

and all — is falling freely in a uniform gravitational field, in which the gravitational force is equal and parallel at all points. This box, and everything in it, will be accelerated at the same rate, so that the relative motions of material bodies within it, even according to ordinary mechanics, will not be affected in the least by the external force.[1] Once more the question has to do with optical (and electrical) phenomena; relativity insists that these also are unaffected; and the full development here of the consequences of this principle leads to strange conclusions.

It appears that, in the vicinity of material bodies which exert a gravitational influence, the geometry of space may be represented as not of the simple type studied by Euclid. The result is that in the vicinity of a large mass the natural path of a freely moving body is not straight but curved, — coinciding, in fact, with the orbit which is ordinarily spoken of as being due to the action of the gravitational force due to the large mass. Thus the motion of a planet in its elliptical orbit, and that of a body in a straight line in empty space, remote from even the stars, are, from the new standpoint, regarded as equally natural, neither requiring any specific explanation or the action of any force. When motion of either type is prevented, as when a stone is hung up by a string or whirled rapidly around in a circle at the end of it, the "gravitational force" which is felt in the first case, and the "centrifugal force" in the second, both arise from the fact that the stone is not permitted to follow its natural path.

In all but a very few instances the consequences of the new theory agree with those of Newton's laws, far within the accuracy of the most refined observations, so that the older and far simpler mathematical methods are still to be employed in practice. But there are three cases in which the results differ measurably, and in each of these the observations are decisively in favor of the new theory.

362. The Motion of the Perihelion of Mercury. According to Einstein the orbit of a planet about the sun (if not disturbed by the attraction of other planets) will be sensibly elliptical in form, with the sun at one focus; the line of apsides, however, will not

[1] It is only because the attractions of the sun and moon on the earth are *not* equal and parallel at all points that they raise the tides.

be fixed in direction, but will slowly advance. (That is to say, the orbit takes the form of an ellipse which is very slowly rotated.) The rate of advance depends only on the planet's mean distance, its period, and the velocity of light, and may be exactly calculated from the theory. The predicted advance should be observable only in the case of Mercury; the orbit of Venus is so nearly circular that its perihelion cannot be found accurately, and the predicted motions for the earth and Mars are too slow.

Now it has long been known that the perihelion of Mercury is moving faster than the older theory indicates. The motion of the perihelion can be determined accurately by observation, especially of the times of the planet's transits, and it is found, after allowing for the secular perturbations due to all the planets, that the perihelion shows an unexplained advance at the rate of $40''.1 \pm 1''.4$ per century. This change of the orbit in a century alters the calculated position of Mercury at transit by more than the planet's diameter,—an amount impossible to ignore. The advance predicted by Einstein is $42''.9$. The perturbations are sensibly the same on the new theory as on the old, so that the agreement between observation and the new theory is striking.

Various attempts have been made, before and after the development of relativity, to account for this motion of Mercury's perihelion by the attraction of a diffused mass of meteoric matter surrounding the sun. This is quite possible as a mathematical exercise, but the quantity of matter required, when illuminated by the sun, would, even on the assumptions most favorable to this theory, reflect an altogether inadmissible amount of light, vastly exceeding in brightness the faint zodiacal light which is actually observed (§ 424). The relativistic explanation therefore holds the field.

363. The Curvature of Light Rays near the Sun. The second, and in some respects the most remarkable, prediction of the new theory is that rays of light in a perfect vacuum, passing by a large gravitating mass, should not be perfectly straight but should bend inward somewhat like the hyperbolic orbits of material particles moving at very high velocity. The computed deflection is insensible for rays passing near planets. For a ray that grazes the sun it amounts to $1''.75$; for other rays it is inversely proportional to the least distance of approach to the sun's center.

The only way to observe this deflection is to photograph the stars surrounding the sun during a total solar eclipse. The bending of the light rays will make the stars appear farther from the sun's center than they otherwise would (Fig. 132). By comparing the photographs taken during the eclipse with others taken under like conditions a few months earlier or later, when the sun is out of the way, the gravitational deflection can be measured.

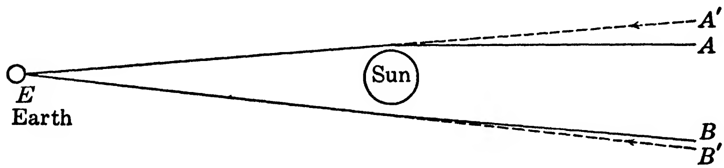

FIG. 132. Deflection of Light Rays in Sun's Gravitational Field

Rays from A and B are deflected inward, so that to an observer on the earth at E the rays appear to come from A' and B', farther from the sun. The ray which passes nearer the sun is deflected more. The effect is enormously exaggerated in the diagram

The observations are extremely delicate. Great care must be used that the camera is properly adjusted, and that the adjustments are the same when the comparison plates are taken. The stars must be at the same altitude in the sky in the two cases, so that the effects of refraction may be the same. When these precautions are employed, and when the number of stars shown on the plates is considerable (thus permitting a determination of the small outstanding differences of adjustment from the plates themselves), reliable results can be obtained.

The test was first made by two English parties at the eclipse of 1919. Observations in Africa were much disturbed by clouds, but those in Brazil were successful in spite of some instrumental troubles. The plates gave conclusive evidence of the existence of the deflection, and determined the amount at the sun's limb (from the observed values farther out) as $1''.98$, which agreed with the predicted value within the error of observation.

Still more definite results were obtained at the Australian eclipse of 1922, especially by the Lick expedition. Every precaution was taken in the construction of the instruments, the exposure of the plates, and their measurement and reduction. More than ninety stars were measured. The mean result from four pairs of plates gave the value $1''.78 \pm 0''.11$ for the deflection at the sun's limb (in almost exact agreement with prediction), and

the theoretical law of variation of the deflection with distance was confirmed. Observations by Canadian and Australian parties likewise confirmed the reality and amount of the deflection.

Here again the observational evidence is decisively in favor of the theory of relativity. (Attempts to explain the observations by peculiarities of refraction in the earth's atmosphere have met with no success.)

364. The Displacement of Spectral Lines to the Red. A third prediction of the theory is that when light is emitted or absorbed by an atom on the surface of a massive star, the vibrations will be slightly slower, and the wave-length of the light greater, than for atoms of the same sort on the earth. Thus the corresponding spectral lines should be shifted toward the red. In the case of the sun the shift is small (§ 591), and its effects are confused with other small displacements not yet fully understood. There is a faint star, the companion of Sirius, for which the predicted Einstein shift is great enough to be readily measurable. The observations of Adams give conclusive evidence of such a shift, of about the predicted amount and not otherwise explained (§ 828).

The astronomical evidence, which alone affords an observational test of the general theory of relativity, is therefore at all points definitely in its favor.

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CHAPTER XI

THE TERRESTRIAL AND MINOR PLANETS

MERCURY, VENUS, EARTH, AND MARS · THE ASTEROIDS · ZODIACAL LIGHT

MERCURY

365. Mercury has been known from remote antiquity, and there are recorded observations running back to 264 B.C. At first astronomers failed to recognize it as the same body on the eastern and western side of the sun, and among the Greeks it had for a time two names, — Apollo when a morning star and Mercury when an evening star. It is so near the sun that it is comparatively seldom seen with the naked eye (Copernicus is said never to have seen it), but when near its greatest elongation it is easily visible as a brilliant star low down in the twilight, varying, according to circumstances, between the stellar magnitudes (§ 688) -1.2 and $+1.1$, that is, from almost the brightness of Sirius to that of Aldebaran. It is best seen in the evening at such eastern elongations as occur in March and April. As a morning star it is best seen at western elongations in September and October.

It is an exceptional planet in various ways. It is the *nearest* to the sun, *receives the most light and heat*, is the *swiftest in its movement*, and (excepting some of the asteroids) *has the most eccentric orbit*, with *greatest inclination to the ecliptic*. It is also the *smallest in diameter* (again excepting the asteroids) and has the *least mass*.

366. Its Orbit. Its mean distance from the sun is 35,950,000 miles, but the eccentricity of its orbit is so great (0.206) that the sun is 7,400,000 miles out of the center, and the distance of the planet from the sun ranges all the way from 28,550,000 to 43,350,000, while the velocity in its orbit varies from 36 miles a second at perihelion to only 24 at aphelion. Its distance from the earth ranges from about 50,000,000 miles at the most favorable inferior conjunction to about 136,000,000 at the remotest superior conjunction.

A given area upon its surface receives on the average nearly seven times as much light and heat as the same area on the earth; and the amount received at perihelion is greater than that at aphelion in the ratio of 9 : 4.

The *sidereal* period is 88 days, and the *synodic* period (from conjunction to conjunction) 116 days. The *greatest elongation* ranges from 18° to 28°, varying on account of the eccentricity of its orbit, and occurs about 22 days before and after inferior conjunction. The inclination of the orbit to the ecliptic is about 7°.

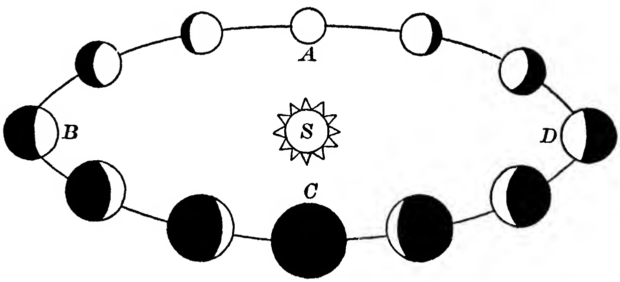


FIG. 133. Phases of Mercury and Venus (Schematic)

In the telescope the planet looks like a little moon, showing phases precisely similar to those of the moon. At inferior conjunction C the dark side is toward us; at superior conjunction A, the illuminated side. At greatest elongation near B, D, the planet appears as a half-moon. It is gibbous between superior conjunction and greatest elongation, while between inferior conjunction and greatest elongation it shows the crescent phase. The form of the crescent is only roughly indicated

367. Diameter, Mass, Density, and Surface Gravity. The apparent diameter of Mercury ranges from 5″ to about 13″, according to its distance from us, and the real diameter is about 3100 miles. This value may be in error by 5 per cent. Its surface is about 15 per cent of that of the earth, and its volume, or bulk, 6 per cent.

The mass of Mercury is extremely difficult to determine. The planet is so small, and so near the powerfully attracting sun, that its effect in disturbing other planets is very slight; and the probable error of Mercury's mass derived from these perturbations is correspondingly great. Occasionally a comet comes near enough to the planet to be sensibly influenced by its attraction, but the same difficulties are met with in this case, — Mercury disturbs its motion very little.

The latest calculated value of the mass of Mercury (de Sitter) is 1/8,000,000 of the sun's mass, or 1/24 that of the earth, with a probable error of about 25 per cent. With this mass, and a diameter of 5000 kilometers, the surface gravity comes out 0.27 that of the earth, and the density 0.70 that of the earth, or 3.8 times that of water, — intermediate between the densities of Mars and the moon, as seems reasonable. These results, however, are subject to very considerable uncertainty.

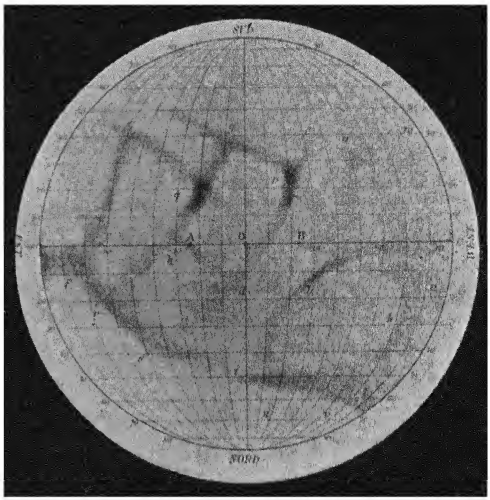


FIG. 134. Surface Detail of Mercury
After Schiaparelli

368. Telescopic Appearance and Rotation. Like the moon and Mars (which also possess solid surfaces), but unlike Jupiter, the illuminated edge of Mercury's disk is brighter than the center. Generally, of course, the planet is so near the sun that it can be observed only by day; but when proper precautions are taken to screen the object-glass from direct sunlight, such daylight observation is not especially difficult. It is only under very favorable atmospheric conditions, however, that any details can be seen on the surface of Mercury. In 1889 Schiaparelli, the Italian astronomer, announced the discovery of certain dark permanent markings there (Fig. 134). As in the case of other difficultly visible planetary details, there are considerable differences between the descriptions of the markings by different observers. Barnard described them as "very much resembling those seen on the moon with the naked eye," while Lowell drew them as almost linear.

Schiaparelli also found that the markings did not change their positions on the planet's disk even in the course of several hours, but remained nearly fixed in their position with respect to the terminator, — the boundary between the illuminated and unilluminated hemispheres of the planet. Granting this permanency, it follows that the planet rotates on its axis only once during its orbital period of eighty-eight days; that is, it keeps the same face always toward the sun, as the moon does toward the earth.

369. Atmosphere and Physical Condition. The low velocity of escape at the planet's surface (3.6 kilometers per second) makes it very probable that any atmosphere that Mercury may ever have had has largely escaped into space, as in the case of the moon (§ 201). If Mercury has any at all, it must be exceedingly rare. The phenomena which prove that Venus has an atmosphere, such as the prolongation of the horns of its crescent by twilight (§ 376), have never been observed in the case of Mercury, and the spectroscopic evidence is inconclusive.

The low value of the albedo, only 0.07, which is somewhat inferior to that of the moon, indicates that the light is reflected from a solid surface rather than from an atmosphere. In the proportion of light given out at its different phases it again resembles the moon, the increase of brightness near the full phase being even more marked. It is therefore probable that its surface has as rough a structure as that of our satellite.

If one side of the planet turns permanently toward the sun, the temperature of this side must be very high, while the opposite face, which never receives any sunlight, must be intensely cold. Between these regions is a space in which, on account of librations, the sun alternately rises above the horizon and drops back again, and in which the variations of temperature must be great. According to Pettit and Nicholson's radiometric measures (§ 618) the temperature of the sunlit side of Mercury is about 350° C., — quite hot enough to melt lead.

370. Transits of Mercury. At the time of inferior conjunction the planet usually passes north or south of the sun, the inclination of its orbit being 7°; but if the conjunction occurs when the planet is very near its node, it crosses the disk of the sun as a small black spot, — not, however, large enough to be seen

without a telescope (Fig. 135). Since the earth passes the planet's node on May 7 and November 9, transits can occur only near those dates.

At the May transits the planet is near its aphelion and much nearer the earth than it is ordinarily, and the transit limit (corresponding to an ecliptic limit) is 2° 40′ on either side of the node. For the November transit, when the planet is nearer the sun, the corresponding limit is 4° 45′, so that transits at this node are about twice as numerous as at the other.

For the November transits the interval between two successive passages of Mercury across the sun is sometimes only 7 years, but is usually 13 years. For the May transits the 7-year interval is impossible. Twenty-two synodic periods of Mercury are nearly equal to 7 years; 41, much more nearly equal to 13 years; and 145, almost exactly equal to 46 years. Hence, 46 years after a given transit another one at the same node is almost certain.

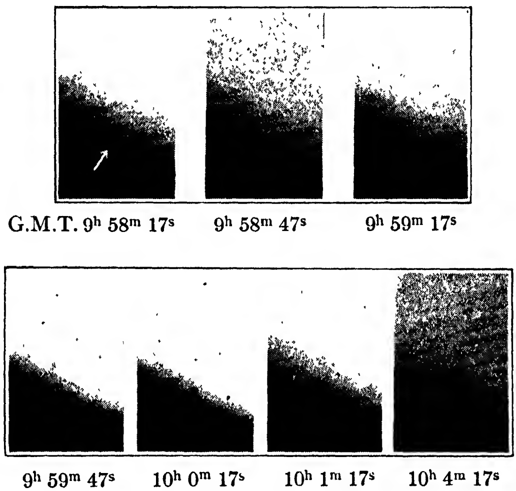


FIG. 135. Transit of Mercury, November 7, 1914

The photograph at $9^h\ 58^m\ 17^s$ shows a slight depression in the limb of the sun, the planet starting to enter on the disk. Second contact occurred shortly after $9^h\ 59^m\ 47^s$. (From photograph by Royal Observatory, Greenwich)

The first and second contacts of the transit of May 7, 1924, were visible in the United States and were well observed. The sun set with the planet well on the disk. The next transit wholly visible in the United States will occur on November 14, 1953.

A transit of Mercury furnishes an opportunity for determining corrections to the planet's place and its orbital elements.

VENUS

The next planet in order from the sun is Venus, by far the brightest and most conspicuous of all in our skies, — the earth's twin sister in magnitude, density, and general constitution, if not in other physical conditions. Like Mercury, it had two names among the Greeks, — Phosphorus as morning star and Hesperus as evening star.

It is so brilliant that it is easily seen with the naked eye in the daytime for several weeks when near its greatest elongation; occasionally it is bright enough to catch the eye at once, but usually is seen by daylight only when one knows precisely where to look for it.

371. Distance, Period, and Inclination of Orbit. Its *mean distance* from the sun is 67,170,000 miles.

The *eccentricity* of the orbit is the smallest in the planetary system (only 0.007), so that the whole variation of its distance from the sun is less than a million miles.

Its *orbital velocity* is 22 miles per second.

The *heat and light* received from the sun are almost exactly double the amount received by the earth.

Its *sidereal period* is 225 days, or nearly seven and one-half months, and its synodic period 584 days, or a year and seven months. From superior conjunction to greatest elongation on either side is 220 days, while from inferior conjunction to greatest elongation is only 72 days, — less than one third as long.

The *greatest elongation* is 47° or 48°.

The *inclination of its orbit* is about 3° 24′.

372. Diameter, Mass, Density, etc. The apparent diameter of the planet ranges from 64″ at the time of inferior conjunction to only 10″ at superior conjunction, the great difference being due to the enormous variation in the distance of the planet from the earth, which is only 26,000,000 miles at inferior conjunction and 160,000,000 at superior conjunction. No other body ever comes so near the earth except the moon, an occasional comet, and one or two asteroids.

Measures during transit make the planet's diameter 7600 miles, and the older measures on a dark sky, about 7800 miles.

The difference is clearly due to irradiation. The real diameter is probably not far from 7700 miles, with a probable error not exceeding 1 per cent.

According to this its *surface* is 0.95 of that of the earth; its volume, 0.92.

By means of the perturbations which the planet produces on the earth and Mars the *mass* of Venus is found to be 1/410,000 of the sun's mass, or 0.81 of the earth's. This value is uncertain by a considerable fraction of 1 per cent. The density comes out 88 per cent, and the superficial gravity 85 per cent, of the earth's. A man who weighs 160 pounds on the earth would weigh 136 pounds on Venus.

FIG. 136. Venus

Photographs showing the various phases and true relative sizes of the planet's disk presented during a synodic period. (From photographs by E. C. Slipher, Lowell Observatory)

373. Brightness, Albedo, and Phases of Venus. Whenever visible at all this planet appears brighter than any other, ranging from -3.3 to -4.3 in stellar magnitude (§ 688). As it moves from superior toward inferior conjunction the increase in apparent diameter at first more than makes up for the diminution of the illuminated portion of the disk, and the planet grows brighter. As the crescent begins to narrow, however, the second influence outweighs the first, and the brightness diminishes again. Maximum brightness is reached about 36 days before and after inferior conjunction, at an elongation of 39° from the sun, when the phase is like that of the moon when it is about five days old (Fig. 136). At this time the planet appears about $2\frac{1}{2}$ times as bright as when near superior

conjunction, casts a strong shadow, and is easily visible to the naked eye in full daylight. When the effects of varying distance are allowed for, it is found that the brightness of Venus, if seen from a fixed distance, would diminish steadily with advancing phase, but much less rapidly than the moon's. According to Müller's observations the brightness for different values of the phase angle (that is, the angle at the planet between lines drawn to the earth and sun) is as follows:

Phase angle	0°	30°	60°	90°	120°	150°
Brightness	100	69	45	25	12	4.3

We may conclude that the reflecting surface of Venus is much less rough than that of the moon or Mercury.

The albedo of the planet is 0.59, — very much what might be expected from a surface completely covered with clouds, but explicable in other ways also.

Photographic measures of the brightness indicate that the color of sunlight is very little altered by reflection from the planet, that is, that its surface is almost white.

According to the theory of Ptolemy, Venus could never show us more than half its illuminated surface, since, according to his hypothesis, the planet was *always between us and the supposed orbit of the sun.* Accordingly, when, in 1610, Galileo discovered with his newly invented telescope that Venus exhibited the *gibbous* phase as well as the crescent, it was a strong argument for the Copernican theory.

Galileo announced his discovery in a curious way, by publishing the anagram, —

Haec immatura a me iam frustra leguntur: o. y.

Some months later he furnished a solution, which is found by merely transposing the letters of the anagram and reads, "Cynthiæ figuras æmulatur Mater Amorum," meaning "The Mother of the Loves [Venus] imitates the phases of Cynthia [the moon]."

374. Surface Markings. These are not at all conspicuous. The usual telescopic appearance is admirably illustrated by Fig. 137, the brightness shading off smoothly toward the terminator without a trace of detail.

Under favorable atmospheric conditions, however, faint and ill-defined markings are often seen. Barnard described them as "large dusky spots," too elusive to reproduce in a drawing, and said that observations on different days gave the impression that they were not permanent.

375. Rotation of the Planet. The rotation period of Venus is not yet determined, mainly because of the difficulty of finding definite and recognizable markings on its surface. It was for a long time supposed, on insufficient evidence, to be a little less than 24 hours; Schiaparelli concluded that the rotation must be very slow, and that it was probable that Venus, like Mercury, keeps always the same face toward the sun.

FIG. 137. Venus in the Crescent Phase

Enlarged from a photograph with the 40-inch refractor. (From photograph by E. E. Barnard, Yerkes Observatory, June 5, 1908)

A more hopeful means of attacking the problem is found in spectroscopic determinations of the radial velocity (§ 579) of points on opposite sides of the disk. Observations at the Lowell and Mt. Wilson observatories agree in showing that the velocity of rotation is too small to measure. A rotation period less than 20 days would very probably have been detected, but one of more than 5 or 6 weeks would have escaped observation.

If the rotation period were comparable with a day, the planet should be perceptibly flattened at the poles, and the difference between the equatorial and polar diameters, when Venus is nearest us, should be about $0''.2$; but the numerous and precise measures made during transits show no evidence of oblateness.

There seems no doubt, then, that the period of rotation is longer than that of any planet except Mercury, but it is not likely that Venus keeps the same face toward the sun. Pettit and Nicholson find that considerable heat is radiated from the dark

side, — enough to indicate that the temperature on that side is fairly uniform and about $-25°$ C. The bright side sends us more heat, but most of the difference is due to the reflected rays of the sun, and the bright part of the visible surface does not seem to be much hotter than the dark part. This is what might be expected above high clouds in a rotating planet; but it would be almost impossible to explain on the hypothesis that the planet always keeps one face toward the sun, for then one side should be very hot and the other very cold. Transfer of heat from one side to the other might diminish this difference, but hardly to the observed degree. Coblentz and Lampland, in 1924, found much more heat coming from the south cusp than from the other. This may indicate that the axis is inclined to the plane of the orbit. Further observations may settle the question within a few years.

376. Atmosphere. The evidence that Venus has an atmosphere is conclusive. Near the time of inferior conjunction the horns of the crescent extend notably beyond the diameter, showing that more than half of the surface must be illuminated by the sun or visible to us. When the planet is very near to the sun the cusps actually run together, forming a complete ring around the disk, which has been seen by several observers at different times. This phenomenon was formerly ascribed to refraction, but Russell, in 1898, showed that it must be due to diffuse reflection of light by the planet's atmosphere, like that which causes our twilight.

From the amount of prolongation of the cusps of the crescent it is possible to calculate the "height of the atmosphere" (§ 119). The observations show, with surprising consistency, that that part of the planet's atmosphere in which the twilight is bright enough to be seen, through the glare of our own atmosphere close to the sun, extends to a height of about 4000 feet above the visible surface. The whole height of its atmosphere must be many times as great.

When Venus is entering upon the sun's disk, or leaving it, at a transit, the portion of its disk outside the sun is surrounded by a line of light, which must be incomparably brighter than the faint extensions of the cusps previously discussed (since it can be seen through the dark glasses used in observing the sun), and is unquestionably due to refraction of light in the planet's

atmosphere. The deviation of this light is, however, but a minute or two of arc, and a rare atmosphere would suffice to produce it.

It may therefore be concluded that the atmosphere of Venus above the visible surface is much less extensive and dense than the earth's, and perhaps comparable with the amount of the earth's atmosphere which lies above the high clouds. (If Venus's apparent surface is a cloud-layer, there may be any amount more below it, and for this very reason inaccessible to our investigation.)

If this atmosphere contained oxygen or water vapor in any considerable quantity, their presence could be detected by certain lines which they would produce in the spectrum of the light reflected by the planet (§ 619). St. John, after very careful observations, finds no evidence of such lines, and concludes that the amount of oxygen in the atmosphere above the visible surface must be less than 1/1000 of the quantity in the earth's atmosphere. The test for water vapor is not so delicate, and a quantity such as prevails above the highest clouds on the earth might escape detection; but 1 per cent as much oxygen as exists above these clouds could have been recognized.

377. Physical Condition and Habitability. The force of gravity is sufficient to retain an atmosphere similar to ours, and it is quite possible that there may be water vapor below the level of the clouds, through which the spectroscope is unable to penetrate. Conclusions regarding the physical conditions which prevail on the surface depend essentially on the assumed period of rotation.

If, as now appears decidedly probable, this period is some weeks in length, and if there are oceans on the planet, as seems likely from its similarity to the earth, then, as Clayden pointed out years ago, there should be ascending air currents and clouds over the sunlit side, with descending currents and possibly clearer sky on the night side. This would account for the brilliant whiteness of the surface, while the vague and fugitive markings which are seen would be naturally interpreted as thinner places in the cloud-layer. If this is the case, there may be much more atmosphere underneath the clouds, and the surface may be much warmer than the upper clouds, as is the case on the earth.

The absence of oxygen is at first very surprising, but on second thought we realize that free oxygen is a very active chemical substance to be present in great quantity in a planet's atmosphere. We know that the store of oxygen in the earth's atmosphere is continually being replenished by the activity of vegetation, and it has often been suggested that all the oxygen in the earth's atmosphere is of vegetable origin, being balanced by a corresponding quantity of carbonaceous or otherwise reduced material buried in the sedimentary rocks. Further confirmation of this belief is found in the fact that igneous rocks and volcanic gases are usually incompletely oxidized. This would suggest that although Venus appears to be better suited than any other planet to be the abode of life such as exists on the earth, yet for some reason life has never developed there. Further investigation, however, may develop new evidence, and may change our estimate of the probabilities, — possibly even within a few years.

378. Absence of Satellites. No satellite of Venus has yet been discovered, and it is certain that the planet has none of any considerable size. It is not impossible, however, that it may have a pygmy attendant, like those of Mars, since its great brilliancy and its nearness to the sun would make the discovery of such a body extremely difficult. There have been several announcements of a satellite, but not one has been verified, and most of them were mistakes, since explained either as observations of stars or as reflections in the eyepiece of the observer's telescope.

379. Transits. Occasionally Venus passes between the earth and the sun at inferior conjunction and transits, or crosses, the disk of the sun from east to west as a round black spot, easily seen by the naked eye through a suitable shade glass. When the transit is central, it occupies about eight hours; but when the track is near the edge of the disk, it is correspondingly shortened. Since the transit can occur only when the sun is within about $1°\,45'$ of the node, the phenomenon is rare and can happen only within a day or two of the dates when the earth passes the nodes, namely, June 7 and December 8.

The special interest of the transits lies in their availability for the purpose of finding the parallax and distance of the sun, as first pointed out by Halley in 1679.

The earliest observed transit, in 1639, was seen by two persons only (Horrocks and Crabtree, in England); but the four which have since occurred, in June, 1761 and 1769, and in December, 1874 and 1882, were extensively observed by scientific expeditions sent out by the different governments to all parts of the world where they were visible. The transits of 1769 and 1882 were visible in the United States.

It is not likely, however, that so much trouble and expense will hereafter be expended upon observations of transits. Other methods of determining the solar parallax have been found to be more trustworthy.

380. Recurrence and Dates of Transits. Five synodic revolutions of Venus are very nearly equal to 8 years, the difference being little more than one day; and 152 synodic revolutions are still more nearly (in fact, almost exactly) equal to 243 years. If, then, we have a transit at any time, another *may* occur at the same node 8 years earlier or later. This will be impossible 16 years before or after, and no other transit can then occur *at the same node* until after the lapse of 235 or 243 years, though a transit or pair of transits may, and usually will, occur *at the other node* in about half that time; thus, the next pair of transits of Venus will occur on June 8, 2004, and June 6, 2012.

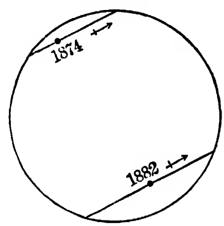


FIG. 138. Tracks of Transits of Venus

If the planet crosses the sun almost centrally, the transit will be solitary, that is, not accompanied by another one 8 years before or after. If, however, the track is more than 12′ from the sun's center, it will be accompanied by another at an interval of 8 years. Transits come thus in pairs at present, and have been doing so for several centuries; after a time this will cease to be true, and they will become solitary for another long period.

THE EARTH

Certain characteristics of the earth as a planet may best be discussed here.

381. Albedo. The only direct way of determining the earth's albedo is by measuring the brightness of the earth-shine on the moon (§ 185) compared with that of the sunlit portion. The observations are difficult; but the rather imperfect ones that have

so far been published show that the full earth, seen from the moon, must be about 40 times as bright as the full moon seen from the earth (with a probable error of some 20 per cent). The resulting value of the albedo is 0.45, — intermediate between those of the cloudless and cloud-covered planets, as might be expected.

The observations also indicate that the earth-lit part of the moon is perceptibly bluer than the sunlit portion, as should be true if any considerable part of the light reflected from the earth comes from its atmosphere, that is, from the blue sky.

382. Appearance from Other Planets. To an observer on Venus the earth, when nearest, would be a very brilliant object, about six times as bright as Venus at brightest appears to us. This great difference arises from the fact that when the two planets are near one another, almost the whole of the illuminated side of the earth is visible from Venus, while very little of that of Venus is visible from the earth.

From Venus the moon would look about as bright as Jupiter, and would oscillate back and forth within half a degree of the earth, so that the two would resemble a *double planet* rather than a planet and satellite, and would form by far the most noteworthy object in the heavens. In all probability there would be a decided difference in the color of their light (the earth's being bluish white and the moon's yellowish), which would add to the effect.

From Mars the earth at its brightest would look about as bright as Jupiter does to us. From the outer planets the earth would appear about one third as bright as Venus at a similar configuration (such as greatest elongation), the main cause of this difference being that the earth receives only half as much light per square mile.

383. Telescopic Appearance. The appearance of the earth as seen with the telescope from Venus, or with the naked eye from the moon, would be very different from that of any of the other planets. The most conspicuous features would be the *clouds*, which, on the average, cover about half the earth's surface and reflect about three times as much light as the surface and sky together in cloudless regions.

As Clayden points out, the clouds would form a brilliant white belt in the region of the equatorial rains, shifting in latitude with

the seasons, flanked on either side by nearly cloudless and therefore much darker belts over the desert zones near the tropics of Cancer and Capricorn, beyond which brighter, partly cloudy regions would extend to the poles.

These belts would exhibit a wealth of ever-changing details, among which the great cyclonic storms of temperate latitudes would be conspicuous as white (cloudy) areas followed by dark (clear) ones, moving eastward and lasting for a number of days. In consequence of this eastward motion the observed average rotation period of the cloudy markings in this zone would be considerably less than twenty-four hours, while nearer the equator, in the trade-wind region, where the winds blow regularly from the northeast or southeast, the observed average rotation period would be more than twenty-four hours.

Except in desert belts there would rarely be an area as much as a thousand miles square entirely free from clouds, and the study of the details of the true surface would be greatly hampered. Even in cloudless weather the surface would be seen through a blue atmospheric veil, like that which intervenes between an observer on the earth's surface and a mountain ten miles or so away, and details could be made out only when the air was free from dust or haze. The most conspicuous surface features would probably be (1) the reflection of the sun from the ocean (under favorable conditions, by far the most brilliant thing on the planet); (2) snow-covered areas (much confused, however, by the overlying clouds); (3) deserts (if practically devoid of vegetation, yellowish or reddish in tone). The darkest parts of the surface would be the oceans (where not directly reflecting sunlight) and the great forest regions, both of which would appear of a dull blue, since most of the reflected light would come from the overlying atmosphere. Cultivated regions and grasslands would appear of a lighter tone (greener), but only the most generalized features of their distribution could be discovered.

384. In the frontispiece, which is from a painting by Howard Russell Butler, the earth is represented as seen from the floor of a crater near the moon's north pole, about three days before new moon. The most conspicuous objects on the surface are the reflection of the sun on the Atlantic Ocean off the coast of Africa,

the belt of clouds in the region of equatorial rains, and a cloud mass over the North Atlantic representing a cyclonic storm. The outlines of parts of Europe and of North and South America may be less conspicuously seen. While, of course, an imaginative representation, the painting has been done with careful consideration of the known facts and is strikingly different from conventional representations of the earth.

A powerful telescope on the moon would reveal a great amount of detail on the rare occasions when the earth's atmosphere above the region under observation was really clear. The view would probably resemble that from an airplane at a great altitude, though on a vastly diminished scale. If so, the coastline would be fairly conspicuous, and even small bodies of water might be seen if they were so placed as to reflect the sun. Mountains might be detected by their shadows, though much less easily than we can see them on the cloudless moon. Painstaking study might lead the hypothetical lunar astronomers to a fairly accurate idea of the topography of the earth's surface. *Seasonal changes* could be easily noticed; the most prominent (after the variations in cloudiness) would be the advance of the winter snows and their poleward retreat in summer, and the change in the color of cultivated lands and prairies from green to brown.

Observers on Venus would be able to see objects fifty miles in diameter, or even smaller, under the best conditions, but would have great difficulty in distinguishing the permanent surface features through the shifting clouds.

Those on Mars could find out much less; for the earth, when nearest, would appear only as a narrow crescent. The prolongation of the horns of the crescent by twilight, and the related phenomena, would probably be much more prominent than in the case of Venus.

385. Whether observers outside could detect any evidence of *human activity* on the earth is an interesting question. The imaginary astronomers on the moon could easily detect such changes as the gradual clearing of the forests in the eastern part of America, and might perhaps notice the formation of the Gatun Lake at Panama. Our great cities would be just visible to them as bright spots on the night side of the earth, and perhaps some of them

as smoky patches by day, and every active terrestrial volcano would look somewhat the same.

Though these evidences of human endeavor would be visible from the moon, it is very doubtful if they could be distinguished from the results of what we usually call natural processes, or would present any recognizable evidence of intelligent design or purpose; and it is practically certain that no definite evidence of the existence of mankind could be detected at all by observations from Venus with instruments such as ours.

MARS

386. This planet, like Mercury and Venus, is prehistoric as to its discovery. It is so conspicuous in color and brightness, and in the extent and apparent capriciousness of its movement among the stars, that it could not have escaped the notice of very early observers.

Its *mean distance* from the sun is a little more than one and a half times that of the earth (141,500,000 miles), and the *eccentricity* of its orbit is so considerable (0.093) that its radius vector varies more than 26,000,000 miles.

At opposition the planet's *average* distance from the earth is 48,600,000 miles. When opposition occurs near the planet's perihelion this distance may be reduced to 34,600,000 miles, while near aphelion it may be as great as 62,900,000 miles. At conjunction the average distance from the earth is 234,400,000 miles.

At an average conjunction Mars is of stellar magnitude + 1.6, — about half as bright again as the polestar; at an unfavorable opposition it is twelve times as bright and of magnitude − 1.1, — not as bright as Sirius; while at the most favorable opposition it is of magnitude − 2.8, — fifty-five times as bright as at the average conjunction and brighter than any other planet except Venus.

These favorable oppositions occur always in the latter part of August (when the earth passes the perihelion of the planet) and at intervals of 15 or 17 years. The last was in 1924.

The *inclination* of the orbit is small, — about 1° 51′.

The planet's *sidereal period* is 687 days, or one year and ten and one-half months; its *synodic period* is 780 days, or nearly

two years and two months. During 710 of the 780 days it moves eastward; and during 70 it retrogrades, on the average, through an arc of 16°.

387. Diameter, Surface, and Volume. The apparent diameter of the planet ranges from $3''.5$ at conjunction to $25''.1$ at the most favorable opposition. Its real diameter is 4215 miles, with a probable error not exceeding 10 miles. This makes its surface 0.283 and its volume 0.150 of the earth's.

388. Mass, Density, and Gravity. Observations of its satellites give its mass as 1/3,085,000 of the sun's or 0.1076 of the earth's. This makes its density 0.72 and its superficial gravity 0.38; that is, a body weighing 100 pounds on the earth would have a weight of 38 pounds on Mars. The velocity of escape from its surface is 5.04 kilometers per second.

389. Phases. When the planet is nearest the earth it is more favorably situated [1] for telescopic observation than any other heavenly body, the moon alone excepted. It then shows a ruddy disk which, with a power of 75, appears as large as the moon does to the naked eye. Since its orbit is outside the earth's, it never exhibits *crescent* phases like Mercury and Venus, but at quadrature it appears distinctly *gibbous*, about like the moon three days from the full. The greatest value which the phase angle (angle sun-planet-earth) can have is a little more than 47°.

390. Albedo. The albedo of the planet, computed from Müller's visual observations, is 0.154, — more than twice that of the moon or Mercury but still comparable with that of fairly dark-colored rocks. The photographic albedo, that is, the reflecting power for the violet light which acts on ordinary photographic plates, is 0.090 and is much less than the visual value, as might be anticipated from the red color of the planet.

With respect to the variations of brightness with phase, Mars behaves very much as Venus does, the light at phase angle 40° (after correction for the effects of varying distance) being 58 per cent of that at the full phase, as against 64 per cent for Venus and 41 per cent for the moon. It is therefore probable that the surface is comparatively smooth.

[1] Venus at times comes nearer, but when nearest is visible only by daylight and shows only a very thin crescent. Eros (§ 420) is too small to be counted here.

391. Rotation. The spots on the planet's disk enable us to determine its period of rotation with great precision. Its *sidereal day* is $24^h\,37^m\,22^s.58$, according to the last determination by Lowell. Observations made a few days or weeks apart give a sufficiently approximate value of the time of rotation to enable one to determine, without fear of error, the whole number of rotations between two observations separated by a much longer interval. Thus a very precise determination can be effected by comparing drawings of the planet made by Huygens and Hooke more than two hundred years ago with others made more recently. The figure given above for the rotation period is not uncertain by more than a few hundredths of a second.

The rotation of the planet gives rise to a regular variation in its brightness, amounting, according to the observations of Guthnick, 1914, to about 15 per cent of the whole. The variation arises from the varying presentation of the lighter and darker spots on its surface.

392. Presentation of Regions of the Surface. The rotation of Mars is so little slower than that of the earth that an observer who notices a given marking near the center of the planet's disk (say at midnight) will see it on the following night at the same hour in almost the same apparent position. This region of the planet's surface will therefore be observable for a number of nights in succession. On account of the greater length of the Martian day, however, the region observable at a fixed hour of the night slowly works backward around the planet, by about 9° per day, and the region first observed gradually passes out of sight, and returns to a new presentation after about 40 days. Midway in this interval the opposite side of the planet is observable, while the side of the planet first considered is visible to observers on the opposite side of the earth.

393. The Inclination of the Planet's Equator to the Plane of its Orbit. This is 25°10′, according to H. Struve's careful study of the satellite orbits. Lowell, from long series of observations of the polar caps, obtains 23° 30′. The discordance between the two determinations shows that systematic errors (§ 122) must affect one or both.

In any event the inclination is nearly the same as that of the earth's equator; so far, therefore, as depends upon that circumstance, the seasons on Mars should be substantially the same as our own.

The celestial pole for an observer on Mars would be in right ascension $22^h\ 10^m$, declination $+54°$, by our reckoning. It follows that when Mars is in perihelion it is a little before midsummer in his *southern* hemisphere, which therefore has a shorter and hotter summer than the northern (compare § 174). It follows, also, that the northern temperate and polar regions of the planet are very poorly, if at all, visible at the favorable oppositions, so that they are not as easy to study as the corresponding southern regions.

394. Polar Compression. There is a slight but sensible flattening of the planet at the poles. Struve's calculations from the perturbations of the orbits of the satellites (which afford the most precise determination), when reduced to the value of the planet's diameter here adopted, make the compression 1/192. Lowell's micrometric measures give 1/190. The larger values found by some older observers are certainly erroneous.

It may seem surprising that the oblateness of Mars is greater than the earth's, though its rotation is slower. The reason for this is that Mars is *less dense* than the earth and *more nearly homogeneous*. The centrifugal force at the equator of the planet is 1/219 of the force of gravity (as may easily be verified). The ratio of the oblateness to this fraction is 219/192, or 1.14, — larger than for any other planet and so near to the theoretical limit for a homogeneous planet (§ 341) that it is clear that the increase of density toward the center must be small.

395. Telescopic Appearance and Surface Markings. With even a small telescope the planet is a beautiful object, showing a surface diversified with markings, dark and light. These markings are, almost without exception, *permanent* features of the planet's surface, observable from year to year; but many of them are subject to *seasonal changes*, which make their study of unusual interest. Temporary markings, such as might be produced by clouds, though they have occasionally been observed, are rare (Fig. 143).

The planet's disk is, as a whole, *ruddy or orange-colored* and is usually brighter around the limb, though not at the terminator, if there is any considerable phase. About three eighths of the

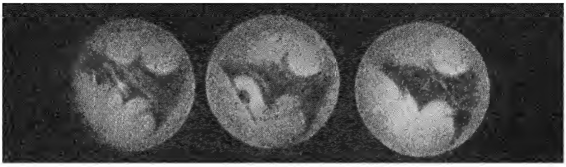


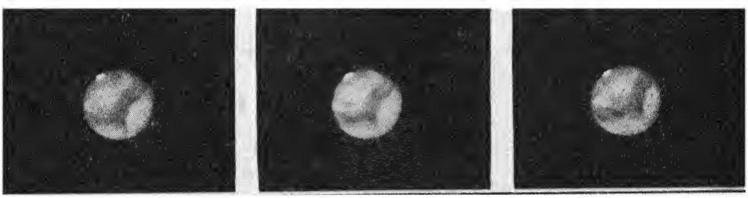
FIG. 139. Mars

From drawings by Barnard

surface, however, is covered by *darker regions* of a bluish-gray or greenish shade, and at the poles appear brilliant white patches, the *polar caps*, which are usually the most conspicuous features of all.

Fig. 140, from a photograph by Professor Barnard, shows these features admirably, and also illustrates the gradual passage of the markings across the disk as the planet rotates on its axis.

South



$16^h\ 46^m$ G.M T. $16^h\ 24^m$ $15^h\ 24^m$

September 28, 1909

FIG. 140. Mars

The rotation of the planet is evident from the comparison of the three photographs. The wedge-shaped extension (toward the north) of the large dark area is one of the most easily recognized features. It is known as *Syrtis Major*. The south pole is tilted toward the earth. It is summer in the southern hemisphere, and the polar cap has receded to a diameter of about 450 miles. (From photographs by E. E. Barnard, Yerkes Observatory)

The *reddish areas* show little or no change with the seasons, and are generally believed to represent the bare and, in itself, almost featureless surface of the planet, upon which the other markings are superposed.

396. The *polar caps* and other white spots, on the other hand, undergo very great and remarkably regular seasonal changes. At the Martian solstices the cap in the winter hemisphere is very large, often extending halfway from the pole to the equator. A little before the spring equinox of this hemisphere (at a season corresponding to the beginning of March in the earth's northern latitudes) the cap begins to shrink, steadily and almost uniformly, so that by the time of the summer solstice it is only five or six hundred miles across, and late in the summer is much smaller.

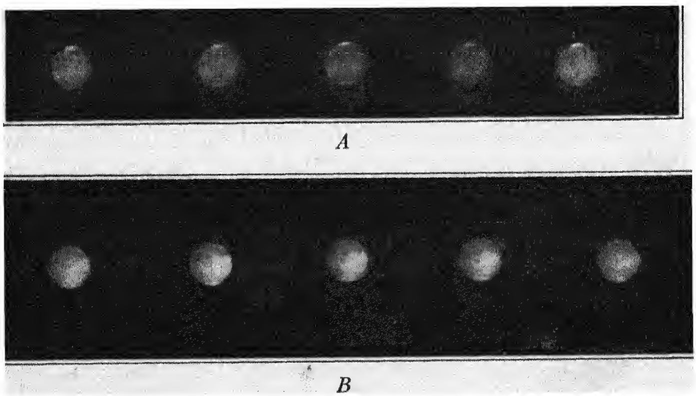


FIG. 141. Seasonal Changes on Mars

A, taken July 28, 1907, corresponding to the Martian date October 25, or shortly after the autumnal equinox, shows both caps. The southern cap is uppermost in the inverted image. Several exposures are habitually made on the same plate to insure obtaining at least one good image; that at the right shows the northern cap the best. In *B*, taken November 14, 1911, for which the Martian date corresponds to the latter part of February, the southern cap is too small to show (though a minute speck could be seen telescopically). The northern has become so large that it extends for fully 60° along the limb, though its center is still out of sight behind it. (From photograph at Lowell Observatory)

Shortly before the autumn equinox (at a date corresponding to about September 1 on the earth) the white material begins to form again, appearing, sometimes overnight, in large patches in various parts of the planet's arctic (or antarctic) zone, which soon coalesce and form a continuous sheet of almost the maximum midwinter dimensions, which remains roughly uniform in size until the following spring.

The south polar cap sometimes entirely disappears toward the end of summer (as in 1894 and 1911); the northern one never

quite vanishes, but shrinks to about 200 miles in diameter and remains of this size for some three months by our reckoning. On the other hand, the maximum size of the southern cap (100° of latitude, or 3700 miles, measured along the planet's surface) is considerably greater than that of the northern cap (85°, or 3100 miles); and the southern cap begins to shrink a little earlier in the spring, and to appear again a little later in the autumn, than the other. These differences in behavior are obviously related to the fact that summer is hotter, and winter colder, in the southern hemisphere than in the northern hemisphere.

397. Other White Regions. The behavior of the polar caps immediately suggests that they are composed of *snow* — or, if not of frozen water, at least of some substance that melts or evaporates as soon as the temperature rises to a certain value in the spring, and that is carried to the other pole in the form of vapor by the planet's atmosphere and is precipitated there as soon as the surface grows cold enough. This theory is further confirmed by the occasional observation of white areas near the sunrise limb, which disappear as the planet's rotation carries them nearer the center of the disk, leaving the ordinary surface visible, — thus behaving exactly like deposits of hoar-frost, formed during the night and disappearing as the sun gets high.

The topography of the planet's surface has a definite influence on some of these phenomena. As the polar caps shrink, isolated white patches, which may last for days or weeks, are left behind in the same places every year. The last remaining portion of the southern cap is not at the pole, but about 7°, or 250 miles, from it, always in the same longitude (though the northern cap is very nearly centered on the pole). The winter "snows" too come down farther over certain definite regions (always reddish areas).

These facts suggest areas of high land, on which the "snow" lies longer than elsewhere. If Mars were proportionately as rough as the moon, the individual mountains would be easily visible at the terminator under favorable circumstances. Lowell estimates that any abrupt elevation much exceeding 2500 feet in height could thus be detected, and none have been found. But there seems to be no reason at all why gradual slopes (like that of the great plains from the Missouri to the foot of the Rocky

Mountains) should not exist on the planet's surface, such that some regions stand many thousands of feet higher than others.

398. The Darker Areas. These lie for the most part in the southern hemisphere, and mainly in its tropical region, forming, in a small telescope, a sort of darkish belt around the planet. They were for a long time supposed to be sheets of water, and received corresponding names (Schiaparelli's designations, derived from classical sources, are now generally adopted). It is, however, now practically certain that they are not water. Among the proofs the following may be mentioned:

(1) The brilliant reflection of the sun, which would be produced by a water surface, has never been observed.

(2) The dark regions are not uniform in tone, but exhibit conspicuous detail within their area (as is illustrated in Fig. 140). This could hardly occur in oceans unless they were so shallow that the bottom showed through almost everywhere.

(3) The shade and depth of color of these areas, and in some cases even their size and shape, vary with the Martian seasons or change from one year to another. The principal markings are fairly permanent in form and position, but vary greatly in intensity, many of them being at times almost or even quite indistinguishable, and at other times dark and conspicuous. Speaking generally, they are most prominent in the spring of the hemisphere in which they lie, while the polar cap is shrinking, and gradually grow smaller or fade out in the autumn, some regions turning from greenish to yellow, while yellow "islands" come out in others. These seasonal effects are perceptible as far as the equator and even beyond it.

Changes in color have also been noticed. Thus, in 1903, and again in 1905, Lowell saw the Mare Erythræum change from blue-green to chocolate-brown shortly after the winter solstice, and return gradually to its former tone as spring approached.

Most of these changes repeat themselves with tolerable regularity in successive revolutions of the planet about the sun, but in some cases more permanent alterations in the outlines appear to have taken place.

399. Finer Details; the Canals. Besides the markings already described, the surface of Mars is rich in finer details, which are of

great interest but difficult to observe. Schiaparelli, in 1877 and 1879, announced the discovery of a great number of fine, dark, straight lines ("canals," as he called them) crossing the ruddy portions of the disk in all directions, and in 1881 he announced further that many of these became double at times, like the parallel tracks of a railway. W. H. Pickering, in 1892, added the detection of numerous small, dark spots connected with the canal system, and of darker markings within the dark areas, which Douglass, in 1894, described as canals similar to those in the reddish regions.

There is now no doubt regarding the real existence of these finer details, but the drawings and description of them by different observers are remarkably discordant.

(1) At one extreme stands Lowell, according to whom the canals, when well seen, are *very narrow* (from 15 to 20 miles wide at the most), *very dark*, *perfectly straight* (lying, with rare exceptions, along great circles on the planet's surface), and of *uniform width and intensity*, although under poor atmospheric conditions they may appear as hazy streaks. He found that they cover the planet's surface with a *complex network*, of *geometrical precision*, extending over both the ruddy and the darker regions, — four, six, or even as many as fourteen canals meeting exactly in one point, which is often marked by one of the dark spots, or *oases*, from 75 to 100 miles in diameter. More than 400 canals and nearly 200 oases have been observed and named at the Lowell Observatory. He found that some 50 of the canals are double, appearing as fine lines from 100 to 200 miles apart and equidistant throughout their whole length. At times only one of the components is strong enough to be visible. Fig. 142 represents typical drawings by Lowell.

(2) At the opposite extreme is Barnard, who, during years of observation with some of the greatest existing telescopes, never saw any trace of such a system of fine geometrical lines, although at times he saw "short, diffused, hazy lines, running between several of the small, very black spots which abound in this region" of the planet's surface, and "two long, hazy, parallel streamers." He said that, with the 60-inch reflector on Mt. Wilson, Mars gave "the impression of a globe whose entire

surface had been tinted a slight pink color, on which the dark details had been painted with a grayish-colored paint, supplied with a very poor brush, producing a shredded or streaky and wispy effect in the darker regions," and added that "no one could accurately delineate the remarkable complexity of detail of the features which were visible in moments of the greatest steadiness."

Antoniadi, with the 32-inch equatorial at Meudon, came to very similar conclusions, stating that, under the best seeing, the canals are neither linear nor uniform, and are sometimes resolvable into a complicated string of finer details.

(3) Other observers take various positions between these extremes. W. H. Pickering, while his drawings show canals in

January 11 South March 25

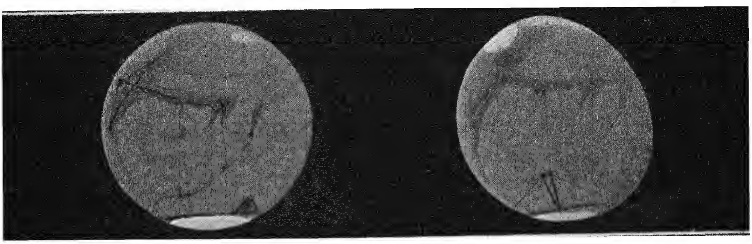


Fig. 142. Mars

Dark markings intensify in the north with advancing summer there and fade in the south with the coming of winter there. (Drawn by Lowell, 1914)

substantially the same position as those drawn independently by Lowell and others, defines a canal as "any long, dark, narrow marking that is straight, or of large radius of curvature, and of fairly uniform breadth and density," and states that "all the larger and more conspicuous canals are curved, though the fainter ones usually appear straight," and that the canals, under the best seeing, are dusky stripes, of different widths up to 150 miles or more, very few being fine lines. He believes the stronger canals to be very nearly continuous and uniform. He has never seen a canal certainly double. Desloges drew the canals as straight streaks, but found some very wide and, under the best seeing, resolvable into a mosaic of intricate details, and most of them diffuse, though a few in the best moments appeared

as fine, straight, sharp, dark lines. He never saw a canal double, though his assistant did, with the same telescope and on the same night, and a collaborator 500 miles away independently confirmed the assistant's observation.

All those observers who see the canals at all find them subject to great variations in visibility, which follow in a general way the similar changes in the dark areas. Lowell, from a study of several hundred of his drawings of the planets, reached very definite conclusions to the effect that the canals are faint or invisible during the Martian spring, and increase in prominence after the polar cap shrinks, — those nearest the pole darkening first, about the time of the summer solstice, while those in lower latitudes, with the oases connected with them, follow successively the "wave of quickening" which advances from latitude 70° to the equator in about 50 days, at the rate of some 50 miles per day, and continues into the opposite hemisphere for 1000 miles or more. Half a Martian year later, when this effect has very nearly faded out, a new "wave of quickening" starts from the opposite pole. Lowell also recorded the apparently capricious disappearance of certain canals for several years at a time, and the appearance of new canals where nothing had been seen for at least fifteen years and probably much longer.

400. All these accounts represent the mature judgment of trained and experienced observers after long and careful study of the planet under favorable conditions. To reconcile their extraordinary divergences is very difficult.

It is quite incredible that any one of them should never once have had the good fortune to see the planet under atmospheric conditions good enough to reveal its surface detail. Some observers maintain that in such work a relatively small aperture is actually better than a very large one; but, however this may be in ordinary conditions of seeing, it is very improbable that it is true of the best moments, and conclusions based on the assumed inferiority of powerful instruments for such study should be regarded with great caution.

The only possible explanation appears to lie in a complex *personal equation*, on the part of the various observers, in seeing and recording these elusive markings, which, as all observers

agree, are forever being concealed and revealed, even on the best nights, by the changing steadiness of the air. There is direct evidence, indeed, that different observers, working independently and by turns with the same telescope, may record in very different fashions what they see, as has happened, for example, in the case of Lowell and Pickering.

Between the formation of the image of a faint marking on the retina of the eye, and the conscious perception, in the mind, of a definite pattern which the hand proceeds to draw, there intervenes a process of extreme complexity, most of which is performed *subconsciously* and probably depends very largely upon the observer's previous experience and training. As Newcomb has pointed out, the telescopic image of a sharp marking cannot, for optical reasons, be as sharp as the marking itself, and the experienced observer learns to correct his judgment of this effect by a process of visual inference which sinks below the level of consciousness. It is quite possible that in some cases this process may be unconsciously overdone, just as an observer with the transit instrument, in trying to avoid the danger of pressing the chronograph key too late, may fall into a fixed habit of making his signal a little too early. One man's mental apparatus may therefore report a faint line as straight, continuous, and uniform, unless there are bends, gaps, or irregularities in it sufficiently prominent to be definitely seen, while another's may refuse to report a marking as straight and narrow unless it is undeniably so. The same principle evidently applies to the convergence of several lines to the same point, and to the whole question of the existence of geometrical figures on the planet's surface.

At the present time it is generally recognized that there exists an objective basis for the canals in the form of fine detail on the surface of Mars, and it is widely believed that these details have, in a general way, the streaky character of the canals; but the existence of a geometrical network is doubted or denied by a large majority of astronomers.

401. Photographs of Mars. Satisfactory photographs of Mars can be obtained only with great telescopes. The direct focal image of the planet, at the most favorable opposition, is not quite one tenth of an inch in diameter in the Yerkes refractor,

and with the Lowell telescope is less than half this. In order to secure a larger image it is necessary to interpose an enlarging lens in the path of the rays. This magnifies the image but makes it correspondingly fainter, so that an exposure of several seconds is necessary even with very large apertures.

Under the best conditions beautiful photographs have been made, notably at Lowell, Yerkes, and Mt. Wilson observatories.

With respect to changes in the larger features of the surface the photographs are likely to have the final word, but with regard to the vexed question of the character of the canals they can give no decisive evidence. That faint markings of some sort are there they prove beyond question, but, owing to the relatively coarse grain of the plate, the best definition obtainable photographically is far below that which can be obtained visually with the same telescope; and, what is still more important, the eye can take advantage of the almost momentary flashes of good seeing, while the photograph records the average conditions during the seconds of exposure. Even though many successive exposures are made on the same plate, and those in which the definition is sharpest are picked out for study, the image of a narrow dark line and that of a diffuse gray band 100 miles or more in width would be identical in appearance on the best of them.

The photographs reproduced in Fig. 143 were made with yellow light. With red light the polar caps are less brilliant and the dark areas and canals are darker, while with green or blue light the opposite is true. On the photographs taken in violet light by Wright (Fig. 144), no detail at all is visible except the polar cap, which is brighter and larger than it is on photographs taken in light of greater wave-length. Except with the shorter wave-lengths there is little or no photographic evidence of the fading out of the surface features in a general luminosity near the limb, which is often noticed visually.

402. Atmosphere. There can be no doubt that Mars possesses some atmosphere, although probably a good deal less than the earth. The principal evidences of this are:

(1) *The behavior of the polar caps*, which can be explained only by the precipitation of some substance which has previously been in a state of vapor.

(2) *The existence of a "twilight arc."* When Mars is distinctly gibbous, the distance from the terminator across to the opposite limb is found to be greater in proportion to the polar diameter

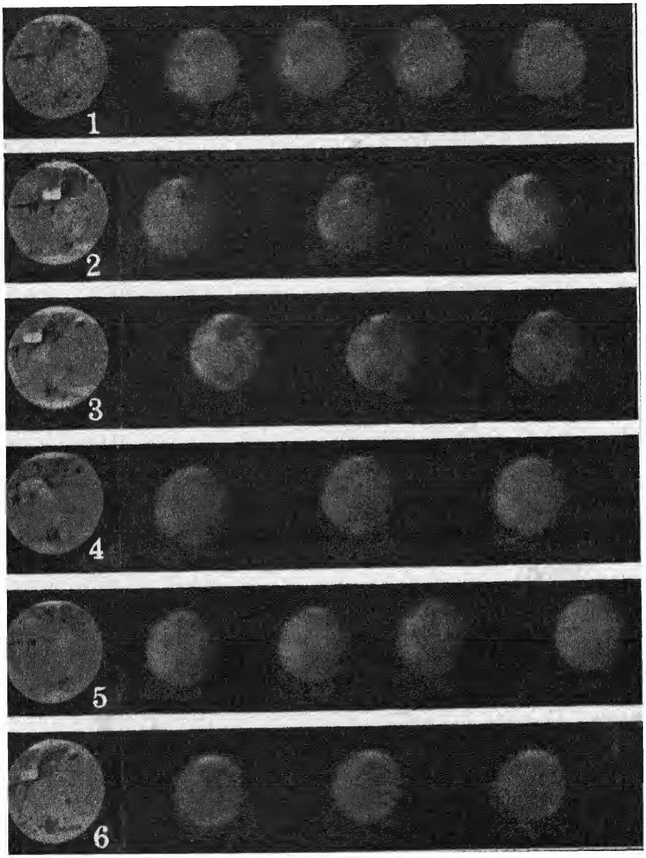


FIG. 143. Clouds on Mars

The successive rows show photographs of the planet made with a yellow color filter: 1, July 8 and June 6, 1922; 2, July 9; 3, later on July 9; 4, July 10; 5, July 12; 6, July 9 (with blue filter). A conspicuous bright marking appeared on July 9, and disappeared within a day or two. It is invisible on the photographs taken with blue light, showing that its color was yellow. The drawings shown at the left were made independently from visual observations. (From photographs by E. C. Slipher, Lowell Observatory)

than the theoretical value, showing that the sunlit portion of the planet encroaches about 8° on the dark hemisphere. It follows that the height of that portion of the atmosphere which

reflects light strongly enough to be visible is about 1/100 of the planet's radius, or 20 miles, as against 40 miles found in similar fashion in the case of the earth (§ 119). It is probable, however, that in the latter case the faint extension of the twilight can be followed a good deal farther than it can on Mars, and that the heights at which the atmospheres of the two planets have comparable densities are much more nearly alike.

Owing to the smaller force of gravity on Mars the density in the atmosphere would increase downward much less rapidly than on the earth; and if the densities were comparable in the two cases, at a height of, say, 20 miles, the Martian atmosphere would contain less material, exert a much lower pressure at the planet's surface, and yet rise higher in its rarefied upper layers, than the terrestrial atmosphere.

(3) *The partial obscuration of the surface markings by a general brightness at the limb.* This is exactly what may be expected if, near the limb, we are looking obliquely through a greater thickness of atmosphere, which reflects some light on its own account. The fact that this limb-light is shown on photographs only when the shorter wave-lengths are used indicates that it is blue scattered light like that of our sky, and also makes it probable that a part of the effect observed visually is a contrast effect arising from the proximity of the dark sky. The earth, similarly viewed, would undoubtedly show a much stronger limb-light.

(4) *The occasional appearance of clouds, fogs, or haze.* Though clouds are very rare on Mars in comparison with the earth, they have been observed many times. The edges of the polar caps, especially of the northern one when it is shrinking, have frequently appeared to be shrouded by a whitish veil, less brilliant than the cap itself. Several observers in recent years have reported rapid changes, almost from night to night, in the visibility of details over large areas of the surface of the planet, which seem explicable only by the formation or clearing of fog or haze. Most conclusive of all are the projections occasionally seen on the terminator, which are evidently the tops of high-lying clouds, catching the sun where all below is in shadow, — not mountains, for they last only a day or two and change their position in that short time, as measurements show.

The altitude of their summits can be calculated in the same fashion as that of a lunar mountain (§ 208), and in some cases has been found to be as much as from 15 to 20 miles.

(5) Perhaps the most striking evidence of the presence of an atmosphere is seen in the photographs by Wright in 1924 (Fig. 144). Photographs were taken through two color screens,

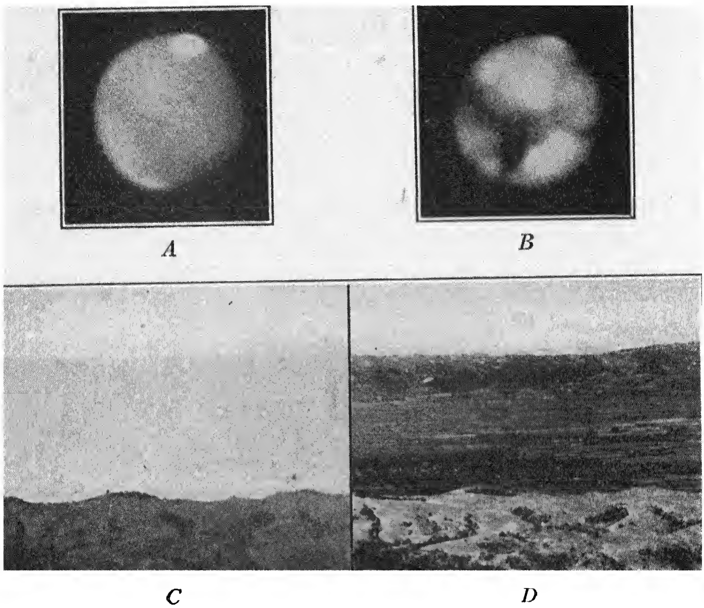


FIG. 144. Mars (September 11) and San José as photographed from Mt. Hamilton *A* and *C*, with violet light; *B* and *D*, with infra-red light. The obliteration in *C* is due to the earth's atmosphere, and the comparison is suggestive of the presence of an atmosphere of considerable density on Mars. San José is distant $13\frac{1}{2}$ miles from Mt. Hamilton. G. M. T. of Mars observations: *A*, $18^h\ 50^m$; *B*, $18^h\ 30^m$

the one allowing the passage of deep red light; the other, violet. In the former there is a wealth of detail; in the latter the planetary disk is of uniform intensity except for the very brilliant polar cap. Comparison of these photographs with those of a mountain range a dozen miles away, through thin summer haze which blots out the distant view completely in the violet photograph and disappears in the red one, indicates very strongly that on Mars also the red light alone has haze-piercing powers.

403. Composition of this Atmosphere. The velocity of escape from the surface of Mars is 5.05 kilometers per second. It follows (§ 201) that the planet could retain an atmosphere of oxygen, nitrogen, and heavier gases, and probably water vapor as well, but not hydrogen or helium. The spectroscopic method (§ 619) is capable of detecting rather small quantities of oxygen and water vapor. All observers agree that the amounts of the latter in the Martian atmosphere must be smaller than in the earth's. The latest investigation (Adams and St. John, 1925) indicates that both are present and that "for equal areas the water vapor above the surface of Mars at the time of observation was of the order of 5 per cent, and the oxygen of the order of 15 per cent, of that normally in the earth's atmosphere."

How much nitrogen or carbon dioxide there may be can only be guessed, as they cannot be detected spectroscopically. The assumption that the amount of atmosphere above a square mile of the planet's surface is somewhere between one tenth and one half as much as on the earth appears to be consistent with the existing data. This would make the atmospheric pressure at the surface roughly between 4 per cent and 20 per cent of that at the earth's surface.

404. Temperature. The vexed question of the temperature of the planet's surface has apparently been conclusively settled by the radiometric observations (§ 618) of the planet made at the Lowell and Mt. Wilson observatories. The observers at both places, though using somewhat different methods, agree that the temperature of the planet's surface rises well above freezing in the equatorial regions at noon, and may reach 10° C. (50° F.) or even a little more. The dark areas are somewhat hotter than the reddish ones. Even at the equator the temperature is well below freezing at sunrise and sunset, and the nights must be very cold. The temperature of the polar cap appears to be as low as − 70° C., but after the southern cap has disappeared in late summer the surface becomes about as warm as at the equator.

405. Nature of the Polar Caps. These investigations appear to answer the long-discussed question of the nature of the polar caps and to make it very probable, to say the least, that they are actually composed of snow, — frozen water. The only

outstanding difficulty is the very low observed temperature above the unmelted polar cap; but if the cap is covered with clouds, like cirrus clouds on the earth, this may be the temperature of their cold upper surface, and the temperature may be much higher below.

Whether this snow actually melts or whether it evaporates into a very dry atmosphere at a temperature below the freezing point, as sometimes happens to a thin snowfall on the Western plains, is quite another story and depends both on the pressure and on the humidity of the atmosphere, which cannot at present be safely estimated. The boiling point of water would be 60° C. at an atmospheric pressure 20 per cent of ours, and 30° C. if the pressure were reduced to 4 per cent. Apparently it is highly probable, therefore, that liquid water can exist on the surface of Mars.

It has been suggested that the caps might be composed of solid carbon dioxide, but this volatilizes, at low pressures, at temperatures much lower than those which the observations indicate for the polar caps.

It has often been pointed out that the deposit forming the polar caps must be very thin, since, unlike the earth's polar snows, it disappears almost completely in summer. A simple calculation shows that the whole amount of heat received at the planet's pole during the time the sun is above the horizon would suffice, if none were lost by reflection or radiation, to convert into vapor a layer of ice only six feet thick.

Since most of the heat received must be lost again into space, the actual thickness of the polar caps must be much less than this, — probably averaging only a few inches, except perhaps in the permanent portion at the north pole. The whole quantity of water formed by their melting would not be enough to fill Lake Erie. Desert conditions must therefore prevail over a great portion of the planet's surface.

406. Explanation of Other Surface Features. Evidently the seasonal changes of the dark areas and markings are intimately related to those of the polar caps, the darkening of the former being apparently a result of the diffusion over the surface, in liquid or gaseous form, of something derived from the latter. It is now very probable that this substance is *water*.

The dark regions may be regarded as vegetation, and the rest of the surface as desert. The enlargement and darkening of the areas as the moisture from the melting polar cap reaches them (whether in the form of streams, rain, or dew), and the changes in their color, toward green at this time and brown or gray in the dry season, are just what might be expected on this view; and minor differences in the course of these changes from year to year are not surprising.

An alternative theory suggested by Arrhenius supposes that the soil of the dark areas is saturated with soluble salts, which absorb moisture from the air, when this is available, and deliquesce, forming a darkish mud, but, when the atmosphere becomes very dry, effloresce, leaving a dry and light-colored surface. Such alkali flats are known in terrestrial deserts.

The presence of oxygen in the planet's atmosphere is strong evidence of the actual existence of vegetation on its surface, in the past at least (compare § 377). The attribution of the present seasonal changes to vegetation appears, therefore, to be decidedly the most reasonable hypothesis.

407. Concerning the nature of the canals, opinions differ even more widely. If the appearance of linear markings arises from the integration by the eye of details too delicate to be seen well (if at all) singly, no further explanation is required.

Arrhenius regards the canals as *cracks* or *fault-lines* in the planet's crust, along which the surface has been stained by escaping vapors, or as *rift-valleys* bounded by such faults, like the valley of the Dead Sea.

The supporters of the vegetation theory (the majority of the observers of Mars) generally follow W. H. Pickering's suggestion and regard the canals as *strips of vegetation bordering watercourses* crossing the arid regions, just as the valley of the Nile would appear to an observer on the moon like a green streak across the yellow African desert. As in this terrestrial example, the watered area may be very much wider than the watercourse, which by itself would be too narrow to be seen.

Seasonal changes in the visibility of the canals are then readily explicable; in fact, the Nile Valley would show just such changes after the annual flood.

408. Is there Life on Mars? The observations of the last few years make it very probable that conditions on Mars are such that vegetable life of the sort that exists on the earth might exist there, and at the present time (1926) it seems more likely than not that it does. These conditions (with the temperature falling so low that everything must freeze up hard every night) would be very unfavorable to most of the forms of life that have developed in the more genial climate of our planet; but there exist on the earth some forms that could very nearly endure such conditions, and there appears to be no real difficulty on this score. It must be remembered, however, that the recent evidence (especially the radiometric measurements of the temperature of Mars) has greatly changed our estimate of the situation, and further evidence may change it again.

That animal life should also exist is not impossible, or, indeed, even improbable. Whether we could detect any evidence of it by telescopic observation at the earth's great distance is quite another question. The late Dr. Percival Lowell maintained that such evidence exists. With the vegetation theory of the dark areas and canals as a starting-point, his arguments may be summarized as follows:

(1) The canals are so straight, so narrow, so uniform in width and in separation (if double), and are connected with one another and with the oases in so definitely geometrical an arrangement, that it is incredible that such a system could have arisen by the chance operation of natural causes. They must, therefore, be the products of intelligent design, whose execution involved engineering skill of a very high order.

(2) The progressive darkening of the canals and oases after the melting of the polar cap shows that they are irrigating channels, along which the water is conducted uniformly in all directions, at the rate of about fifty miles a day, from the pole to the equator and beyond. Water might flow naturally down some of these channels, but hardly down them all; much less could it flow in opposite directions, according to season, along the same canals in the equatorial region. Hence this water must be propelled artificially, and the intelligent beings who constructed the canal system are still operating it.

(3) Though it is obviously useless to speculate concerning the physical organization or appearance of these inhabitants, it may be inferred, from the fact that the canals form a single system extending from pole to pole over the planet's whole surface, that they have established a world civilization embracing their race as a whole.

This theory has naturally aroused great popular interest, but it rests largely on conclusions regarding the surface details which are not generally accepted, and is far from being a necessary consequence of these.

Everything depends upon the geometrical character of the network of canals, and, as has already been pointed out, this may be the product of personal equation in the subconscious operation of the observer's mind. Again, the changes in the visibility of the canals may be explained in quite other ways; for example, as Lau has suggested, on the assumption that the planet's atmosphere becomes hazy in spring, concealing the finer details, and clears gradually in summer, beginning near the pole, so that the canals which have all already darkened under the haze come out successively in lower latitudes.

It is therefore necessary to render a verdict of "not proven" with regard to this theory. It should nevertheless be remembered that the development of a reasoned argument, showing how the existence of intelligent inhabitants on a planet fifty millions of miles away could be detected by such observations as can be made with existing telescopes, is in any event an admirable example of constructive scientific imagination.

409. Satellites. The planet has two satellites, discovered by Hall at Washington in 1877. They are extremely small and can be seen only with very large telescopes and when Mars is near opposition. The outer one, Deimos, is at a distance of 14,600 miles from the planet's center and has a period of $30^h\ 18^m$, while the inner one, Phobos, is at a distance of only 5826 miles and has a period of $7^h\ 39^m$, less than one third of the planet's day. (This is the only known case of a satellite with a period shorter than that of the rotation of its primary.) Phobos, therefore, *rises in the west*, as seen from the planet's surface, and *sets in the east*, completing its strange backward diurnal revolution

in about 11 hours. Deimos, on the other hand, rises in the east but takes nearly 132 hours in its diurnal circuit, which is more than four of its months. Their orbits are nearly, but not quite, circular, and are inclined to the plane of the planet's equator by $1\frac{3}{4}^\circ$ and 1° respectively. The attraction of the equatorial bulge of the planet causes their apsides to advance and

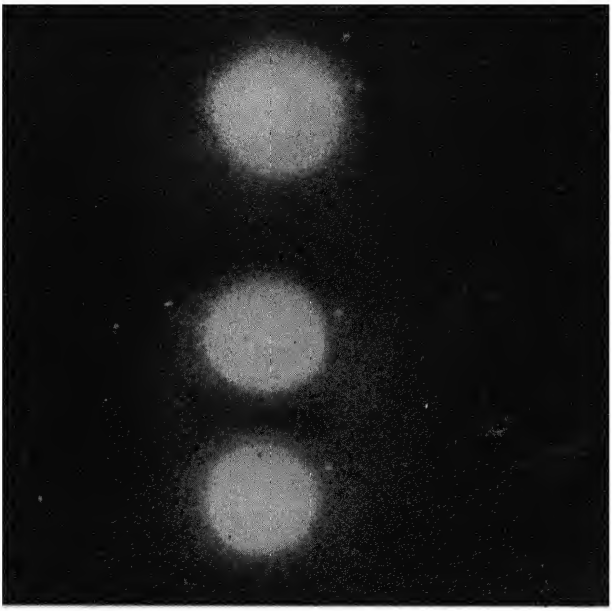


FIG. 145. Satellites of Mars

Photographed with the 24-inch refractor, August 21, 1924. Phobos is close to the planet and Deimos is faintly shown about $\frac{3}{4}$ inch farther to the right. Graduated exposures show Phobos to be about one and one-fourth magnitudes brighter than Deimos. (From photograph by E. C. Slipher, Lowell Observatory)

their nodes to retrograde (at the same speed), completing a revolution in 56 years for Deimos and $2\frac{1}{4}$ for Phobos (§ 342).

Their diameters are too small to be measured with any micrometer, but may be estimated from their apparent brightness as seen from the earth, on the assumption that the surfaces have the same reflecting power as that of the planet. The results of different observers are very discordant, as is natural for such faint objects, but they indicate that Phobos, which is the brighter of the two, is

about 10 miles in diameter, and Deimos about 5 miles. On these assumptions Phobos, seen in the zenith from a point on the planet's surface directly beneath it, would appear about 1/3 the diameter of the full moon and 1/25 as bright, while Deimos would be 1/40 as bright as Phobos and only 80″ in diameter, and would appear to eyes like ours as a brilliant planet, like Venus.

According to Lowell neither satellite shares the red color of the planet.

THE ASTEROIDS

410. The asteroids, or minor planets, are a host of small bodies circulating around the sun between the orbits of Mars and Jupiter. The name "asteroid" (starlike) was suggested by Sir William Herschel early in the nineteenth century, as indicating that, though really planets, they appear like stars.

Kepler had noticed the wide gap between Mars and Jupiter and had tried to account for it, though unsuccessfully, and when Bode's law (§ 269) was stated, in 1772, the impression became very strong that there must be a missing planet in the vacant space, — an impression greatly strengthened by the discovery of Uranus in 1781, at a distance almost precisely corresponding to that law. The first discovery was made by the Sicilian astronomer Piazzi, who was then engaged in forming his extensive catalogue of stars.

On the first night of the nineteenth century (January 1, 1801) he observed a small star where there had been no star a few days earlier; the next day it had obviously moved, and it continued to move. He named the new planet *Ceres*, after the tutelary divinity of the island, and observed it carefully for several weeks, until he was taken ill; but before he recovered, the planet was lost in the evening twilight. It was rediscovered at the close of the same year by means of the calculations of Gauss, who invented, for the purpose, the method of determining a planetary orbit from three observations (§ 319).

In 1802 Pallas was discovered by Olbers while he was searching for Ceres. Juno was found by Harding in 1804, and in 1807 Olbers discovered Vesta, the only asteroid ever visible to the naked eye. The hunt for others was kept up for several years

longer without success, because those engaged in it did not look for objects sufficiently small.

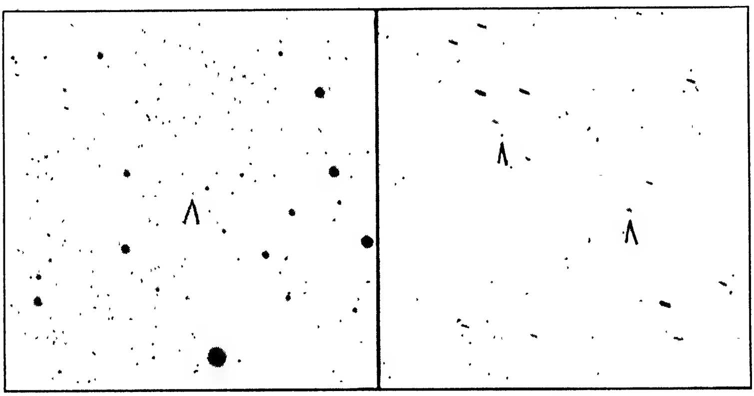
The fifth asteroid, *Astræa*, was discovered in 1845 by Hencke, an amateur, who had resumed the search afresh by studying the smaller stars and after fifteen years of fruitless labor was rewarded by the new discovery. In 1847 three more were found, and not a year has passed since then without the discovery of from 1 to 100. The number of those which have been sufficiently well observed to permit the calculation of reliable orbits passed 1000 in 1924. More than 500 others have been discovered but lost again because not enough observations were obtained to determine their orbits. Most of the brighter ones have evidently been already picked up, and nearly all the more recent discoveries are of very faint objects, appearing like stars of the eleventh to the fourteenth magnitude, which require a large telescope to make them even visible; but these are still being found in large numbers. There are at least 1500, and very likely more, which are bright enough to be observed with present-day instruments.

411. Method of Search. Formerly the asteroid hunter conducted his operations by making special telescopic star charts of regions near the ecliptic and from time to time comparing the chart with the heavens. If an interloper appeared on the chart, a few hours' watching would show whether it moved or not, that is, whether it was a planet or merely a variable star. The work, especially that of chart-making, was very laborious.

In 1891 a new method was introduced by Max Wolf, of Heidelberg. A camera with a wide-angle lens of several inches aperture is mounted equatorially and moved by clockwork; with this are made photographs of portions of the sky from 5° to 10° in diameter. If the telescope is carefully guided during the exposure, the *stars* show on the negative as small black *dots*; but a *planet*, if present, will move among the stars during the two or three hours of exposure, and its image will be a *streak* (Fig. 146*A*) instead of a dot, and so recognizable at once. It often happens that several asteroids are found on one negative.

An effective method for the photography of the fainter asteroids was practiced by the Reverend J. H. Metcalf, an American

amateur. While the telescope follows the stars by its driving-clock the plate-holder is given a slow motion, the same as that of an average asteroid in the region under observation. The image of an asteroid remains nearly stationary on the plate during the entire exposure, and is many times as intense as if it had been allowed to trail. The subsequent search is now for small black dots among a multitude of star-trails.



A B

FIG. 146. The Asteroid (508) Princetonia

The photograph at the left was exposed for 182 minutes while the guiding star was kept carefully bisected by the cross-wires. The images of the stars are round, whereas that of the asteroid is a trail. (The cut was retouched to strengthen this; the trail was well shown on the original photograph.) Two asteroids appear on the photograph at the right. They are indicated by ink marks — as is customary. The plate was moved in the direction and with the speed calculated for *1911 NA*, which appears as a dot. The image of *Princetonia*, which had a different motion, is a short trail, and those of the stars are still longer trails. *Princetonia* was discovered and named by R. S. Dugan. (From photographs by Max Wolf, Königstuhl-Heidelberg)

When first announced, each asteroid is designated provisionally by the year followed by two letters, as 1924 RJ. The list of discoveries from 1893 to 1924 numbered 1726. These discoveries do not, however, refer to as many different objects. A number of the older asteroids have not been observed since the year of discovery, and are adrift and practically lost. Now and then they are picked up as new; thus, (132) Æthra was rediscovered in 1922 after having been lost since 1873. It is sometimes found, on calculating backward, that an asteroid whose orbit has

just been computed has been discovered once, or even twice, before, but that the earlier discoverers did not make observations that were sufficient to get an orbit.

When a reliable orbit has been computed, and it is certain that the planet is new, the director of the Recheninstitut at Berlin (the international center for this subject) assigns it a permanent number, and the discoverer gives it a name. The earlier asteroids received mythological appellations, but at present the legendary lore of all lands is very nearly exhausted, and planets have been named not only for cities, colleges, and friends of their discoverers but even for ocean steamers, pet dogs, and favorite desserts! All the names are given the feminine form, except those of a few outlying but important objects at the extreme outer and inner limits of the group.

A complete list of the asteroids, with elements of their orbits, and with short ephemerides giving their positions in the heavens near the time of opposition, is published annually by the Recheninstitut. They are usually referred to by both number and name, as illustrated below.

412. Mean Distances and Periods. The mean distances of the different asteroids from the sun differ widely, and the periods correspond. The nearest to the sun, of those so far known, are (433) Eros and (434) Hungaria, whose mean distances are respectively 1.46 and 1.94 astronomical units, and whose periods are 1.76 and 2.71 years. The most remote is (944) Hidalgo, with a mean distance of 5.71 and a period of 13.7 years. Seven eighths of them, however, have mean distances lying between 2.3 and 3.3 astronomical units, and, therefore, periods between $3\frac{1}{2}$ and 6 years.

The average of the mean distances of the asteroids bearing the numbers 1–807 is 2.805, almost exactly that indicated by Bode's law. For the larger ones, which are bright enough to stand a good chance of being observed even if near the outer limits of the group, the average is 2.91, and this is probably nearer the true mean.

The mean distances are not distributed at all uniformly through their range. There are several marked gaps, doubtless due to the action of Jupiter, since they come just where the period of the asteroid would be exactly commensurable with that of the great planet, that is, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{5}$, etc. of Jupiter's period.

Curiously enough, the few asteroids whose periods are more than seven years seem to follow the opposite rule, and to cluster around the distances corresponding to periods $\frac{2}{3}$ and $\frac{3}{4}$ of Jupiter's or to Jupiter's period itself.

413. Other Orbital Elements. The inclinations average about 9° 30′, and the eccentricities about 0.15, — much greater than for the principal planets. The orbit of (944) Hidalgo is inclined 43° to the ecliptic; and that of (2) Pallas, nearly 35°. Of the first thousand, 3 others have inclinations exceeding 30°, and there are 17 more with inclinations between 25° and 30°. The eccentricity is also very large in some cases, — 0.53 for (887) Alinda, 0.54 for (719) Albert, and 0.65 for (944) Hidalgo (greater than for some comets), while it exceeds 0.30 in 25 other cases. There is a definite tendency for large eccentricity and large inclination to go together. The longitudes of perihelion again show the influence of Jupiter, nearly twice as many falling within 90° of Jupiter's perihelion as in the opposite half of the circle.

The orbits so cross and interlink that if they were material hoops or rings, the lifting up of one would take all the others with it, and those of Mars and Jupiter as well.

414. Diameter, Albedo, etc. These bodies are so small that micrometrical measurements, even of the largest ones, are very difficult, and of the smaller ones impossible. Barnard, with the Lick and Yerkes telescopes, has obtained measures of the disks of the four brightest and presumably largest, with the following results: Ceres, 480 miles; Pallas, 304 miles; Juno, 120 miles; Vesta, 240 miles. It is rather surprising that Vesta, which, if placed at the same distance, would look fully twice as bright as Pallas, and 20 per cent brighter than Ceres, should be so much smaller, but measures taken by Hamy, by an altogether different method (§ 824), confirm Barnard's value.

From these diameters, and from the observed brightness of the asteroids, it appears that Ceres, at the full phase, reflects nearly the same proportion of the incident light as does the moon; Pallas, about as much as Mars; and Vesta, almost as much as Venus.

The brightness of all four, and of all the other asteroids which have been photometrically observed (twenty or more in number),

falls off very rapidly as the phase increases, so that it is probable that their surfaces are even rougher than the moon's. When this is taken into account (§ 204), the albedo of Ceres comes out 0.06; of Pallas, 0.07; of Juno, 0.12; and of Vesta, 0.26.

The diameters of the fainter asteroids may only be estimated from their observed brightness. On the assumption that their albedo is equal to the average of that of the four brightest asteroids, it appears that there are probably a dozen others of diameters between 150 and 100 miles and perhaps one hundred and fifty more exceeding 50 miles, while the majority range from 50 to 10 miles in diameter and a few are even smaller, — mere "mountains broken loose."

These estimates are necessarily somewhat uncertain, and, as the measures of the brighter asteroids show, may be in error by one third of their amount in some cases; but they must at least give the true order of magnitude of these little bodies.

415. Mass, Density, etc. It seems probable that the density of the asteroids does not differ much from that of the crust of the earth or the mean density of the moon. If this is so, the mass of Ceres is about 1/8000 that of the earth. On such a planet the force of superficial gravity would be about 1/30 of gravity on the earth, and a body projected from the surface with a velocity of 1700 feet per second (less than that of an ordinary rifle-ball) would fly off into space and never return to the planet. On the smallest asteroids, of diameter less than ten miles, it would be quite possible to throw a stone from the hand with velocity enough to send it off into space. It is obvious, therefore, that even the largest asteroids cannot possess any atmosphere.

416. Aggregate Mass. On the basis just described the aggregate mass of the thousand asteroids under discussion may be estimated as about 1/3000 that of the earth, Ceres and Pallas accounting for about half of the whole. From the distribution of brightness, and therefore presumably of size, among those in the inner part of the group, it appears that the undiscovered smaller ones, although exceedingly numerous, are probably so small that their combined mass is considerably less than that of those already known. In the outer part of the zone, corresponding to periods longer than six years, only the larger objects have

so far been observable, and estimates are more uncertain; but from existing evidence it seems that 1/1000 of the earth's mass is a reasonable estimate, and 1/500 a liberal estimate, for the mass of all the asteroids, known and unknown.

Even a much larger amount of matter circulating between the orbits of Jupiter and Mars would be too small to produce perceptible perturbations of the orbit of Mars.

417. Form, Rotation, etc. In the case of such small bodies the gravitational forces, which compel a large planet to be nearly spherical in form, would be relatively inefficient, and it is not impossible that some of them may be of irregular shape.

A number of them show periodic fluctuations in brightness, such as might be caused by the rotation of such a body or of a spherical body covered with large bright and dark spots. Among these are (7) Iris, with a period of $6^h\,12^m$; (15) Eunomia, $3^h\,2^m$; (116) Sirona, $9^h\,40^m$; (345) Tercidina, $8^h\,47^m$; and (433) Eros, $5^h\,16^m$. It is natural to suppose that these are the rotation periods of the bodies.

This has been proved for Eunomia by Pickering and Wendell, who found that the apparent period of variation was longer when the planet was advancing than when it was retrograding. This is exactly what should happen if the rotation of the planet is direct; for, as can be seen from the general equations of synodic motion (§ 268), the apparent period should then be longer than the true period of rotation in the first case, and shorter in the second. They showed also that the variation was due to differences in albedo of opposite sides of the planet, and not mainly to irregularities of shape. In the latter case there would be two maxima of brightness in each rotation; in the former, but one. The observed change in period agreed with the former hypothesis.

418. The number of these bodies already known is so great, and the prospect of increase so indefinite, that it is a serious problem to take care of them.

Photographic methods now make it easy enough to observe all but the faintest asteroids; but to follow the motion of one of these little rocks by calculation is more troublesome, on account of the great perturbations produced by Jupiter, than to do the same for one of the great planets. Indeed, an exact solution of

the problem would be, in many cases (according to Professor Brown), much more difficult and laborious than the lunar theory.

Methods by which the perturbations of the majority of these bodies may be approximately calculated, without prohibitive labor, have been developed, however, and "mean elements" and tables have been published for about ninety, by means of which the planets' positions can be predicted at any time within a minute or two of arc. Many more will doubtless be added to this list in the near future, but certain cases present difficulties, notably those in which the period is nearly one half or one third that of Jupiter. With proper coördination of the work of observation and computation there is, however, no present danger that planets for which reliable orbits have once been obtained will have to be turned adrift again for want of attention.

419. Remarkable Asteroids. *The Trojan group.* The asteroids (588) Achilles, (617) Patroclus, (624) Hector, (659) Nestor, (884) Priamus, and (911) Agamemnon are noteworthy because their mean distances and periods are very nearly identical with Jupiter's. They are of great theoretical interest, since it is probable that they represent actual examples of the Lagrangian solutions of the problem of three bodies (§ 329), and keep always at approximately the same distance from Jupiter, as well as from the sun, circulating about one or another of the points in the plane of Jupiter's orbit which form, with Jupiter and the sun, the vertices of equilateral triangles. All six are at present within 20° of these points, Patroclus and Priamus following Jupiter, and the other four preceding it.

They are very faint objects, of the twelfth to the fourteenth magnitude, observable only with powerful instruments; but this is because they are so far off. They average perhaps 80 miles in diameter, and there may be many more of the group undiscovered.

Exceptional orbits. A few asteroids have orbits of extraordinary eccentricity, — even greater than that of some comets. The most remarkable is (944) Hidalgo, which has a major axis of 5.71 astronomical units and a period of 13.7 years, — much the longest so far known. The eccentricity is 0.65, so that the perihelion distance is 2.0 and the aphelion distance 9.4, — as

great as the mean distance of Saturn. The orbit plane is highly inclined (43°.06), and the planet can never come anywhere near Saturn. But at its descending node its orbit comes within 26,000,000 miles of Jupiter. This suggests that it may have been thrown into its present extraordinary orbit by perturbations at a close approach to Jupiter (compare § 322). Its brightness corresponds to a diameter between 15 and 30 miles. At a perihelion opposition it is of about the eleventh magnitude; when in aphelion at opposition it is fainter than the nineteenth, so that it can only be observed at a favorable opposition.

The asteroid (719) Albert has a mean distance of 2.58 and an eccentricity of 0.54. At perihelion it can come within about 18,000,000 miles of the earth. Even at this distance it is only of about the twelfth magnitude. In aphelion it is of the twentieth magnitude and probably not observable with any existing telescope. It is probably only 2 or 3 miles in diameter.

420. Eros. This little planet, insignificant in size but of great astronomical interest, deserves special attention. It was discovered in August, 1898, photographically, by Witt, of Berlin, and at once attracted attention on account of its short period. Thanks to the numerous observations which have since been made, its elements are now known with very great precision. Its sidereal period is 643.23 days, or very nearly $1\frac{3}{4}$ years, and its synodic period 845 days, — the longest known.

Its mean distance from the sun is 1.458 times the earth's, or 135,430,000 miles; but the eccentricity of its orbit, 0.223, is so considerable that at aphelion it is 165,630,000 miles from the sun, well outside the orbit of Mars and within the asteroid region, while at perihelion its distance is 105,230,000 miles. The inclination of the orbit is 10° 49′, the perihelion within little more than 2° of the descending node and only 21° from the earth's perihelion, so that the least possible distance between the two planets is only *13,840,000 miles*. This is only a little more than half the least distance of Venus, and at such oppositions (which always happen about January 22) the parallax of Eros is nearly 60″.

Observations made on the planet at such time of close approach determine its parallax, and hence with the aid of our knowledge

of its orbit (§ 287) the parallax and distance of the sun, with a far smaller proportional error than any other direct method.

Unfortunately these favorable oppositions are very rare; and, by worse luck, one happened in 1894, just before the discovery of the planet. The nearest approach since then occurred in 1901, when the minimum distance was a little less than 30,000,000 miles. Thousands of observations, visual and photographic, were made at that time at many observatories, and their discussion has led to one of the most accurate values of the solar parallax ever determined (§ 218).

There will be a better chance in 1931, when the least distance will be about 16,200,000 miles (on January 30). The planet will be favorably placed and is likely to be extensively observed.

The theory of the planet's motion is also noteworthy, on account of the large perturbations produced by the attraction of the earth. These permit a very accurate determination of the ratio of the earth's mass to the sun's, and hence of the solar parallax. This method, even at present, surpasses all others in accuracy.

Eros is probably about 15 miles in diameter, as estimated from its apparent brightness. It appears ordinarily like a star of the eleventh or twelfth magnitude, and is visible in a small telescope only if one knows just where to look for it. At a perihelion opposition it would be almost visible to the naked eye (of magnitude 7.2) and brighter than any other asteroid ever becomes, except four or five of those first observed.

Eros is also one of the asteroids that show periodic variations in brightness which are very probably due to rotation. The period is $5^h\ 16^m$, and there are two maxima and two minima of light in this interval. In February, 1901, the greatest light was three times the least, but within the next three months this range diminished to almost nothing. The variation was large again in 1903 and almost entirely absent in 1907. These remarkable changes can be partly explained by a high inclination of the planet's equator; sometimes we view the body almost from the direction of the pole, where, of course, the rotation should lead to no change in brightness. The details, however, remain to be worked out.

421. Origin of the Asteroids. Two alternative but not mutually exclusive hypotheses have been advanced for the origin of this remarkable swarm of tiny planets. One is that they represent a planet spoiled in the making, that is, a mass of material which never coalesced to form a single body. The other and older theory is that they represent fragments of a planet which, for some reason, has exploded. No single explosion could account for the present tangle of orbits; for some of them are so much larger than others that, turn them about as one might, they could not be made to intersect. But this hypothesis has nevertheless been greatly strengthened by a remarkable recent discovery.

422. Families of Asteroids. If such a catastrophe should occur, and if the fragments were dispersed with relatively small velocities, their orbits would remain very similar, having nearly, though not quite, the same period, eccentricity, and inclination. At first they would all pass nearly through the point of explosion, but the perturbations, due mainly to Jupiter, would gradually shift the orbits, and after a few hundred thousand years this would no longer be true. It can be shown, however, that these perturbations would not alter, in the long run, the mean distances of the planets or their inclinations to the plane of Jupiter's orbit. Moreover, although the eccentricities and longitudes of perihelion would be altered by perturbations, they would change in such a manner that the centers of their orbits, when plotted in space, would all be equidistant from a certain definite point on the line joining the sun with the center of Jupiter's orbit.

K. Hirayama has shown that there exist several groups of asteroids which satisfy these conditions in a very striking manner. Five of these groups, containing from 15 to 44 asteroids apiece, have been recognized, and it seems impossible that the simultaneous agreement with all three conditions can be due to chance. It appears probable that the asteroids of these groups have actually been formed from a single central mass, although the details of the process remain obscure. A more violent explosion would produce orbits less like one another, and its effect would be much harder to trace, so that it is not at present known to what extent such processes may be responsible for the general run of the asteroids.

Certain other matters relating to the region of the inner planets may appropriately be mentioned in closing this chapter.

423. Absence of an Intra-Mercurial Planet. It has been supposed that there might be one or more planets with orbits lying inside that of Mercury. These would be practically invisible under ordinary conditions, but might be seen either when in transit across the sun or, especially, at the time of a total solar eclipse. Such a planet has several times been reported as discovered, notably in 1859 (when it was even named Vulcan), and in 1878; but there is now no doubt that these reports arose from mistakes of some sort, such as taking a small round sun-spot for a planet in transit.

The question has apparently been settled in the negative by photographic observations made during recent total eclipses, and especially by the Lick Observatory expeditions in 1901, 1905, and 1908, whose photographs, covering a region extending 12° east and west from the sun, show great numbers of stars (506 on the plates of 1908, most of which are too faint to be seen with the naked eye) and *not a single object which cannot be identified as a known star*. The observations of 1922 fully confirmed this.

It seems practically certain, therefore, that there are no intra-Mercurial bodies brighter than the eighth magnitude, that is, more than about thirty miles in diameter.

424. The Zodiacal Light. On any clear, moonless evening, after the twilight has disappeared, a faint beam of light is visible in the west, stretching upward along the ecliptic. Near the horizon it is wider and brighter than the Milky Way, but it grows narrower and fainter at higher altitudes.

It can be seen best in the spring, when the ecliptic is most nearly perpendicular to the horizon. A similar beam of light, extending upward along the ecliptic from the eastern horizon, is visible before sunrise and is seen best in the autumn. These regions of diffused luminosity *move around the ecliptic with the sun* and are evidently the opposite extremities of a luminous region surrounding the sun and elongated along the ecliptic. From observations at midnight in midsummer it appears that this light extends about 45° north of the sun (and presumably as far south of it). Along the ecliptic it can usually be followed nearly 90° from the sun, and

in very clear air, especially in the tropics, its extremities are seen to extend entirely around the ecliptic, forming a complete ring (the zodiacal band), on which, just opposite the sun, is a slightly brighter patch 10° or so in diameter, called the Gegenschein, or counterglow.

The zodiacal light, 30° or 40° from the sun, is very conspicuous, and it is probable that the regions near the sun, which are concealed from us by the twilight, are even brighter, though not bright enough to be seen through the diffused light which fills the air during even the longest total eclipse.

The spectrum of the zodiacal light has been photographed by Fath, who finds it identical with that of sunlight, as far as can be determined with the very low dispersion necessary in photographing so faint an object. Its light is partially polarized, as it would be if it were reflected, in part at least, from very fine particles, or molecules of gas.

Van Rhijn's work at Mt. Wilson indicates that the zodiacal light is not confined to the zodiacal belt but extends faintly over the whole heavens and accounts for nearly 60 per cent of the light of the sky on a moonless night. About 15 per cent more of the "sky light" originates in the earth's atmosphere and is apparently due to a faint permanent aurora (§ 658). If we could get rid of these two illuminations, the sky would be much darker and the Milky Way far more conspicuous.

The observations make it almost certain that the zodiacal light is *reflected sunlight* from innumerable small bodies, scattered throughout a region shaped like a lens or a much flattened ellipsoid of revolution, having its greatest diameter nearly in the plane of the ecliptic, and extending well beyond the orbit of the earth. Each individual particle, unless small enough to be held up by the sun's radiation pressure (§ 320), must be moving in its own independent orbit about the sun.

The brightening of the zodiacal band which forms the Gegenschein may be explained by the greater brightness of the individual particles at the full phase (as Searle suggests), or (following Moulton) by the concentration of these particles which would take place, under the combined attractions of the earth and sun, in the neighborhood of a point on the line through these two

bodies and about a million miles outside the earth's orbit. Both factors probably contribute to the observed effect. The quantity of reflecting matter necessary to produce the observed illumination of the sky is surprisingly small. Calculation shows that it would be accounted for if, inside the earth's orbit, there were particles one millimeter in diameter, of the low albedo of the moon, and at an average distance of about five miles apart; or if, in case the light is scattered by molecules of gas, the whole amount of gas in the thickness of nearly two hundred million miles would, if compressed to atmospheric pressure, form a layer somewhat less than one centimeter thick.

So rarefied a medium would be without sensible retarding influence on the motion of planets or comets traversing it, and the attraction of the whole mass, even on the most extreme reasonable assumptions, would be far too small to produce any sensible effect upon the motions of the planets.

CHAPTER XII

THE MAJOR PLANETS

JUPITER · SATURN · URANUS · NEPTUNE

JUPITER

Jupiter, the nearest of the major planets, is usually next to Venus in order of brilliancy among the heavenly bodies. It is occasionally surpassed by Mars when that planet is nearest, but except when near conjunction it appears brighter than Sirius, the most brilliant of the stars. It is not, like Venus, confined to the twilight sky, but at the time of opposition dominates the heavens all night long.

425. Jupiter's orbit presents no marked peculiarities. The *mean distance* from the sun is 5.20 astronomical units (483,200,000 miles), and the *eccentricity* of the orbit is not quite 1/20, so that the distance from the sun varies about 47,000,000 miles between perihelion and aphelion.

At an average opposition the planet's distance from the earth is about 390,000,000 miles, while at conjunction the distance is about 576,000,000 miles; but it may come as near to us as 367,000,000 miles and may recede to a distance of nearly 600,000,000 miles.

The relative brightness of Jupiter at an average conjunction and at the nearest and most remote oppositions is respectively as the numbers 10, 27, and 18, and its corresponding stellar magnitudes are -1.4, -2.5, and -2.1.

The *sidereal* period is 11.86 years, and the *synodic* period is 399 days, a little more than a year and a month.

426. Dimensions. The planet's equatorial diameter, at mean distance, as found by Sampson from the durations of the eclipses of its satellites, is $37''.84$, corresponding to 88,640 miles. Measures of the disk with the filar micrometer give a value about $0''.75$, or 1700 miles, greater. The discrepancy is undoubtedly

due to irradiation in the latter case (§ 288), and the former value is probably very near the truth (see § 441).

The apparent diameter varies from 50″ at an October opposition (or 44″ at an April one) to 32″ at conjunction. The oblateness is considerable, so that the eye notices at once the elliptical form of its disk. Struve finds, from the perturbations of the innermost satellite, the value 1/15.4. The direct measures give substantially the same value.

This makes the polar diameter 82,880 miles, and the mean diameter (§ 144) 86,720 miles, or 10.95 times that of the earth, whence its surface is 120 times, and its volume 1312 times, that of the earth. It is by far the largest of the planets in the solar system; in fact, whether we regard its bulk or its mass, it is larger than all the rest put together.

427. Mass, Density, etc. Its mass is very accurately known, both from the motions of its satellites and from the perturbations of the asteroids, and is $1/1047.40 \pm 0.03$ of the sun's mass (de Sitter, from all observations to 1915), or 316.94 times that of the earth. Comparing this with its volume, we find its density 0.242 that of the earth, very nearly equal to that of the sun, and 1.34 times that of water.

Its mean superficial gravity comes out 2.64 times that of the earth, but on account of the rapid rotation of the planet and its ellipticity there is a very considerable difference between the force of gravity at the equator and at the pole, amounting to 15 per cent of the equatorial gravity. (On the earth the difference is only 1/189.)

When, however, allowance is made for the low density and rapid rotation of the planet, it is found that its ellipticity is *less*, in proportion to the magnitude of the centrifugal force which produces it, than in the case of the earth, — the ratio of the oblateness to the centrifugal force at the equator (§ 341) being 0.76, as against 0.97 for the earth. This shows that the excess of the density near the planet's center above the mean density, and the corresponding deficiency in the density of the surface layers, must be much more pronounced than in the case of the earth.

428. Phases and Albedo. The orbit is so much larger than that of the earth that the planet shows no sensible phases, even at

quadrature, though at that time the edge farthest from the sun shows a slight darkening. The albedo is high, — 0.44 (Schönberg, 1921), — but not quite equal to that of Venus. The change of Jupiter's brightness with phase is small. Within the range of 12° of phase angle accessible from our point of observation on the earth the brightness (corrected for the variations in the distance) changes by about 10 per cent, while that of Mars, under similar circumstances, would change by 15 per cent, and that of the moon by 21 per cent.

It appears, therefore, that the surface of Jupiter must be much smoother than that of the other bodies.

The center of the disk is much brighter than the edge, — as is true in the case of the sun and Saturn, but not of Mercury, Mars, or the moon. The falling off in intensity is most rapid close to the limb, which, according to photometric measures by Schönberg, is only one eighth as bright as the center.

The contrast between the limb and the dark surrounding sky tends to obscure this effect in an ordinary telescopic view of the planet, but it is conspicuous in photographs and in visual observations in twilight. This darkening of the limb is readily explicable by the absorption of light in the atmosphere overlying a uniformly reflecting surface.

It has sometimes been supposed that the planet might be self-luminous to some extent; but this cannot be the case, for its satellites, when eclipsed by entering its shadow, become totally invisible.

429. Axial Rotation. Jupiter rotates on its axis more swiftly than any other planet, — in about $9^h\ 55^m$. The time can be given only approximately, not because it is difficult to find, and to observe with accuracy, well-defined objects on the disk, but because different results are obtained from different spots, according to their nature and their distances from the planet's equator. The rotation period is shortest near the equator; but in place of a gradual variation with the latitude, as in the case of the sun, there appear to be a number of different zones, rather sharply bounded and each with its own rate of rotation.

There is a great equatorial current, covering a zone from 10,000 to 15,000 miles wide, whose rotation period is a little more

than $9^h\ 50^m$. Outside this the periods range from $9^h\ 55^m\ 5^s$ to $9^h\ 55^m\ 42^s$, with no apparent relation to latitude (the distribution of the various zones is different in the northern and southern hemispheres), and well-marked features near one another on the planet's surface drift by one another, sometimes at the rate of 200 miles an hour.

The positions of these zones, and their rates of rotation, vary from year to year. Thus, the period in the equatorial zone increased from $9^h\ 49^m\ 59^s$ in 1879 to $9^h\ 50^m\ 30^s$ in 1889 (according to Williams), and is now decreasing again, while that of the "great red spot" was $9^h\ 55^m\ 34^s.9$ in 1879 and $9^h\ 55^m\ 41^s.9$ in 1899. These changes are unmistakable and not due to uncertainty in the observations.

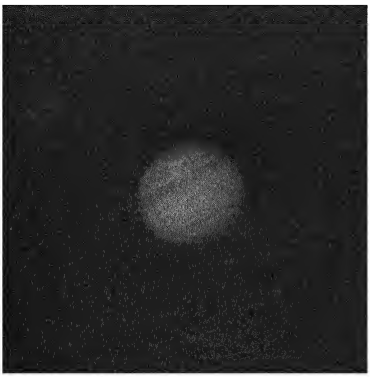


FIG. 147. Jupiter

Enlargement of photograph taken with the 40-inch refractor. (From photograph by E. E. Barnard, Yerkes Observatory)

The plane of the equator nearly coincides with that of the orbit, the inclination being only 3°, so that there can be no well-marked seasons on the planet due to causes such as produce our own seasons.

430. Telescopic Appearance. In even a small telescope the planet is a fine-looking object, since a magnifying power of only 60 makes its apparent diameter, even when remotest, equal to that of the moon. The axial rotation can be well observed with a telescope of 9 inches aperture, or even less. With a large instrument and a magnifying power of 300 or 400 the disk is covered with an infinite variety of detail, interesting in outline, rich in color (mostly reds and browns, with here and there an olive green), and these details change continually as the planet turns on its axis.

For the most part the markings are arranged in belts more or less parallel to the planet's equator, as shown by Fig. 147 from a photograph by Barnard. These belts vary in number, breadth, and position from year to year.

Most of the markings are short-lived, lasting only a few weeks or months at most. From the manner in which they change their shapes and positions it is evident that they are atmospheric, like clouds.

431. Semi-Permanent Markings. There are, however, some markings which are at least semi-permanent and continue for

FIG. 148. Jupiter

October 19, 1915. The bay of the great red spot is seen near the right margin of the disk, above the middle. (From photograph by E. C. Slipher, Lowell Observatory)

years with only slight changes. The most remarkable of these is the great red spot, shown in Fig. 148.

This was first noted in 1878, was extremely conspicuous for several years (being about 30,000 miles long by 7000 wide and brick-red in color), and then gradually faded away, losing its red color and slightly changing its form and becoming rounder. Even yet, while scarcely visible itself, the place which it occupies is clearly

marked by a notch, or hollow, in the edge of the great southern belt. Denning, from a study of earlier drawings of the planet, has identified this same hollow as far back as 1831, and the spot itself since 1859. It might be supposed that this spot, which has lasted for at least seventy-five years, was permanently attached to a solid nucleus below, if it were not for the unquestionable changes in its rotation period. During the last thirty years this has always been within a second of $9^h\ 55^m\ 40^s.3$; but, even so, the spot has drifted, during this interval, with respect to a hypothetical nucleus rotating uniformly at this rate, to a distance of 20,000 miles on each side of its mean position, and also through several thousand miles in latitude. Between 1879 and 1885 it gained more than half a revolution on this supposititious solid nucleus, and its motion in earlier years was still more rapid.

FIG. 149. Jupiter
December 19, 1917. (From photograph by E. C. Slipher, Lowell Observatory)

In the belt just south of the red spot a dark region, about 60° (45,000 miles) in length, has persisted since 1901, and is called the south tropical disturbance. Its rotation period is $9^h\ 55^m\ 19^s.5$, so that it overtakes the red spot at intervals of about two years, streaming by it at the rate of about 16 miles an hour. At these times the motion of the "disturbance" is accelerated, while the red spot appears to be dragged bodily in the direction of the neighboring current through several thousand miles, dropping back after the encounter.

It is evident that both these markings are floating in a fluid medium, and not rooted to anything solid within the planet.

432. Atmosphere and Spectrum. The spectrum of Jupiter is in general that of reflected sunlight, but there are strong bands in the orange and red which evidently arise from absorption in the planet's atmosphere.

They are diffuse in character (not resolvable into fine lines), and are evidently identical with similar and still stronger bands in the spectra of Saturn, Uranus, and Neptune (Fig. 208).

Their origin is still unknown, but their strength and character are sufficient to show that the atmosphere must be dense. The heaviest band of all, in the extreme red near λ 7200, is practically coincident with the strongest band of water vapor; but since the other bands of water vapor do not appear in the spectrum, Slipher, the discoverer of the band in question, believes that it must be due mainly to some other unknown constituent.

Spectroscopic observations upon the relative shift of the dark lines in the spectrum at the eastern and western limbs give a very fair determination of its rotation period.

433. Temperature and Physical Condition. The characteristics and rapid changes of the surface markings of Jupiter make it almost certain that they are clouds of some sort. The rapidity of their motions and transformations suggests that there is a vigorous circulation in the planet's atmosphere, with rapid exchange of material between the surface and the underlying depths. Until recently it was supposed that this demanded a fairly high temperature, and even that the surface might be almost red-hot, although not quite hot enough to be perceptibly self-luminous; but observations by Coblentz in 1914 and 1922 show that the radiation which comes to us from the planet is almost entirely reflected solar radiation, and indicate that the temperature of the surface is near $-140°$C., which is about what might be expected if very little heat comes up from the interior, so that the surface is warmed only by the sun's radiation. The atmosphere above the visible surface must therefore consist of the permanent gases, and the clouds may be of condensed particles of carbon dioxide or other substances which are familiar to us as gases, and which boil vigorously at temperatures far below zero.

The low mean density of Jupiter, as well as of the other major planets, presents a difficult problem of interpretation. We know that the inner portions are much denser, and the outer layers less dense, than the mean, and that there is a shell of atmosphere of unknown depth which is included in the measurement of the diameter of the planet. It could be explained on the assumption

that the inner part of the planet is very hot and even gaseous, as in the case of the sun. This may be true, but Jupiter is probably as old as the earth, and it is hard to see how, in this long interval, it can have escaped cooling sufficiently to liquefy or even solidify. So long as it was liquid or gaseous, the material cooled at the surface would continually sink, and new material would come up to be cooled. If it has cooled, it must contain a larger proportion of relatively light materials than the earth — a reasonable hypothesis (§ 543). Jeffreys suggests that the planet may have a core of dense, rocky material surrounded by a deep layer of ice and then by an extensive atmosphere. The problem is intricate and deserves further investigation.

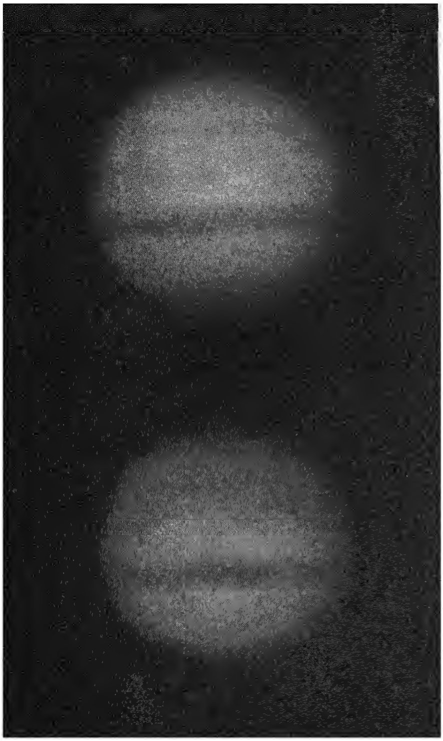


FIG. 150. Jupiter

Photographed March 7 (top) and March 19, 1920, with the Lowell 24-inch refractor. Note the remarkable change in the south tropical belt (above equator) during the twelve-day interval. (From photograph by E. C. Slipher, Lowell Observatory)

It is probable that the visible markings on the planet are at different levels, — the rapidly changing ones being in the rarefied outer gaseous layers, while the more permanent ones lie deeper and probably originate from eruptions from denser layers, where disturbances may maintain themselves for a long time. The rotation period at these lower levels must be nearly uniform, — for fluid friction would soon smooth out any considerable irregularities, — and is probably about equal to that of the great red spot. The shorter periods, on this view, correspond to currents in the upper atmosphere, running in the direction of the planet's rotation. The great equatorial current, which

has fairly well-defined gaseous banks, runs eastward at the rate of 250 miles an hour. Winds in the earth's upper atmosphere — also eastward — have often been observed to go half as fast.

434. Satellite System. Jupiter has nine satellites, so far as is known at present. Four of them (Fig. 151) are so large as to be easily seen with a common field-glass, and were, in a sense, the first heavenly bodies ever discovered, having been found by Galileo

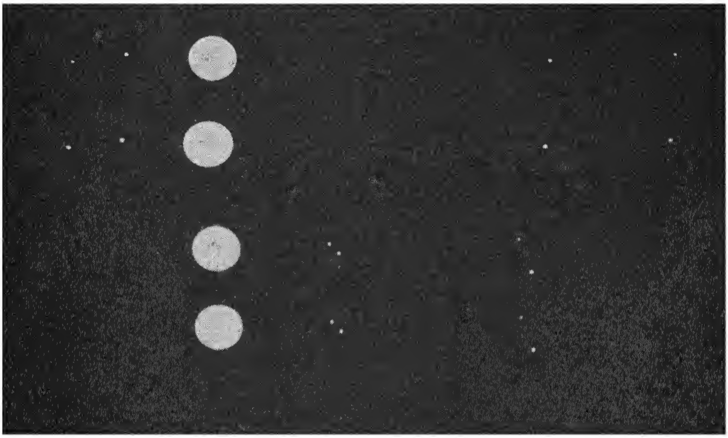


FIG. 151. Satellites of Jupiter

Photographed February 16, 1921 (top pair), and May 25, 1923, with the 24-inch refractor. The oblateness of the planet's disk is well shown. (From photographs by E. C. Slipher, Lowell Observatory)

in January, 1610, with the newly invented telescope. The others are exceedingly small and are visible only with very powerful telescopes. The fifth, the nearest of all to the planet, was discovered visually by Barnard in 1892. The remaining four have been found photographically since 1905, and are very remote from the planet. It is not improbable that other faint, distant satellites may remain to be discovered.

435. The Galilean Satellites. This name is often given to the four large satellites in honor of their discoverer, who, within a few weeks after his first observation, ascertained their true character and determined their periods with surprising accuracy. They are usually known as the first, second, etc., in the order of

distance from the primary, but they also have names, — Io, Europa, Ganymede, and Callisto, respectively, — which, however, are seldom used.

Their periods are approximately $1\frac{3}{4}$, $3\frac{1}{2}$, 7, and $16\frac{2}{3}$ days, and their distances from the planet range from 262,000 to 1,169,000 miles, or from 5.9 to 26.4 equatorial radii of Jupiter. Their orbits are very nearly circular and very nearly in the plane of Jupiter's equator, — the greatest inclination to this plane being 28′, in the case of the second satellite, — and the greatest eccentricity 0.0075, in the case of the fourth.

436. Masses and Perturbations. The satellites disturb each other's motions considerably, and from these perturbations their masses can be ascertained in terms of the planet's mass. The third satellite, which is much the largest, has a mass of about 1/12,000 of the planet's, or a little more than double the mass of our own moon. The first satellite is about half as massive as the third, while the masses of the second and fourth are each about 2/3 that of the moon, or 1/40,000 that of Jupiter. The theory of their mutual perturbations (which must also take into account the attraction of the sun and of the equatorial protuberance of Jupiter) is very intricate, and the masses of the first and fourth satellites are still rather uncertain.

In consequence of these perturbations a curious relation (discovered by Laplace) exists between the longitudes of the first three satellites, so that they cannot all come into conjunction with one another at one time, or into simultaneous opposition or conjunction with the sun.

437. Diameters; Densities. All four satellites show sensible disks in telescopes of moderate aperture, and their diameters are easily measurable.

Barnard, from observations with the great Yerkes refractor, found their respective diameters at mean distance (5.20 astronomical units) to be respectively 1″.05, 0″.85, 1″.51, and 1″.43, corresponding to 2460, 2000, 3540, and 3350 miles. From the length of time it takes them to enter the planet's shadow during eclipse the diameters come out 2320, 1960, 3200, and 3220 miles, — according to Stewart's discussion (1916) of the very numerous photometric observations made at Harvard.

The first and second satellites are therefore not far from the same size as our moon, while the other two are more than half as large again, — larger, indeed, than Mercury. With the eclipse diameters, which are probably the least affected by the systematic errors of observation, the densities of the four satellites come out 0.88, 0.87, 0.65, and 0.17 times the moon's density, or 2.9, 2.9, 2.2, and 0.6 times that of water. It is therefore probable that the first two are masses of rock, like our own satellite. Jeffreys suggests that the third and fourth may be composed largely of ice or solid carbon dioxide. The velocity of escape from the surfaces of all four is about the same as that from the moon, so that, unless they have always been very cold throughout their history, they can hardly possess atmospheres. Slipher finds that the atmospheric band, conspicuous in the spectrum of Jupiter, is absent from that of the third satellite.

438. Brightness and Albedo. The third satellite is the brightest as well as the largest and most massive. The first and second are nearly equal in brightness, and are about two thirds as bright as the third, while the fourth is little more than half as bright as these two.

All four would be visible to the naked eye on a clear, dark night if they were not so near the planet, and the third would be an easy object, like a star of the fifth magnitude. But since the third satellite is never more than 6′, nor the fourth more than 11′, from Jupiter, which is more than 800 times as bright as the brighter of the two, they cannot be seen without optical aid, except perhaps by extraordinarily keen eyes, under very favorable conditions.

The albedo of the first satellite appears to be very nearly equal to that of Jupiter; that of the second, greater by about 20 per cent than the average for the planet's disk; that of the third, about as much less; while that of the fourth is less than one third that of Jupiter. These values are confirmed by the appearance of the satellites when in transit in front of the planet. The second always shows bright; the first, bright except against the brightest parts of the planet's disk; the third, darkish except when near the limb; and the fourth, grayish even near the limb, and almost black at mid-transit.

As seen from the planet's surface the first satellite would give about one fifth as much light as our moon; and the other three

together, half as much as the first. The reason for this is that sunlight on Jupiter is only 1/27 as intense as ours. If set up beside our moon under equally strong illumination, even the fourth satellite would be fully twice as bright as the moon. The third, if placed beside Mercury, or even Mars, would exceed them in brightness, thus appearing as a very respectable planet.

439. Surface Markings and Rotation. All the satellites show markings on their surfaces when viewed with large telescopes and under the best conditions. Those on the third are the easiest to see, while those on the second are very faint and difficult. From observations of these markings by Barnard, Douglass, and Innes it appears certain that the third satellite behaves like our moon and always keeps the same face toward its primary. Douglass has shown that the same is very probable for the fourth satellite.

According to Barnard the first satellite, when between us and the planet, shows a bright equatorial belt nearly parallel to the belts of Jupiter, and darker caps at the poles. When in front of one of the brighter parts of the planet's surface, only the polar regions are visible as two separate dark spots; but when the satellite is in front of one of Jupiter's darker belts, its polar caps merge with the background, and the satellite appears like a narrow white line.

Photometric observations, especially a long series by Guthnick (1914), indicate that all four satellites vary regularly in brightness with their position in their orbits, as would be true if they were spotted and their rotation and revolution periods were identical. The same conclusion follows from the Harvard observations of the eclipses, which show, by the forms of the curves of variation in brightness, that the faces of all four satellites which are turned toward the planet during eclipse are spotted. Stebbins (1926) finds a decided diminution in brightness with phase, indicating that the satellites — especially the fourth — have rough surfaces, like the moon.

440. Eclipses and Transits. The orbits of the satellites are so nearly in the plane of the planet's orbit that, with the exception of the fourth, which at certain times escapes, they all pass through the shadow of the planet, and suffer *eclipse* at every

revolution (Fig. 152). At conjunction they cast their own shadows upon the planet, and these shadows can easily be seen in the telescope as black dots on the planet's disk (Fig. 153),—*shadow-transits*. The satellites themselves, which *transit* the disk about the same time, are much more difficult to observe.

When the planet is exactly in opposition, the shadow, of course, is directly behind it, and we observe an *occultation*

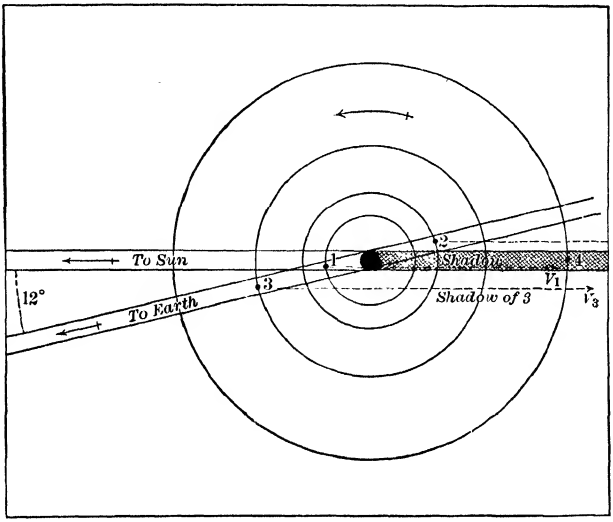


FIG. 152. Eclipses, Transits, and Occultations of Jupiter's Satellites

The planet is supposed to be at western quadrature. The shadow of the planet projects so far to one side that the whole eclipse of satellites II, III, and IV takes place clear of Jupiter's disk. Satellite I is in transit, preceded by its shadow; II is occulted; III is in transit; and IV is in eclipse

(§ 244) instead of an eclipse. At other times we ordinarily see only the beginning or the end of eclipse; but when the planet is at or near quadrature, the shadow projects so far to one side that the whole eclipse of every satellite except the first takes place clear of the disk.

The times of occurrence of all the phenomena of the satellites — eclipses, occultations behind the planet, and transits of the satellites and their shadows across its disk — are given in the *Nautical Almanac* each year. These four types of phenomena provide very interesting observations for even a small telescope.

441. Photometric Observations of the Eclipses. The eclipses are gradual phenomena, the loss of light proceeding continuously from the time when a satellite first strikes the penumbra of the planet's shadow until it is completely immersed in the umbra. The first and second satellites move through distances equal to

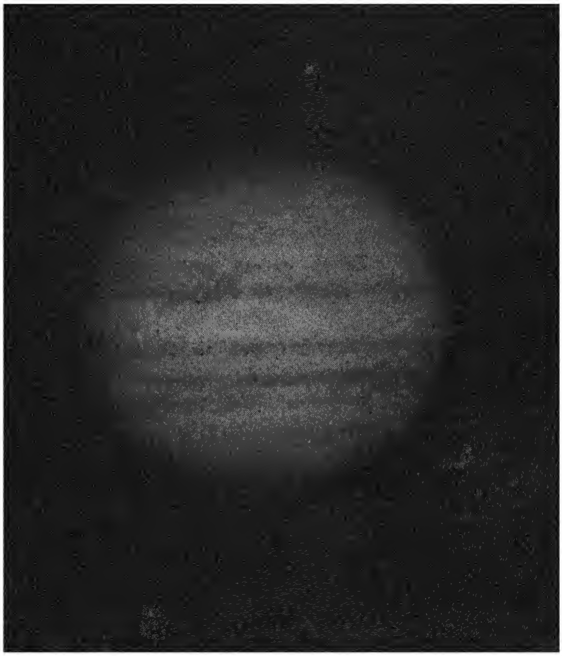


FIG. 153. Shadow-Transit of Ganymede

This photograph was taken soon after the beginning of the transit of the shadow of Ganymede. The satellite itself had completed its transit (Fig. 152 shows how this can happen). The series of faint white spots in the south tropical belt is especially to be remarked. (From photograph by E. C. Slipher, Lowell Observatory, 1914, September $23^{d}\ 15^{h}\ 14^{m}$ G.M.T.)

their own diameters in between $3\frac{1}{2}$ and 4 minutes; the third satellite takes nearly 8 minutes to do this; and the fourth, $10\frac{1}{2}$. The existence of the penumbra modifies these durations very little, but when the satellite's track passes well above or below the center of the shadow, and hence intersects its edge at an oblique angle, the duration of the decrease or increase of its light may be greatly prolonged (though the whole duration of the eclipse is shortened).

The moment during these gradual changes at which the satellite is last seen (or first glimpsed when coming out of the shadow) depends on the state of the air and of the observer's eye, and on the power of his telescope; so that observations of the time of disappearance or reappearance sometimes differ by a minute or two in the case of the first satellite, and by five or even ten minutes for the fourth, making them practically useless for the determination of longitude (§ 107). More precise results can be obtained by measuring the brightness of the satellite with a photometer, using one of the other satellites as a standard of comparison. From a series of such observations, twenty or more of which can sometimes be made during one eclipse, the moment when the satellite is of just half its normal brightness (that is, when it is just half immersed in the planet's shadow) can be determined within a very few seconds.

A very large number of such observations of the eclipses of all four satellites have been made at Harvard by Professor Pickering and others. They are of great value both in determining the exact elements of the orbits and motions of the satellites and for finding the diameters of the satellites and of Jupiter, and have formed the observational basis for Sampson's new tables, by which their motions may be accurately predicted.

Sampson finds that the eclipses sometimes come early or late. He suggests that the diameter of Jupiter is variable, the clouds in its atmosphere lying higher at some times and places than at others, and ranging as much as 100 miles on either side of their mean level. Some such explanation seems to be demanded by the facts.

442. The Equation of Light. In 1675 Roemer, a Danish astronomer (the inventor of the transit instrument, meridian circle, and prime-vertical instrument, — a man almost a century in advance of his day), found that the eclipses of Jupiter's satellites show a peculiar variation in their times of occurrence, which he explained as due to the *time taken by light to pass through space*. His bold and original suggestion was neglected for more than fifty years, until long after his death, when Bradley's discovery of *aberration* (§ 162) proved the correctness of his views.

When we observe a celestial body, we see it, not as it *is* at the moment of observation, but as it *was* at the moment when the

light which we see left it. The time-interval involved is known as the *equation of light.* An increase in the distance between the object and the observer increases the equation of light and further delays the observation of any phenomenon which is occurring on the object. Thus the observed times of the eclipses of Jupiter's satellites are affected by the change in the distance of the earth from Jupiter (Fig. 154). From such observations it is possible to find the time required for light to traverse the diameter of the earth's orbit, — about $16\frac{1}{2}$ minutes.

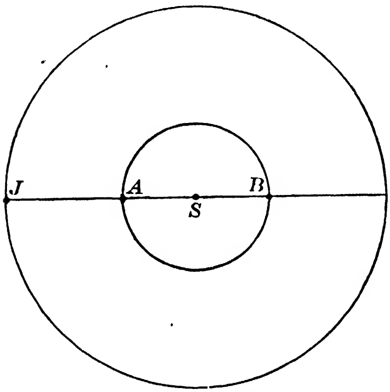


FIG. 154. The Light Equation of the Eclipses of Jupiter's Satellites

When the earth is at A, the phenomena of the satellites are observed relatively about $16\frac{1}{2}$ minutes earlier than when the earth is at B. (J represents Jupiter, and S represents the sun)

Until 1849 our only knowledge of the velocity of light was obtained from the observations of Jupiter's satellites and the constant of aberration, the earth's distance from the sun being found by other methods. But at present the case is reversed. The velocity of light has been determined experimentally with very high accuracy (§ 556), and from this, knowing the "light equation," we may deduce the distance of the sun. Unfortunately, owing to the difficulty in determining the exact moment of mid-eclipse, the result, even from the Harvard photometric observations, cannot compete in precision with other methods. It is, however, in excellent agreement with them, Sampson finding, for the time that light takes to traverse an astronomical unit, the value $498^s.6 \pm 1^s.3$, which leads to the solar parallax $8''.80 \pm 0''.02$.

443. The fifth satellite was discovered by Barnard at the Lick Observatory in September, 1892. It is so faint (its discoverer estimated it as of the thirteenth magnitude, or about 1/1000 as bright as the first satellite), and so near the planet, that it is invisible with telescopes of less than 18 or 20 inches aperture, and a difficult object even with the largest instruments except under

very good conditions. Its distance from the planet's center is 112,600 miles, and its period, $11^h\,57^m\,22^s.7$. Its orbit is very nearly circular and almost in the plane of the planet's equator, but the eccentricity (0.003) and inclination to this plane (27′) have been detected by careful observations. The influence of the planet's ellipticity causes the line of apsides to advance, and the nodes to retrograde at the enormous rate of 916°, or more than $2\frac{1}{2}$ complete revolutions per year (according to H. Struve). The ellipticity of the planet can be very accurately determined from these motions.

The satellite itself is far too small to show a disk, and its diameter can be estimated only from its brightness (which has never been accurately measured). If of the albedo of the first satellite, it would be about 75 miles in diameter; but if similar to the fourth, 150 miles. The first figure is perhaps the more probable.

444. The Outer Satellites. The sixth and seventh satellites (in order of discovery) were found by Perrine at the Lick Observatory, in December, 1904, and January, 1905, on photographs made with the Crossley reflector. The sixth, which is the brighter of the two, is of magnitude 13.7 (according to visual estimates by Barnard) and is probably about 100 miles in diameter. The seventh is much fainter and may be 40 miles in diameter.

The periods of these two satellites are nearly equal (250 and 260 days), and their mean distances from the planet are 7,100,000 and 7,300,000 miles. Both their orbits have large eccentricities (0.15 and 0.21), and their planes are inclined 29° and 28° to that of Jupiter's orbit, but in different directions, so that they make an angle of about 28° with one another. The two orbits, though nearly of the same size, interlock like two links of a chain, and nowhere come within 1,800,000 miles of one another.

Both satellites are subject to very large solar perturbations which may affect their longitudes by several degrees in either direction.

The eighth satellite was discovered photographically by Melotte at Greenwich in February, 1908. It is a very faint object, about equal to the seventh satellite in brightness and, presumably, in size.

Fig. 155, from a photograph made at Greenwich, shows the sixth, seventh, and eighth satellites — and how faint they are.

The orbit of the eighth satellite is extremely remarkable. Its period is a little more than two years, and its motion retrograde, — in the opposite sense from that of the seven inner satellites. Its mean distance from the planet is 14,600,000 miles, but on account of the great eccentricity of the orbit the actual distance ranges from 9,100,000 to 20,100,000 miles. The inclination of the orbit is high, about 32° to the plane of the planet's orbit, or, more properly speaking, 148°, since the motion is retrograde (§ 281); but, like all the other orbital elements, it is subject to great changes on account of the enormous perturbations produced by the sun. The sun's disturbing force (§ 334) is at times as much as 1/9 of the planet's attraction, and the orbit has so far been calculated by the method of mechanical quadratures (§ 323).

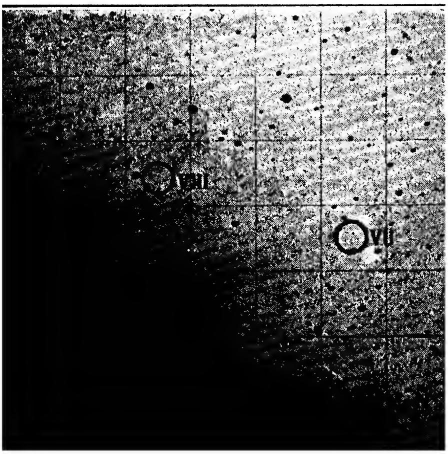


FIG. 155. Jupiter's Satellites VI, VII, and VIII

Reproduced from the negative (February 28, 1908, exposure 80 minutes) on which satellite VIII was discovered. The telescope was made to follow Jupiter, thus causing the stars to trail slightly. The images of the satellites are small dots within the small circles. A part of the overexposed image of Jupiter appears in the corner. A réseau (§ 83) was photographed on the plate for purposes of measurement. (From photograph at Royal Observatory, Greenwich)

During the four revolutions from 1908 to 1916 the period (measured from a given longitude back to the same longitude again) varied from 713 to 768 days; the eccentricity, from 0.29 to 0.45; and the inclination to the plane of the planet's orbit, from 28° to 34°; and it is improbable that the limits of variation of any of these quantities were reached in this short interval.

The ninth satellite, discovered by S. B. Nicholson in July, 1914, on photographs taken at the Lick Observatory, is still fainter than

the eighth, and may be 25 miles in diameter. Its motion is retrograde, like that of the eighth satellite, and its mean distance from the planet a little greater (15,000,000 miles), with a period of two years and two months, eccentricity 0.25, and inclination 156°, according to the elements derived from the observations of 1914 and 1915. Its perturbations are also very large.

The average apparent distances of the eighth and ninth satellites at elongation exceed 2° (when Jupiter is near opposition), and under favorable circumstances they may be greater; thus the eighth satellite was 2° 47′ from the planet in July, 1913. When these satellites have been observed through half a dozen revolutions, a study of their motions will give a very precise determination of the mass of Jupiter (and will present a laborious and difficult problem to the mathematical investigator).

445. Are these Satellites Captured Asteroids? If, in the course of the changes of its orbit, the eighth satellite should ever get about twice as far from Jupiter as it did in 1913, and should do this near the time of its opposition or conjunction with the sun, the sun's disturbing force upon it would equal the whole attraction of the planet and counteract it almost completely, so that the satellite would move away out of the range of the planet's attraction altogether, and circulate in an independent orbit of its own about the sun, becoming a faint asteroid. It is equally conceivable that, reckoning backwards, the reverse process may have occurred at some past time, — an asteroid approaching the planet so slowly and in such a manner that under the combined attraction of the planet and sun (the planet's attraction alone could not do it) it was "captured," and settled down to move around the planet as a satellite.

The question whether any of the distant satellites of the planets have thus been captured (and are therefore likely to be lost again in the future) is of great interest. The problem has been solved mathematically in the case where the mass of the satellite is negligible and the planet's orbit circular.

Calculation shows that if the planetary orbits were circular, the moon, Jupiter's sixth and seventh satellites, and the ninth satellite of Saturn (soon to be described) would be rigorously confined to closed regions surrounding their respective primaries. Since the actual orbits of the planets are elliptical, the calculations do not *prove* that this is actually the case; but the "margin of safety" is in all cases great enough to make it exceedingly probable that these satellites have been circulating about the planets for an indefinite period, and will continue to do so indefinitely.

For the eighth and ninth satellites of Jupiter the calculations show that no such necessary restrictions exist; but, with their retrograde motion, Moulton considers it reasonable to believe that their motions are stable.

Moulton has also shown that a satellite of Jupiter with *direct* motion, and with a period as long as two years, would probably, though not in all possible cases, be unstable, and would eventually be lost to the planet.

SATURN

446. Saturn is the most remote of the planets which were known to the ancients. It is a conspicuous object of the first magnitude, outshining all but the brightest stars, with a steady yellowish radiance, not varying much in appearance from month to month, though in the course of fifteen years it alternately gains and loses about 70 per cent of its brightness with the changing phases of its rings (§ 457). It is unique among the heavenly bodies, — a great globe attended by eight, perhaps nine, satellites and surrounded by a system of rings which has no known counterpart elsewhere in the universe.

447. Orbit. Its mean distance from the sun is about nine and one-half astronomical units, or 885,900,000 miles; but the distance varies nearly 100,000,000 miles on account of the considerable *eccentricity* of the orbit (0.056). Its nearest opposition approach to the earth is about 745,000,000 miles, while at the remotest conjunction it is 1,027,000,000 miles away.

The *sidereal period* of the planet is about 29½ years, the *synodic period* being 378 days. The *inclination of the orbit* to the ecliptic is about 2½°.

448. Diameter, Volume, and Surface. The apparent *mean* diameter of the planet varies from 20″ to 14″ according to the distance. The planet is more flattened at the poles than any other, its oblateness being fully 10 per cent. Struve's direct measures and his study of the perturbations of the satellites lead to the value 1/9.5; Lowell's recent measures, to 1/9.2.

The equatorial diameter is about 17″.25 at mean distance, or 74,100 miles, — the older observations, which give a value near 17″.5, being probably somewhat increased by irradiation. With the ellipticity the polar diameter comes out 66,300 miles, and the mean diameter 71,500 miles, — very slightly more than 9 times that of the earth. Its surface is therefore about 81 times, and its volume 734 times, that of the earth; but owing to the difficulty of correcting the measures for irradiation, the diameter is probably

uncertain by 1 per cent, and the surface and volume by 2 and 3 per cent respectively.

449. Mass, Density, and Gravity. The planet's mass is well determined both by the observations of its satellites and by the perturbations which it produces upon Jupiter, and may be taken as 1/3499 that of the sun, or 94.9 times the earth's mass, with a probable error of perhaps one part in a thousand.

The remarkable fact follows that its mean density is only 0.13 times that of the earth, or *0.715 times the density of water.* It is by far the least dense of all the planets. The superficial gravity averages 1.17 times that at the earth's surface, but varies by 30 per cent between the equator and the pole.

450. Axial Rotation. The surface of Saturn is marked by belts, parallel to its equator, as in the case of Jupiter, but it is only very rarely that spots appear which are well enough defined to permit a determination of the rotation period. A white spot near the equator, which suddenly appeared in 1876 and continued visible for several weeks, gave a period of $10^h\ 14^m\ 24^s$, according to Hall, its discoverer. One discovered by Barnard in 1903, in latitude 36° north of the planet's equator, gave the much longer period of $10^h\ 38^m$. It is evident that on Saturn, as on Jupiter, the rotation periods must be different in different latitudes, and it is significant that in this case too the rotation is most rapid near the equator.

The centrifugal force at the equator is 0.17 of the force of gravity. The ratio of the planet's ellipticity to this quantity is only 0.62, — so near to the theoretical limit (0.5) for a body whose whole mass is concentrated at the center as to make it certain that the superficial layers must be of very low density, much of the mass being probably concentrated into a deep-lying nucleus.

The position of the equatorial plane is accurately determined by the perturbations which the planet's ellipticity produces in the planes of the satellites' orbits. It is inclined 28° 6′ to the plane of the ecliptic, or 26° 45′ to that of the planet's orbit.

451. Surface Markings, Color, and Spectrum. As in the case of Jupiter, the edges of the disk are much less brilliant than the center. The belts are less sharply defined and less variable than those of Jupiter. There is usually a brilliant yellowish zone at the equator, and a darkish cap, of greenish hue, at the pole.

Lowell has reported the discovery of delicate darkish "wisps" or "lacings" crossing the bright belts diagonally. These are, however, too numerous and too similar to one another to be available for determining the exact rate of the planet's rotation.

R. W. Wood, in 1915, secured photographs with various kinds of plates and colored screens, which show extraordinary differences in the reflecting

FIG. 156. Saturn

This photograph, as well as those of Fig. 157, was taken with the 60-inch reflector, using isochromatic plates and a yellow screen. It reproduces closely the relative visual brightness of the various regions. The rings of Saturn were wide open when this photograph was taken. The shadow of Saturn falls on ring *B*, while the dark band just above the rings, in front, is an effect produced by the crape ring and the shadow of the rings. (From photograph by E. E. Barnard, Mt. Wilson Observatory)

power of the various regions for light of different wave-lengths. With deep red light (of about 7500 angstroms) hardly any surface markings are visible. With yellow light the equatorial belt is bright and the polar regions dark, but with violet light (about 4250 angstroms) the equatorial belt is also very dark and the polar cap almost black. The darkening at the planet's limb is much less conspicuous in the violet, showing that the light from this region of the planet is bluer than that coming from the middle of the disk. The rings, compared with the planet, are faintest in the deep red and brightest in the violet.

The planet's spectrum shows bands in the orange and red, similar to those in Jupiter's spectrum, but stronger. These bands are absent in the spectrum of the ring, which presumably has no atmosphere.

452. Brightness and Albedo. Saturn appears to the naked eye to be comparable in brightness to the brighter fixed stars. In addition to the changes in its apparent brightness arising from the variations due to its distance from the earth and sun, there are large variations due to the different aspects under which we see its rings. When these are edgewise toward the earth, and practically invisible, the planet in opposition appears like a star of magnitude 0.9, — about as bright as Altair, — and in conjunction one third fainter. But when the rings are most widely displayed they reflect to us about one and two-thirds times as much light as the ball of Saturn, and its brightness is correspondingly increased. This phase occurs near the planet's perihelion and aphelion. At opposition in the former case it appears of magnitude -0.4, — brighter than any star visible in the latitude of New York, except Sirius, and $3\frac{1}{2}$ times as bright as at opposition with "ring invisible." At aphelion it is about one third fainter.

The albedo of the ball of Saturn is 0.42 (according to Schönberg), — a little less than that of Jupiter, and suggesting a similar physical condition. The photographic albedo is 25 per cent less, in good accordance with the yellowish color which the planet presents to the eye.

When the ring is invisible, the planet's light shows a small change with phase, which indicates that its reflecting surface, like Jupiter's, is effectively a smooth one. When a large part of the light comes from the rings, however, much greater variation with phase is revealed by the observations. This leads to important conclusions regarding the nature of the rings (§ 459).

453. The physical constitution of the planet presents problems similar to those offered by Jupiter, but more acute. There can be no doubt that the visible surface is gaseous and that the atmosphere is very deep, but how the mean density of the interior can be so low is not yet fully understood. Coblentz's observations indicate that the temperature of the surface is about $-150°$ C. Even this is some 30° higher than the sun's radiation could keep it.

454. The Rings. The most remarkable peculiarity of the planet is its *ring system.* The globe is surrounded by three thin, flat, concentric rings in the plane of Saturn's equator, like circular disks of paper perforated through the center. They are generally referred to as *A*, *B*, and *C*, *A* being the exterior ring.

Galileo *half* discovered them in 1610; that is, he saw with his little telescope two appendages on each side of the planet, but he

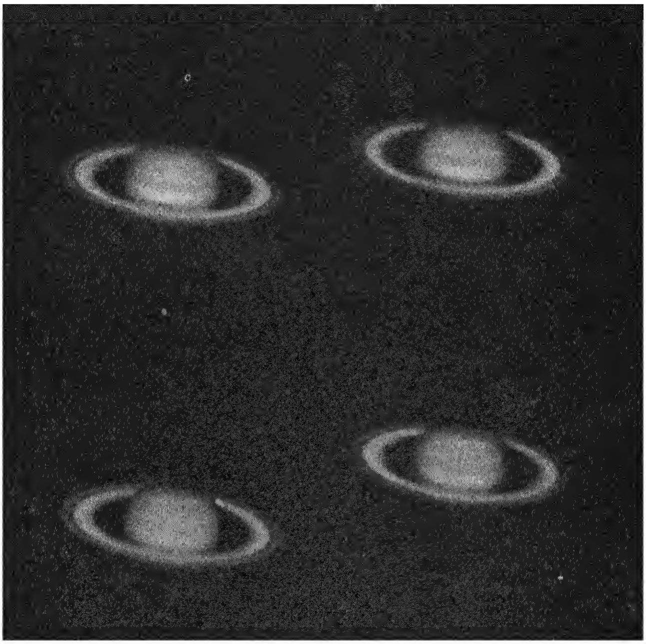


FIG. 157. Saturn

We are looking more obliquely at the ring system here than in Fig. 156. Saturn hides the rings on the far side. The planet is in opposition, and its shadow extends directly away from the earth. (From photograph by E. E. Barnard)

could make nothing of them, and after a while he lost them, to regain them again some years later, greatly to his perplexity.

The problem remained unsolved for nearly fifty years, until Huygens explained the mystery in 1655. Twenty years later D. Cassini discovered that the ring is *double*, that is, composed of two concentric rings, with a dark line of separation between them, and in 1850 Bond, of Harvard, discovered the third "dusky"

or "gauze" ring between the principal ring and the planet. (It was discovered a fortnight later, and independently, by Dawes in England.)

455. Dimensions of the Rings. The outer ring, *A*, has an exterior diameter of 171,000 miles, according to Lowell's recent measures, and is a little more than 10,000 miles wide. "Cassini's division" between it and ring *B* is probably about 3000 miles wide (although, owing to irradiation, most measures make it narrower) and appears to be perfectly uniform all around. Ring *B* is 145,000 miles in outer diameter, and a little less than 16,000 in width. It is much brighter than *A*, especially at its outer edge; indeed, as Fig. 156 shows, it is as bright as the brightest parts of the planet's surface. According to Lowell it is separated by a narrow gap, perhaps 1000 miles wide, from ring *C*, which is sometimes known as the crape ring, because it is only feebly luminous and is semi-transparent, allowing the edge of the planet to be seen through it. This innermost ring is about 11,500 miles wide, making the width of the whole ring system 41,500 miles, and leaving a clear space of 7000 miles between its inner edge and the planet's equator. The thickness of the rings is very small indeed, probably not exceeding 10 miles. If we were to construct a model of them on the scale of 10,000 miles to an inch, so that the outer one was fully 17 inches in diameter, they would be thinner than the thinnest tissue paper. This extreme thinness is proved by the appearance presented when the plane of the ring is directed toward the earth, as it is once in every fifteen years. At that time the ring becomes invisible for several days even in the most powerful telescopes.

456. The outer ring, *A*, is occasionally seen divided by a narrow dark line known as "Encke's division," but more usually there is only a darkish streak upon it.

Lowell has seen and measured several divisions in ring *B*, — fine linear markings, concentric with the ring. No markings, however, which would permit a determination of the rotation period of the rings have ever been observed.

The shadow of the planet on the rings is of course conspicuous, except at opposition, when it is concealed behind the planet. That of the ring on the planet, also, can frequently be seen as a narrow dark band bordering the ring where it crosses the disk.

The dark marking on the inner side of ring *B* in Fig. 156 is the crape ring, which, though invisible against the sky with the short exposures employed, shows dark against the planet (where its own shadow and that of ring *B* are probably seen through it, as it is semi-transparent).

457. Phases of the Rings. The rings lie exactly in the equatorial plane of the planet, so far as can be determined by careful observation; and in the planet's revolution around the sun the plane of the equator and of the rings keeps parallel to itself. Twice, therefore, in the planet's revolution, when the plane of the ring passes through the earth, we see it edgewise, and twice at its

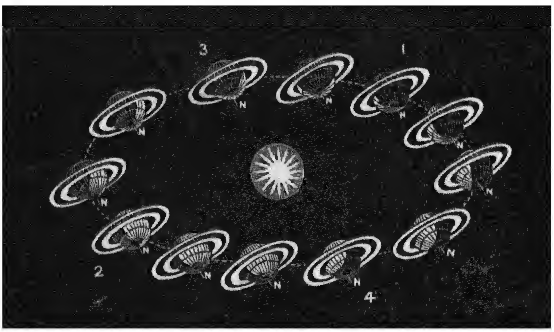


FIG. 158. The Phases of Saturn's Rings

maximum width, when it is at the points halfway between the nodes (Fig. 158). The former phases occur when the planet's longitude is 172° or 352° in the constellations Leo and Aquarius; the latter when its longitude is near 82° (in Taurus and Gemini) or 262° (in Sagittarius). The maximum elevation of the earth above the plane of the rings is 27°, so that the greatest apparent width of the ring system is a little less than half its length, and about one sixth more than the polar diameter of the planet.

Near the time of disappearance the ring appears like a thin needle of light projecting on each side of the planet to a distance nearly equal to its diameter. On this the satellites are seen threaded like beads when they pass behind or in front of it.

The plane of the rings takes nearly a year to traverse the earth's orbit, and during this time the earth may cross it either

once or three times, according to circumstances. In the former case the disappearance of the rings occurs when the planet is near conjunction, under unfavorable conditions for observation, as in 1878 and 1891. In the latter, two at least of the three disappearances are well observable, as was the case in 1921.

458. Appearance of the Dark Side of the Rings. When the rings are exactly edgewise toward us, they are invisible for a day or two even with the great Yerkes telescope; but when the earth and sun are on opposite sides of the plane of the rings, so that we see the dark side of the rings (greatly foreshortened, of

South

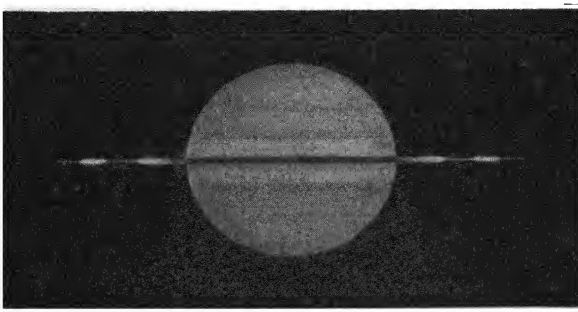


FIG. 159. Saturn

As seen with the 40-inch refractor of the Yerkes Observatory, December 12, 1907, when the earth and the sun were on opposite sides of the plane of the rings. (From drawing by E. E. Barnard)

course), they are visible, and present a very remarkable appearance, first described by Bond in 1849 and observed in great detail by Barnard and others in 1907 (Fig. 159).

The whole surface of the rings is visible, though very much less brightly illuminated than the planet's disk, so that where the rings cross the disk they appear as a narrow dark band. On the faint hazy strip outside the planet appear two brighter regions, or condensations, on each side, which are brighter but, according to Barnard, no wider than the remainder of the rings. These are permanent phenomena, having been seen by many observers throughout the time when the dark side of the rings was visible. Measures of their distances from the planet show that the inner ones coincide in position with the crape ring, while the outer ones

include the Cassini division and also the bright portion of ring *B* just inside it.

The observed illumination may be explained by the combination of two factors:

(1) *Sunlight reflected from Saturn to the surface of the rings.* Calculation shows that the light received in this way at the position of the outer condensations would be about 30 times our full moonlight — strong enough to illuminate the rings very perceptibly.

(2) *Sunlight transmitted through the thinner parts of the rings,* which, as is shown by direct observation in the case of the crape ring, are partially transparent, or translucent.

This must be the main factor in producing the condensations which fall exactly upon those portions of the system (the crape ring and the Cassini division) which, under full sunlight, reflect the least light, but which, as these observations show, must transmit the most. The brighter and denser portions of the rings are probably very nearly, if not quite, opaque.

459. Structure of the Rings. It is now universally admitted that the rings are not continuous sheets of either solid or liquid matter, but a *flock or swarm of separate particles*, little "moonlets," each pursuing its own independent circular orbit around the planet, though all moving almost exactly in the same plane.

This theory is confirmed in a remarkable way by entirely different lines of evidence.

(1) *Direct observations.* The transparency of the crape ring, and the illumination of the dark side of this and of the Cassini division by transmitted sunlight, can be reasonably accounted for only on the hypothesis that they are composed of particles so thinly scattered that the open sky is visible between them.

Again, the distant satellite Iapetus was observed by Barnard in 1889 while it was passing through the shadow of the system. It vanished entirely in the shadows of the ball and the bright ring *B*, but remained visible, though fainter than normal, when immersed in that of the dusky ring. Barnard's estimates of its brightness show that about 90 per cent of the sunlight passed through this ring near its inner edge, and nearly 50 per cent at the outer edge, where it joins the bright ring. At this time the

sun's rays traversed the ring very obliquely, at an angle of 11°. If they had gone through squarely, the loss of light would have been less than one fourth as great; whence it appears that if the crape ring could be viewed at right angles to its own plane, the solid particles, even in its densest portion, would cover only about one eighth of the background. Ring *B*, which has an apparent albedo fully as great as that of the planet, must be much more closely packed and is probably quite opaque. Ring *A*, which is fainter, is partially transparent. This was proved in 1917 by the observations of Ainslie and Knight, who watched a star of the seventh magnitude pass behind the outer ring and the Cassini division. Behind the outer ring the star was much fainter than normal, but it brightened up twice for a few seconds, presumably while it was crossing two of the narrow divisions sometimes observed in ring *A*. The star apparently shone with its full brightness while crossing the Cassini division.

(2) *Photometric observations of the planet's brightness.* It has already been mentioned that the brightness of the light reflected from the rings falls off considerably as the phase angle increases, though that of the planet's ball does not. When the rings are widely open, the combined light at quadrature (phase angle 6°) is 79 per cent of that at opposition (after correction for the effect of changes in distance). This shows that the light from the rings, which forms rather more than half of the whole, has diminished by 35 per cent.

Seeliger has shown that this is a direct consequence of the meteoric constitution of the rings. At different points our line of sight penetrates to very different depths between the particles of which the ring is composed, — rarely, however, getting clear through without striking something. The sun's rays do the same, and the particles lying nearest the sunlit side of the rings cast shadows on those which lie deeper. When the earth is exactly in line between the sun and Saturn, each particle *hides its own shadow*, and the whole surface of the rings appears bright. But when we are even a degree or two out of line, the shadow of each particle begins to come out from behind it, and the multitude of tiny shadows diminishes the total light. From the observed behavior Seeliger concluded that, on the average, the particles

occupy about 1/16 of the whole volume of rings *A* and *B*. Schönberg (1921) makes the fraction much less, and concludes that the larger particles are accompanied by much fine dust.

460. Keeler's Demonstration of the Meteoric Theory of Saturn's Rings. In 1895 Keeler, then at Allegheny, obtained *spectroscopic proof* that the *outer* edge of the ring revolves *more slowly than the inner*, as the theory requires, but as would not be true if the ring were a continuous sheet. The observations were delicate, involving the small shifts in the positions of the spectral lines produced by motions of approach or recession, according to Doppler's principle (§ 564).

At the inner edge of the ring Keeler's observations (compare Fig. 188) indicated a velocity of 20 kilometers a second; at the outer edge, only 16, — precisely the velocities that satellites of Saturn ought to have at corresponding distances from the planet.

It may be noted also that the lines in the spectrum of the ball of the planet indicated a velocity, at the edge of the planet, of 10 kilometers a second, corresponding to a rotation period of $10\frac{1}{4}$ hours, almost exactly agreeing with that deduced by Professor Hall from the observation of the spot. Keeler's results have since been fully confirmed by several other observers.

461. Stability of the Rings. Before the photometric and spectroscopic proofs of the constitution of the rings became available, it had been shown by Pierce and Clerk Maxwell that neither a solid nor a liquid ring could continue permanently to revolve around a planet. The least disturbance would cause a solid ring to oscillate more and more wildly until it dashed itself to pieces against the planet, and would cause a liquid ring to break up into numerous parts. Maxwell, in 1859, proved that a ring composed of a vast number of small satellites would be *stable*, maintaining its general character even if subjected to moderate disturbances from outside, such as would be produced by the attractions of the large satellites, — provided that the aggregate mass of the particles were sufficiently small in comparison with that of the planet.

H. Struve has since shown that the whole mass of the rings cannot exceed 1/27,000 of that of Saturn (or about 1/4 of the mass of the moon) and is probably much less. This is determined from the perturbations of the satellites produced by the attraction

of the rings, — which are very similar to those arising from the ellipticity of the planet, but fall off more rapidly with distance.

It is extremely probable that the rings, as a whole, are stable and form a permanent feature of the Saturnian system. The finer divisions may, and probably do, change somewhat from time to time.

Kirkwood has shown that the divisions in the rings are very probably due to the perturbations produced by the satellites, since they occur at distances from the planets where the period of a small body would be precisely commensurable with that of some one of the inner satellites. Thus, the distances at which the period would be 1/2 that of Mimas, the innermost satellite, and 1/3 and 1/4 of those of the next two satellites, all fall within Cassini's division; the boundary between rings *B* and *C* corresponds to 1/3 of the period of Mimas, Encke's division to 3/5 of it, and the divisions recently observed by Lowell to various other values, — 2/5, 3/8, and so on.

It will be remembered that there are similar gaps in the distribution of the asteroids, in places where their periods would be commensurable with that of Jupiter. In both cases the gaps are nearly but not quite bare of small bodies.

In 1850 Roche proved that a liquid satellite of any planet, if at a distance greater than a certain definite limit, would be merely distorted by the tide-raising forces resulting from the planet's attraction; but that if it were nearer than this limit, these forces would overcome the mutual gravitation of its parts, and actually tear the satellite to pieces. For a satellite of the same density as the planet this limit is 2.44 times the planet's radius. Now the distance of Mimas, the nearest satellite, is 3.11 times Saturn's radius, so that it lies well outside Roche's limit, while the outer radius of ring *A* is 2.30 times that of the planet, placing the whole ring system in the region in which any satellite-forming material would, if liquid, have been torn to bits by the tidal forces, and would, if composed of separate solid particles, have been prevented by these forces from aggregating into larger masses.

462. Satellites. Saturn has nine of these attendants, the largest of which, named Titan, was discovered by Huygens in 1655. It is easily seen with a 3-inch telescope. D. Cassini,

with his long-focus telescope (§ 54), found four others before 1700; Sir William Herschel, in 1789, discovered the two that are nearest the planet; and an eighth was discovered in 1848 by W. C. Bond, and independently by Lassell two days later. The ninth was discovered photographically, in 1898, by W. H. Pickering. As the order of discovery does not agree with that of increasing distance from the planet, it has been found convenient to adopt names for these satellites (of which those of the first seven, in order of discovery, were assigned by Sir John Herschel, and those of the last two by their discoverers).

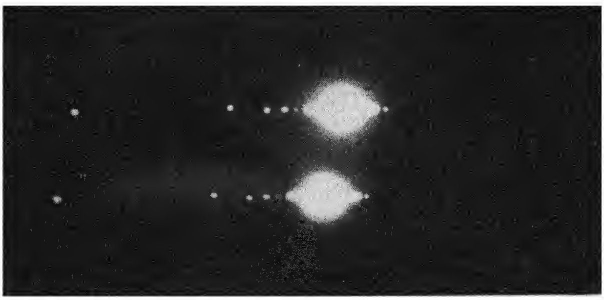


Fig. 160. Satellites of Saturn

Photographed March 2, 1921, with the 24-inch refractor and color filter. From left to right they are Titan, Rhea, Dione, Tethys, Mimas, and Enceladus. The image of the planet and the ring-system is much overexposed. (From photograph by E. C. Slipher, Lowell Observatory)

The range of the system is very great (though exceeded by that of Jupiter's attendants, so far as our present knowledge goes). Mimas, the nearest to Saturn of the satellites, coasts around the edge of the rings at a distance from them of only 30,000 miles (115,300 from the planet's center), in a period of $22^h\ 37^m$. The next four, Enceladus, Tethys, Dione, and Rhea, follow closely outside it, with periods ranging from $1^d\ 9^h$ to $4^d\ 12^h$, and with distances from 148,000 to 327,000 miles. Farther out is Titan (much the largest of the family, as his name suggests), with a period of $15^d\ 23^h$ and a mean distance of 759,000 miles. The faint satellite Hyperion, with a period of $21^d\ 7^h$, comes next; then, beyond a great gap, is Iapetus, with a period of $79^d\ 8^h$ and a mean distance of 2,210,000 miles. Finally, far outside the others,

is the tiny retrograde satellite Phœbe, at a mean distance of 8,034,000 miles, and with a period of 550^d.

The five inner satellites move in nearly circular orbits, and nearly in the plane of the rings; the orbit of Mimas is inclined $1\frac{1}{2}°$ to that plane, and that of Tethys 1°. The orbit planes of Titan and Hyperion deviate slightly from the plane of the rings, in the direction of the orbit-plane of the planet; and that of Iapetus is not far from halfway between the two.

Phœbe, the faint and distant satellite which was discovered by W. H. Pickering in 1898, has a *retrograde motion*, unprecedented at the time of its discovery, but later matched by the eighth and ninth satellites of Jupiter. The orbital eccentricity (0.17) and inclination (5°.3, or, more properly, 174°.7) are considerable, but not so great as in the case of Jupiter's outer satellites.

Pickering, in 1905, reported the discovery of another very faint satellite with a period of 20.85 days, which he named Themis, but the discovery has not been confirmed. It may be that one or more faint satellites of about that period exist.

463. Diameters, Albedo, Rotation. Titan shows a distinct disk in large telescopes, and the measures of Barnard and Lowell make its diameter, at mean distance, about 0″.60, or 2600 miles. Its brightness is a little more than 1/1000 that of the planet (not including the rings), from which it follows that its albedo is about four fifths of the average for the planet's disk. This agrees well with observations of its transits by Struve, who finds that near the middle of the disk Titan appears as a dark spot, while it is indistinguishable near the limb.

The remaining satellites are too small to show perceptible disks. Rhea, the brightest after Titan, takes about four minutes to enter or leave the planet's shadow during an eclipse, which shows that its diameter must be about 1100 miles and its albedo about equal to that of the planet. Its shadow on the planet has been seen at the Lowell Observatory.

The diameters of the minor satellites can be estimated only from their measured brightness. If they are comparable in albedo with the planet and the denser part of the rings (as seems probable), Dione and Tethys must be 700 or 800 miles in diameter, Enceladus about 500, and Mimas perhaps 400.

For the outer satellites the estimates are more uncertain. Iapetus is probably about 1000 miles in diameter, Hyperion about 300, and Phœbe perhaps 150.

It is worth remarking that Hyperion and even Phœbe, if they could be brought to the same distance from the sun as the asteroids, would rank among the brightest asteroids.

Several of the satellites show regular changes in brightness, which are repeated when the satellite returns to the same point in its orbit. In the case of Iapetus the range of variation is extraordinary. Near its western elongation it is almost as bright as Rhea, but at eastern elongation it is less than one fifth as bright, and fainter than Enceladus. The average albedo of the side which precedes in the orbital motion, and faces us at eastern elongation, must be less than one fifth of that of the opposite face. Measures by Guthnick and Wendell show that Titan and Rhea likewise vary in brightness, but only by 25 or 30 per cent. Similar variations in Mimas and Enceladus have been reported by Lowell, and in Phœbe by W. H. Pickering. It is probable that all the satellites, like our moon, always keep the same face toward their primary.

464. Masses and Densities. From the perturbations the masses of several of the satellites have been accurately determined, and rough estimates or superior limits obtained for the others. The mass of Titan is 1/4150 of Saturn's or 1.86 times that of our moon, which, with the diameter given above, makes the satellite's density slightly greater than the moon's, or $3\frac{1}{2}$ times that of water.

The masses of Dione, Tethys, and Mimas come out respectively 1/70, 1/120, and 1/2100 of the moon's mass. These are by far the smallest celestial masses which have ever been measured by their gravitational effects.

Struve has pointed out that the masses of the inner satellites are very much smaller, in comparison with that of Titan, than might have been expected from their brightness. Mimas, in particular, has 1/33 of Titan's brightness but only 1/4000 of its mass. Even if its albedo were double that of Titan (which is improbable, since the latter is already high), its density would be only 1/8 that of the larger satellite, or 2/5 that of water. It is clear that the inner satellites must, at the same time, be of very high albedo and of low density.

URANUS

465. Discovery of Uranus. Uranus was the first *planet* ever "discovered," and the discovery created great excitement and brought high honors to the astronomer. It was found accidentally by the elder Herschel on March 13, 1781, while "sweeping" the heavens for interesting objects with a 7-inch reflector of his own construction. He recognized it at once by its disk as something different from a star, but, never dreaming of a new planet, supposed it to be a peculiar kind of comet; its planetary character was not demonstrated until nearly a year had passed, when Lexell showed by his calculations that it was doubtless a planet beyond Saturn, moving in a nearly circular orbit.

It is distinctly visible to a good eye on a dark night as a faint star, almost exactly of the sixth magnitude, but is very far from being conspicuous.

The name "Uranus," suggested by Bode, finally prevailed over other appellations that were proposed (Herschel had called it Georgium Sidus, in honor of the king).

It was found, on reckoning backward, that the planet had been many times observed as a star and had barely missed discovery on several previous occasions. Twelve observations of it had been made by Lemonnier alone, and later they proved extremely valuable in connection with the investigations which led to the discovery of Neptune.

466. Orbit. The mean distance of Uranus from the sun is 19.19 astronomical units, or 1,782,300,000 miles. The actual distance varies 84,000,000 miles on each side of this, owing to the eccentricity of the orbit (0.047). The inclination of the orbit plane to the ecliptic is small, only 46′. The planet's periodic time is 84.01 years, and the synodic period 369.16 days. The orbital velocity is $4\frac{1}{4}$ miles per second.

It is so remote that its apparent brightness varies by only 20 per cent from opposition to conjunction, and there is no perceptible difference in its appearance at opposition and quadrature.

467. Dimensions, Mass, Density. Uranus, under sufficient telescopic power, shows a sea-green disk about 3″.75 in diameter (at mean distance), corresponding to a real diameter of some

32,400 miles; but its small and rather faintly illuminated disk is a very difficult object to measure, and the results of different observers range through 10 per cent on either side of this mean value.

The disk, under favorable circumstances, shows a decided ellipticity, — about 1/12 according to the measures of Schiaparelli, Young, Barnard, and Lowell, which are all in good agreement.

Bergstrand's study of the motion of the line of apsides of the innermost satellite leads to a smaller ellipticity, 1/18. As both types of observation are delicate, the discrepancy is not surprising. With 1/14 as a compromise, the planet's mean diameter comes out almost exactly 4 times, its surface 16 times, and its volume 64 times that of the earth.

The planet's mass is much more accurately determined than its diameter (by the motion of its satellites and by the perturbations which it produces on Saturn), and is 14.7 times that of the earth (with a probable error of less than 1 per cent). With the diameter adopted above, the density comes out 0.23 of the earth's, or 1.27 times that of water, and the mean surface gravity 0.92 of the earth's.

468. Albedo and Spectrum. The albedo is about equal to that of Saturn or Jupiter. The change of brightness with phase cannot be found by observation, as the earth never gets more than 3° out of line between the planet and the sun. If it is assumed to be similar to that found for Jupiter and Saturn, the albedo comes out 0.45. It should be remembered, however, that sunlight at Uranus is only 1/14 as intense as at Jupiter's distance, so that the disk of the planet appears in the telescope to be less than one tenth as bright as a portion of Jupiter's disk of the same apparent size.

The planet's *spectrum* exhibits heavy bands in the red, orange, and green, similar to those found in Jupiter and Saturn, but far more intense (Fig. 208), doubtless owing to some unidentified substance in its atmosphere. These bands explain, at least in part, the greenish tinge of the planet's light.

It is probable that the physical condition of the planet is generally similar to that of Jupiter and Saturn. The low density raises the same problems (§ 433). The high albedo indicates that the visible surface is covered with clouds; and the heavy bands in the spectrum, that the overlying atmosphere is extensive and dense.

469. Rotation. The planet's ellipticity shows that it must be in rapid rotation, but the period remained undetermined until 1912, when Lowell and Slipher, by the spectroscopic method (compare § 579) derived the value *$10\frac{3}{4}$ hours*. Though the planet's spectrum was necessarily narrow, the inclination of the lines was unmistakable, and the period given, though possibly uncertain by half an hour or more, must be substantially correct. The *direction of rotation* agrees with that of the motion of the satellites, and the equatorial plane must coincide almost exactly with that of their orbits, for otherwise the planes of their orbits would undergo considerable perturbations, and such have not been observed.

A striking confirmation of the spectroscopic period was furnished in 1917 by the photometric observations of Leon Campbell, which showed a variation in the brightness of Uranus amounting to 0.15 magnitude in a period of 10 hours and 49 minutes. It is probable that an accurate value of the period of rotation can be obtained by continuing the observations.

Several observers have reported extremely faint bands or belts upon the planet, much like the belts of Jupiter seen with a very small telescope. The earlier observers found that these belts were inclined to the satellites' orbit plane at a considerable angle (though they were so faint that it was difficult to be sure about it), but Lowell's later observations make the directions of the two practically coincident. No distinct markings, permitting a direct determination of the rotation period, have ever been observed.

470. Satellites. The planet has four satellites, — Ariel, Umbriel, Titania, and Oberon, — Ariel being the nearest to the planet. The two brightest, Oberon and Titania, were discovered a few years after the discovery of the planet, by Sir William Herschel, who thought he had discovered four others also; he may have glimpsed Ariel and Umbriel, but it is very doubtful. They were first certainly discovered and observed by Lassell in 1851.

These satellites are telescopically the faintest bodies in the solar system and the most difficult to see (excepting some of the recently discovered satellites of Jupiter and Saturn). Oberon and Titania are of about the fourteenth magnitude, the latter being a little the brighter, and they can be just glimpsed under the best conditions, with a telescope of ten or twelve inches

aperture. Ariel is much fainter, and Umbriel fainter still, and they are observable only with the very largest telescopes.

In real size they are probably comparable with the inner satellites of Saturn, which they would closely match in brightness if placed at the same distance from the sun. Titania may perhaps be 1000 miles in diameter, and Umbriel 400.

Their orbits are very nearly circular, and all lie in one plane, which, as has been said above, must be coincident with that of the planet's equator. They are very close-packed also, Oberon having a distance of 364,000 miles, with a period of $13^d\ 11^h$, and Ariel a distance of 119,000 miles and a period of $2^d\ 12^h$.

S

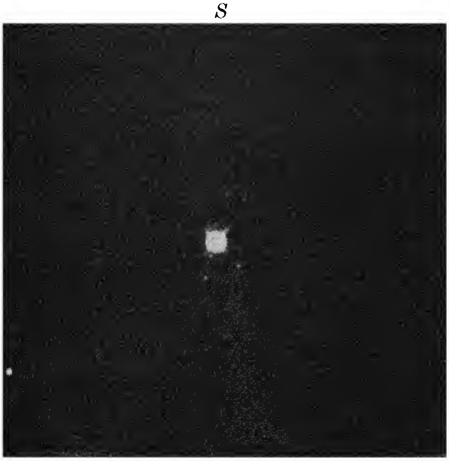


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FIG. 161. Uranus and its Four Satellites

Ariel and Umbriel (which is the fainter) are close to the planet and below it, Titania a little farther out in almost the same direction, and Oberon to the right of the other three. The remaining objects in the field are stars. The planet's disk is much enlarged by overexposure. (The photograph was made on September 6, 1915, by C. O. Lampland, at Lowell Observatory)

471. Plane of Revolution. The most remarkable thing about the system remains to be mentioned. The plane of their orbits is inclined 82°.2 to that of the ecliptic, and in that plane they revolve backward; these two statements may be combined (§ 281) by saying that the inclination is 97°.8 to the plane of the ecliptic, or 98°.0 to that of the planet's orbit.

When the line of nodes of the orbit plane passes through the earth, as it did in 1882 and again in 1924, the orbits of the satellites are seen edgewise and consequently appear as straight lines. On the other hand, in 1861 and 1903 these orbits were seen almost *in plan* as nearly perfect circles. Near the latter dates the planet's pole was about in the center of the disk, which

appeared perfectly round, while the former epochs were best for determining the inclination of the orbits, the position of the nodes, and the polar compression of the planet.

NEPTUNE

472. Discovery of Neptune. This is reckoned as the greatest triumph of mathematical astronomy since the days of Newton. After Uranus was discovered, it was very soon found impossible to reconcile the old observations of that planet, by Lemonnier and others, with any orbit that would fit the observations made in the early part of the nineteenth century; and, what was worse, the planet almost immediately began to deviate from the orbit computed from the new observations, even after allowing for the disturbances due to Saturn and Jupiter. It was misguided by some unknown influence to an amount almost perceptible by the naked eye; the difference between the actual and computed places of the planet amounted, in 1845, to the "intolerable quantity" of nearly two minutes of arc.

By modifying the elements of the planet's orbit, on which the computations were based, it proved possible to represent the observations from 1782 to 1845 with errors nowhere exceeding 20″. One might think that such a discrepancy between observation and theory was hardly worth minding; but the way in which observations by different astronomers, in successive years, agreed with one another and disagreed with theory made it certain that some unknown force must be acting on the planet. In exact science such unexplained but unquestionably real "residuals" are often the seeds from which new knowledge springs, and in this case the discrepancies supplied the data which sufficed to determine the position of a great world, theretofore unknown.

As a result of a most skillful and laborious investigation, Leverrier, a young French astronomer, wrote in substance to Galle, then an assistant in the Observatory at Berlin:

"*Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude 326°, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disk.*"

The planet was found at Berlin on the night of September 23, 1846, in accordance with this prediction, within half an hour after the astronomers began looking for it and within 52′ of the precise point that Leverrier had indicated.

An Englishman, J. C. Adams, fairly divides with Leverrier the honors for the mathematical discovery of the planet, having solved the problem and deduced the planet's approximate place even earlier than his competitor. Before Galle found the planet, Challis, at Cambridge,— to whom Adams had communicated his discovery,— was searching for it by the laborious method of observing the right ascensions and declinations of all the stars in the suspected region, and identifying the planet by its motion. He had actually secured two observations of it, and would have anticipated Galle by several weeks if he had mapped his observations as fast as they were made. The Berlin astronomers had the great advantage of a new star chart covering that region of the sky, so that they recognized the planet as an interloper at once, and confirmed its character by its sensible disk, and, within twenty-four hours, by its motion.

After the first approximate orbit of the new planet had been computed from a few weeks' observations, it was found, on reckoning backward, that it had been observed as a star several times by different astronomers in previous years, just as Uranus had been. (Neptune's disk is so small that its planetary appearance is revealed only by keen scrutiny.) These old observations were of use in determining the planet's orbit with accuracy.

Both Adams and Leverrier, besides calculating the planet's position in the heavens, had deduced elements of its orbit and a value for its mass, both of which turned out to be seriously incorrect. The reason was that they assumed that the new planet's mean distance from the sun would follow Bode's law, a supposition quite warranted by all the facts then known, but which, nevertheless, is not even roughly true. As a consequence their computed *elements* were erroneous, and that to an extent which has led some critics to declare that the mathematically computed planet was not Neptune at all, and that the discovery was merely a "happy accident." But this is not so. The methods of solution of the problem were not sufficient to derive, from the available data, elements of the planet's orbit which would give even roughly correct positions for all past and future time; but they were sufficient to give, with a very good degree of approximation, its *distance* and *direction* from the sun during the *time covered by the bulk of the observations*, and hence to inform observers where to point their telescopes, and this was all that was necessary for finding the planet. In a similar case the same thing could be done again.

473. The Planet and its Orbit. The planet's mean distance from the sun is 30.07 astronomical units, or 2,792,700,000 miles (instead of 3,600,000,000 as predicted by Bode's law). The orbit is very nearly circular, its *eccentricity* being only 0.0086. Even this, however, corresponds to a variation of 48,000,000 miles in the planet's distance from the sun. The *period* is 164.8 years (instead of 217, as indicated by Leverrier's computed orbit), and the *orbital velocity* is about $3\frac{1}{3}$ miles per second. The *inclination* of the orbit is $1° 47'$.

Neptune is invisible to the naked eye but can easily be seen with a good field-glass, appearing as a faint star of magnitude 7.7. With a small telescope it can only be distinguished from the neighboring stars by means of its motion from night to night, but with larger instruments it shows a greenish disk a little more than $2''$ in diameter. As in the case of Uranus the older measures made the diameter too large. Abetti, from a discussion of all the observations up to 1912, concludes that the diameter is $2''.3$, corresponding to 31,000 miles, or 3.92 times the earth's diameter.

The planet's mass is known with far greater percentage accuracy, both from its satellite and from its action on Uranus, and is 1/19,350 of the sun's mass, or 17.16 times that of the earth. With the diameter given above, the density comes out 0.29 times the earth's, or 1.6 times that of water, and the superficial gravity 1.12 times our own. The albedo, calculated from the observed brightness and this diameter, is 0.52, — a high value, and somewhat uncertain on account of the difficulty of measuring the diameter.

The *spectrum* shows the same bands which appear in the case of the other major planets, but much stronger (Fig. 208), indicating a very extensive atmosphere.

Menzel has suggested that the unknown substance responsible for these bands may be some compound which is completely decomposed at ordinary temperatures and can exist in quantity only where the atmosphere is very cold. This would account for the steady increase in the strength of the bands from Jupiter to Neptune, and might perhaps also explain why they have not been found in the laboratory.

The planet's disk is sensibly circular and shows no markings, so that there is no direct evidence concerning its rotation. It will be noticed that Uranus and Neptune form a "pair of twins,"

being remarkably alike in size, mass, density, and all other characteristics, and doubtless in physical constitution as well. They resemble each other much more closely than do the earth and Venus, — the other twins in the sun's family.

474. Satellite. Neptune has one satellite, discovered by Lassell within a month after the discovery of the planet itself. It moves in a sensibly circular orbit with a radius of 220,000 miles and a period of $5^d\ 21^h\ 2^m\ 38^s$. Its orbit is inclined about 40° to that of the planet, and its motion is *retrograde.* It is a very faint object (between the thirteenth and fourteenth magnitudes), though somewhat brighter than any of the satellites of Uranus. Its real size, however, must be considerable, for if brought to the distance of Saturn or Jupiter it would be nearly as bright as Titan, or as the first or second satellite of Jupiter. It is probable, therefore, that it is comparable with these bodies both in diameter and in mass, which would make it fairly similar in both respects to our own moon. Neptune's satellite has not been given a proper name.

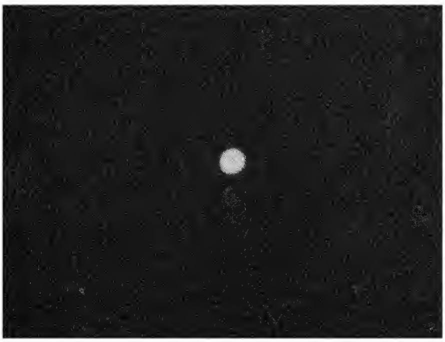


FIG. 162. Neptune and its Satellite

Photographed with the 40-inch refractor. The satellite is shown beneath the planet. (From photograph by E. E. Barnard, Yerkes Observatory)

The *plane of the satellite's* orbit shows a steady and remarkable motion, its position in 1900 making an angle of fully 10° with that which it occupied in 1850. The only known force that could cause so great a perturbation (in the absence of other satellites of any size) is the *attraction of an equatorial protuberance upon the planet.* That this explanation is correct is confirmed by the fact that the pole of the orbit plane is evidently describing a small circle in the heavens, that is, that the plane itself always makes the same angle with a fixed plane, on which its nodes move uniformly in the opposite direction to the satellite's motion. This fixed plane must

be the plane of the planet's equator (§ 342). From the observations so far available it appears that the satellite's orbit is inclined about 20° to the planet's equator, and the latter about 29° to Neptune's orbit, while a revolution of the nodes takes about 580 years (Eichelberger and Newton, 1926).

From these data it is possible to calculate the planet's oblateness, and its period of rotation, by making various assumptions regarding the ratio of the ellipticity to the centrifugal force at the equator. This ratio depends on the planet's internal constitution (§ 341).

If the internal constitution is assumed to be like that of Saturn, the oblateness comes out 1/32 and the rotation period 12 hours; if it is like Jupiter's, the results are 1/57 and $17\frac{1}{2}$ hours; if it is like the earth's, the results are 1/81 and $23\frac{1}{2}$ hours. The truth probably lies within this range and nearest to the middle set of values, and it is almost certain that Neptune rotates more slowly than the other three major planets and faster than the earth. The calculated ellipticity would be imperceptible in so small a disk, but the matter may soon be tested spectroscopically, as in the case of Uranus.

475. The Solar System as seen from Neptune. At Neptune's distance the sun itself would have an apparent diameter of a little more than 1′ of arc, — only about the diameter of Venus when it is nearest to us, and too small to be seen as a disk by the unaided eye (if there were eyes like ours on Neptune). The light and heat received from the sun by Neptune are only 1/900 of what the earth receives. Even so, the intensity of sunlight at Neptune would be 520 times that of full moonlight on the earth, or equal to that of a thousand-candle-power electric lamp at a distance of about ten feet, and hence abundant for all visual purposes. In fact, as seen from Neptune, the sun would look very much like an electric arc-lamp at a distance of a few yards.

As a source of heat, however, the sun would not amount to much. If the planet's surface were a good absorber and radiator, the solar radiation would only suffice to keep it at a mean temperature of about 51° absolute, or − 222° centigrade, and under these circumstances nitrogen would be a solid and oxygen a solid or a dense liquid (§ 612). The actual temperature of the planet's surface may be considerably raised by the escape of heat from the interior, but nevertheless is probably very low.

None of the other planets of the system could be seen nearly as well from Neptune as from the earth. To eyes like ours Jupiter

and Saturn, near elongation, would perhaps be visible as stars of the third and fifth magnitudes 10° and 17° from the sun. Venus and the earth would be almost as bright, but since they could at most get only $1\frac{1}{2}°$ and 2° from the sun, they would be invisible except during a total eclipse of the sun by Neptune's satellite. Mercury, Mars, and Uranus would never be visible to the naked eye, but the first two might be detected on photographs made during such an eclipse, and the last would be observable telescopically, since its greatest elongation would be 40°.

476. Possible Ultra-Neptunian Planets. A planet farther from the sun than Neptune, if of any size, would surely be found sooner or later, either by means of the disturbances that it would produce in the motions of Uranus and Neptune or by the methods of the asteroid hunters, — modified by comparing photographs taken on different days, since the trail during a single exposure would be very short.

Several attempts have been made to locate such a planet by study of the perturbations of Uranus. Neptune is unfortunately yet unavailable for such an investigation, for it has completed little more than a third of the circuit of its orbit since its discovery. Whatever might be the perturbations due to an unknown body, it is possible to modify the elements of the orbit in such a way that the motions of an imaginary planet, following the modified orbit without perturbations other than those due to the known planets, and of the real Neptune affected in addition by the attraction of the unknown body, will be almost exactly the same over the relatively short arc of the orbit covered by the observations, though the two may deviate widely from one another a revolution or two earlier or later.

Since the elements of a planet's orbit are determined so as to represent the existing observations as closely as possible, after allowing for the perturbations of known origin, the deduced results, if any unknown disturbing force exists in the case of Neptune, will be the modified elements just described; and the true character, and even the existence, of the perturbations may be concealed until the observations cover a complete revolution or more.

Uranus has been observed over nearly $2\frac{1}{2}$ revolutions, and there appear to be small unexplained irregularities in its motion, not exceeding 5″, but probably real. From these Gaillot, Lowell, and others have been led to suspect the existence of a planet distant from the sun about $1\frac{1}{2}$ times the distance of Neptune. Lowell finds that in this case there are two possible positions for the unknown body, almost 180° apart in longitude, showing that the

solution of the problem is not always unique, as it was in the case of the discovery of Neptune. The mass of the hypothetical planet comes out less than half that of Neptune; but, even on the most unfavorable assumptions regarding its density and albedo, it ought to be of about the twelfth magnitude, and a conspicuous object on long-exposure photographs, so that, if it really exists, it is rather surprising that it has escaped discovery so long.

Gaillot suspects another and larger planet, rather more than twice as far from the sun as Neptune.

One thing, at least, is definitely proved by these investigations. If there were a planet as large as Neptune within twice the distance of Uranus, or one as large as Jupiter within $2\frac{1}{2}$ times the distance of Neptune, it would produce much larger disturbances in the motion of Uranus than have been observed; and hence no unknown planets exist unless they are considerably smaller.

EXERCISES

1. When Jupiter is visible in the evening, do the shadows of his satellites precede or follow the satellites as they cross the planet's disk?

2. On which limb, the eastern or the western, do the satellites appear to enter upon the disk?

3. How would the brightness of sunlight at the distance of Saturn compare with sunlight on the earth?

4. What would be the greatest elongation of the earth from the sun as seen from Jupiter? from Saturn? from Uranus?

5. What would be the apparent angular diameter of the earth when transiting the sun as seen from Jupiter?

6. What is the rate in miles per hour at which a white spot on the equator of Jupiter, showing a rotation period of $9^h\,50^m$, would pass a spot with a period of $9^h\,55^m$?

7. Find the diameter, volume, density, and surface gravity of Neptune, taking the apparent diameter of the planet as $2''.3$, the planet's mass as 17 times that of the earth, the solar parallax as $8''.80$, and the mean distance of Neptune from the sun as 30.07.

CHAPTER XIII

COMETS AND METEORS; ORIGIN OF THE SOLAR SYSTEM

COMETS: NUMBER AND DESIGNATION · ORBITS AND ORIGIN · CONSTITUENT PARTS, DIMENSIONS, AND MASS · ORIGIN OF THEIR LIGHT · CONSTITUTION AND DEVELOPMENT · METEORS: SHOOTING STARS AND FIRE-BALLS · NUMBER, VELOCITY, AND LENGTH OF PATH · EXPLANATION OF LIGHT AND HEAT · METEORITES, THEIR APPEARANCE AND ANALYSIS · METEORIC SHOWERS AND RADIANTS · ORIGIN OF METEORS AND THEIR CONNECTION WITH COMETS · ORIGIN OF THE SOLAR SYSTEM: ITS REGULARITIES · ATTEMPTED EXPLANATION BY THE NEBULAR HYPOTHESIS · HYPOTHESIS OF DYNAMIC ENCOUNTER · PLANETESIMAL AND TIDAL THEORIES · ORIGIN OF THE MOON

477. Comets are very different from the stars and planets. They appear from time to time in the heavens, remain visible for some weeks or months, pursue a longer or shorter path, and then fade away in the distance. They are called *comets* (from *coma*, "hair") because when one of them is bright enough to be seen by the naked eye, it looks like a star surrounded by a luminous fog and usually carries with it a large stream, or tail, of hazy light.

Large comets are magnificent objects, sometimes as bright as Venus and visible by day, with a dazzling nucleus and a nebulous head as large as the moon, accompanied by a train which extends perhaps halfway from the horizon to the zenith and sometimes is really long enough to reach from the earth to the sun. Such comets are rare, however; the majority are faint wisps of light, visible only with the telescope.

In ancient times comets were always regarded with terror, either as actually exerting malignant influences or, at least, as ominous of evil; and the notion still survives in certain quarters, although the most careful research fails to show, or even suggest, the slightest physical reason for it.

478. Number of Comets. Up to 1925 we have on our lists nearly 900 comets, including the different returns of the periodic ones. About 400 of these were recorded before the introduction of the

telescope in 1609, and therefore must have been bright. With the improvement of telescopes and the multiplication of observers

FIG. 163. Donati's Comet

A naked-eye view, October 4, 1858. (From a drawing by Bond)

the rate of discovery has steadily increased. During the last half of the eighteenth century it averaged about one comet per year. Since 1880 the average rate has been a little over five per year, of which 70 per cent were "new" and 30 per cent were returns of

comets previously known. The greatest number detected in a single year was eleven, in 1925. Most of these later discoveries are faint objects, often visible only with large telescopes.

The total number of comets must be enormous, for, even with the telescope, we can see only those which are favorably situated for observation, and many of the fainter ones certainly escape discovery. It may be estimated that at least a thousand, and probably more, visit the neighborhood of the sun every century. There is seldom a night when one is not telescopically visible somewhere in the sky; often there are several.

479. Designation of Comets. A remarkable comet generally bears the name of its discoverer or of someone who has acquired ownership of it, so to speak, by some important research respecting it. Thus, we have Halley's, Encke's, and Donati's comets. The common herd of comets are distinguished only by the year of discovery with a letter indicating the order of *discovery* in that year, as comet 1895*a*, *b*, *c*; or, again, by the year with a Roman numeral denoting the order of *perihelion passage*. Thus, Donati's comet, which is "comet 1858*f*" is also "comet 1858 VI." The latter is the more useful designation and is generally used in catalogues of comets.

Comet *a* is not always comet I, for comet *b* may outrun it in reaching the perihelion. It often happens that a comet's perihelion passage does not occur in the same year as its discovery.

In some cases a comet bears a double name: as, for instance, the Pons-Brooks comet, which was first discovered by Pons in 1812, and on its return in 1883 by Brooks.

480. Discovery of Comets. As a rule these bodies are first found by observers who make a business of searching for them. For this purpose they usually employ a telescope (known as a "comet-seeker") carrying an eyepiece of low power, with a large field of view, with which they "sweep" backward and forward across the sky in such a way that each strip of sky overlaps a little upon that last reviewed. When first seen, a comet is usually a mere roundish patch of faintly luminous cloud, resembling the nebulæ with which the heavens are strewn (§ 2). If really a comet, it will reveal its true character within an hour or two by its motion.

Some observers have found a great number of these bodies, the record being 27, by Pons. Many have been found by amateurs with small telescopes. Occasionally a bright comet is picked up with the naked eye by someone not an astronomer at all, a notable instance being the great comet of January, 1910, which was first seen by three South African railroad workmen.

Recently several have been discovered by photography, — the first by Barnard at the Lick Observatory in 1892, — and in more than one case, upon searching in the position calculated after the orbit became known, images of a comet have been found on photographs taken before its discovery. Halley's comet was photographed in 1909 before it could be observed visually.

481. Duration of Visibility and Brightness. The great comet of 1811 was observed for seventeen months, and that of 1861 for nearly a year. With the more powerful telescopes of the present day, comets can be followed longer. Comet 1889 I was followed for more than two years and a half, up to a distance of more than eight astronomical units from the sun, and the observations of comet 1905 IV, including photographs taken more than a year before its discovery, extend over three years and a half. In most cases, however, comets are lost to sight after a few months; and when one is not discovered until it is receding from the earth or the sun, it is sometimes observable for only a few weeks or days.

On two occasions a comet near the sun has been photographed during a total eclipse, and never seen before or after the few minutes of totality.

As to *brightness*, comets differ widely. Some are barely visible with large telescopes. The majority known are observable with smaller instruments, — which might be expected, since most of the discoverers have worked with relatively small telescopes. About one in five (to judge by the experience of the past thirty years) becomes visible to the naked eye for at least a few days. Fifteen or twenty in a century become so conspicuous that a casual star-gazer could hardly fail to notice them, and a very few (perhaps two in a century, on the average) are visible even in broad daylight when near the sun. The last comets so observed were 1843 I, 1882 II, and 1910 I.

Eighteen comets are recorded as having been visible to the naked eye during the first twenty-five years of the twentieth century, and at least six of these were conspicuous. Three of them were magnificent objects: 1901 I (visible only in the southern hemisphere), 1910 I, the great "daylight comet," and Halley's comet in 1910.

ORBITS OF THE COMETS

482. The ideas of the ancients as to the motions of these bodies were very vague. Aristotle and his school considered them to be merely exhalations from the earth, inflamed in the upper air, and therefore meteorological bodies and not astronomical at all. Seneca, indeed, held a more correct opinion, but it was shared by few, and Ptolemy, in his *Almagest*, fails to recognize them as heavenly bodies.

Tycho was the first to establish their rank as truly celestial by comparing the observations of the comet of 1577, made in different parts of Europe, and showing that its parallax was less, and its distance therefore greater, than that of the moon.

Kepler supposed that they moved in straight lines, and seems to have been more than half disposed to consider them as living beings, traveling through space with will and purpose, "like fishes in the sea."

Hevelius in 1675 was the first to suggest that their orbits might be parabolas, and his pupil Doerfel proved this to be true for the comet of 1681. The theory of gravitation had now appeared, and Newton soon worked out and published a method by which the elements of a comet's orbit can be determined from the observations.

483. Character of the Orbits. A large majority of comets move in orbits which are very nearly *parabolas*. Out of nearly four hundred orbits computed up to 1910 almost three hundred are of this sort. About a hundred are distinctly *elliptical*, and some twenty are perceptibly *hyperbolic*, though all the latter differ only very slightly from parabolas. This statement, however, is likely to lead to misconceptions unless two things are borne in mind.

First, the orbits listed as parabolic are not exactly so. All that can be said is that the comet did not deviate from motion in

the computed parabola by an amount sensible to observation during the time that it was visible. Now most comets are visible only in those very small portions of their orbits which lie near the earth and the sun, and in this portion of the orbit a parabola, an elongated ellipse, and a hyperbola may almost coincide (Fig. 164).

When an "observed arc" is short, it is therefore usually possible to satisfy the observations, within reasonable limits, by means of a variety of orbits, — elliptical, parabolic, or hyperbolic. In this case the parabolic orbit, which is the easiest to compute, is adopted. An orbit is described as elliptical or hyperbolic only when the observations cannot be satisfied by a parabola. Evidently, the longer the observed arc is, the more favorable is the opportunity of finding the real character of the orbit; and this is fully confirmed by experience. Among the best-observed comets elliptic or hyperbolic orbits are found to be the rule, and those indistinguishable from a parabola the exception.

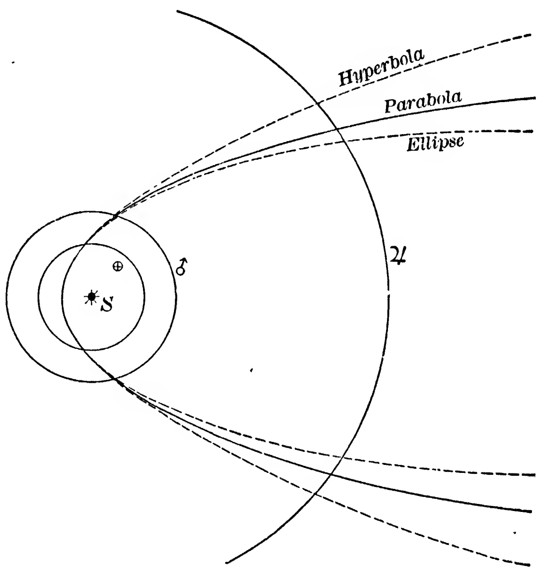


FIG. 164. The Close Coincidence of Different Species of Cometary Orbits within the Earth's Orbit

The earth's orbit is indicated by the sign ⊕

Second, the orbits given by the computers are what are called "osculating" orbits; that is, they are the orbits that each comet would pursue if, at some specified date near the middle of the observations, *all the planets should cease to attract the comet*, leaving it free to move under the influence of the sun's attraction alone.

As things actually are, the attraction of the planets is continually altering the comet's motion, — perhaps even changing the orbit from an ellipse into a hyperbola, or vice versa.

The orbit along which a comet approached the sun will therefore always be somewhat different from the osculating orbit near perihelion, and may be very decidedly different as regards the major axis, eccentricity, and period. The other orbital elements are usually little changed. The orbit along which it finally leaves the sun will again be different from either of the others.

Unless the calculations to determine these differences have been made (which has been done in very few cases), conclusions drawn from the tabulated aphelion distances or periods of comets whose calculated periods are more than a few thousand years are liable to grave uncertainty.

Thus, for example, the osculating orbit of Coggia's comet, 1874 III, gives a period of 13,700 years, but according to Fayet's calculations the orbit in which it approached the solar system corresponded to a period of 5100 years.

The exact determination of the period, and the discrimination between a parabola, an elongated ellipse, and a hyperbola, require a careful and very laborious discussion of all the available observations, with proper consideration of all the perturbations (§ 332). The calculation of such a definitive orbit is a heavy piece of work, which is not undertaken till the comet has disappeared and all the observations of it have been published.

484. The Elliptic Comets. Comets which leave the neighborhood of the sun and planets in parabolic or hyperbolic orbits return no more to the solar system. Of the comets that have distinctly elliptical orbits, about fifty have been found to have periods less than a hundred years, and twenty-five of these have actually been observed (up to 1925) at two or more returns to perihelion, — two of them at 25 and 34 returns, respectively. There are about twenty other comets with calculated periods ranging from 100 to 1000 years, and thirty more with periods between 1000 and 10,000 years.

These longer periods are, of course, very uncertain. When a comet has been well observed with modern instruments over a period of two or three months, it is possible to determine its period. If the period is short, — say six or eight years, — the

uncertainty is a week or two; for a period of fifty or a hundred years it is a year or more; and for a thousand-year period it is likely to be a century. It is therefore not surprising that most of the periodic comets observed at a second appearance have been independently discovered, as new objects, and only recognized latér as returns of known ones. The recognition often takes some time, for a comet has no individual identity by which it may be recognized merely by looking at it — no striking individual peculiarities like those of the planets Jupiter and Saturn. It is identifiable only by its path.

485. The Short-Period Comets. Of the 54 comets of period less than a century which were known in 1925, 41 have periods between three and nine years in length, and the periods of 36 fall in the much narrower interval between five and seven and one-half years. These short-period comets form a distinct group. Their motions are all direct, and the inclinations of their orbits small, — only three exceeding 30° and the average being 14°. They are all relatively faint objects, few of them being visible to the naked eye, and that only when they come unusually near the earth. A few have at times developed short tails, but most of them have no tails at all.

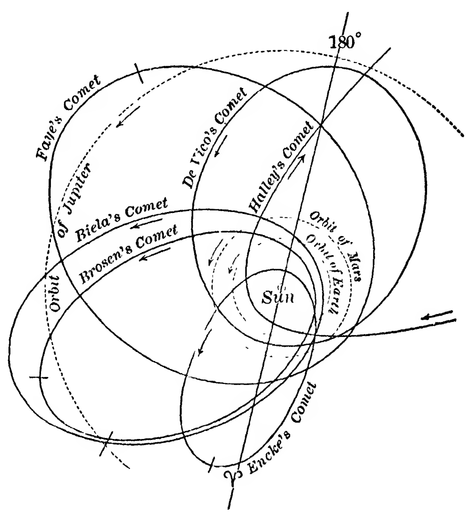


FIG. 165. Orbits of Short-Period Comets whose Aphelia and Nodes are near the Orbit of Jupiter

Fig. 165 shows the orbits of several of these comets (it would cause confusion to insert all of them). It will be seen that in every case (except Halley's comet) the comet's aphelion is not far from Jupiter's orbit, and that one of the nodes (which are marked on the orbits by short cross-lines) is still nearer. It follows that the orbit of each of these comets comes close to that of Jupiter in space, so that if the two bodies pass near the point of

closest approach at the same time, they will be very near one another. This relation holds good not merely for the orbits shown in the figure but for all comets with period less than nine years. No less than twenty-four of these have orbits which pass within 15,000,000 miles of Jupiter's orbit. Eleven others come within 50,000,000 miles. For the remaining six the minimum distances are from 51,000,000 to 85,000,000 miles. The average distance for all is 22,000,000 miles.

486. The Capture Theory. There must clearly be some reason why there should be so many comets with periods of about this length, and why their orbits should show such distinctive characteristics. Jupiter must be responsible in some fashion, for the relations between their orbits and his cannot be due to chance. (If the orbits were distributed at random in space, only one in ten would pass within 25,000,000 miles of Jupiter's orbit.)

Laplace was the first to suggest that these comets have been "captured" by Jupiter, that is, that they were originally moving in parabolic or nearly parabolic orbits, and have been diverted into their present paths by the effects of an encounter with the planet.

If a comet, moving in a sensibly parabolic orbit, passes very near to Jupiter, its velocity, relative to the sun, will be considerably altered by the planet's attraction. If it is accelerated, it will go off in a hyperbola; but if it is retarded, the orbit will become elliptical, and the comet will return at regular intervals, moving in a path which, of course, passes through the point where the disturbance took place (§ 322).

H. A. Newton has shown that a parabolic orbit might in this way be transformed, at a single encounter, into an elliptical orbit with a period as short as five years, if only the comet passed within about 200,000 miles of Jupiter. Such close approaches would obviously be rare. The change might take place, however, in two or more stages; a comet might first be diverted into an ellipse of long period, and later, at a subsequent encounter, again retarded, with a further shortening of the period. Newton has shown that a parabolic comet having direct motion is much more likely to be captured than is a retrograde comet. This would explain why the short-period comets have direct motion, as a sort of survival of the fittest, parabolic comets with retrograde motion usually escaping.

There are several groups of two or more short-period comets whose orbits are so situated that it would be possible for all the

comets of one group to be simultaneously very near one another and near Jupiter also. It is not unlikely that each of these groups has been produced by the disruption of a single comet at some past epoch.

487. Capture of Brooks's Comet. Several instances of the profound alteration of a comet's orbit by an encounter with Jupiter have actually occurred. Brooks's comet (1889 V) was a faint but well-observed telescopic comet, and was seen again in 1896, 1903, 1910, and 1925, its period being 7.10 years. The calculations of Chandler and C. L. Poor show that on July 20, 1886, the comet made an exceedingly close approach to Jupiter, — passing inside the orbit of the fifth satellite and only 55,000 miles from the planet's surface. Before this encounter its period was 29.2 years and its perihelion distance 5.5 astronomical units, — almost exactly the same as the aphelion distance afterwards. At its first appearance it was accompanied by four faint companions, which appear to have separated from it at the time of the close approach to Jupiter.

It is clear from the foregoing that most of the short-period comets can hardly be regarded as permanent members of that group, since their orbits may be radically altered by encounters which are almost certain to occur, sooner or later.

488. Acceleration of Encke's Comet. The motions of almost all the comets appear to be just what would be expected of masses moving in free space under the laws of gravitation. But there is one remarkable exception. Encke's comet, a rather faint object, just visible to the naked eye under the most favorable conditions, is noteworthy for its very short period of 3.30 years, — the least so far known. It was seen in 1786, and again in 1795 and 1805, but its periodicity was first recognized by Encke in 1819. Since that date it has been observed at every one of the thirty-one succeeding returns (up to and including 1924). Encke found that after exact allowance was made for the perturbations due to the attraction of the planets (which sometimes alter the period by as much as a week) there remained outstanding a steady shortening of the period, which could not be explained by the attraction of any known body; and this has been fully confirmed by later researches. Between 1819 and 1914 the period has shortened by

almost two days and a half, corresponding to a diminution of the mean distance by 275,000 miles. The only reasonable explanation of this change is that the comet does not move freely through interplanetary space, but meets with resistance of some sort which retards its motion.

At first sight it seems almost paradoxical that a resistance should shorten the comet's period, but any diminution of the velocity must necessarily diminish the major axis (§ 315), and this leads to a diminution of the period. Indeed, the percentage change in the period is greater than that in the circumference of the orbit, so that, over most of the orbit, its speed is actually increased. It gains more speed *by falling nearer the sun* than it loses by the direct effect of the resistance.

North

FIG. 166. Encke's Comet

Exposure, 1^h 53^m with the 10-inch Bruce doublet, October 29, 1914. It is difficult to guide on a comet, — to keep the patch of nebulous light accurately bisected by the cross-wires, — and the star-trails, therefore, are sometimes ragged. The stars trail, of course, because of the orbital motion of the comet. (From photograph by E. E. Barnard, Yerkes Observatory)

Backlund, from a very careful study of all the apparitions of the comet, finds that the resistance to its motion has decreased, apparently almost abruptly, several times, and he has also shown that the retardation does not take place uniformly all around the orbit, but occurs in a relatively narrow region not far from perihelion.

The comet's aphelion is at present 0.9 astronomical unit inside Jupiter's orbit, so that it cannot now come near enough to Jupiter to undergo any very great perturbations. But Backlund has

shown that (if the resistance to its motion was the same in the past as at the time of discovery) the comet's aphelion was close to Jupiter's orbit about 6000 years ago. It may have been captured by Jupiter at that time, and then saved, by the slow shrinkage of its orbit, from the danger of being sent away into space by another encounter.

489. The comets of longer period differ in several respects from the group just discussed. Many of them have been *conspicuous objects*, far brighter than any short-period comet. Their orbital *inclinations are high*, and the *eccentricities are large*. Several of them are *retrograde*.

Not one of the thirty-six comets whose periods lie between 10 and 1000 years comes nearer than 19,000,000 miles to Jupiter's orbit, and only eight approach it within 50,000,000 miles. The situation with respect to the other major planets is similar.

There appears, therefore, to be little or no evidence that these comets of longer period owe the elliptical character of their orbits to encounters with the planets, unless, indeed, they were captured so long ago that the gradual accumulation of minor perturbations in the interval has shifted their orbits clear away from the original points of encounter. They are probably much more nearly permanent members of our system than are the short-period comets.

490. Halley's Comet. This, in many respects the most famous of all comets, deserves special description.

It was the first periodic comet whose return was predicted. Halley based his prediction upon the fact that he found its orbit in 1682 to be nearly identical with those of the comets of 1607 and 1531, which had been carefully observed by Kepler and Apian; and he also found records of the appearance of bright comets at similar intervals in 1456, 1301, 1145, and 1066. He noticed, of course, that the two intervals between 1531 and 1607, and between 1607 and 1682 were not quite equal, but he had sagacity enough to see that the differences were no greater than might be accounted for by the attractions of Jupiter and Saturn.

Though it was not then possible to compute just what the effect of these attractions would be upon the return of the comet, he saw that Jupiter's action would retard it, and he

accordingly fixed upon the early part of 1759 as the time at which the comet might be expected. Before that date, however, mathematics had so advanced that the necessary calculations

FIG. 167. Halley's Comet

Photographed May 13, 1910, with Cooke lens of 8 inches focal length; exposure, 35 minutes. This photograph shows the planet Venus (on the right) and a faint meteor trail across the comet's tail about 7° from the head. The long trails in the right-hand corner are from artificial lights. Length of comet, 52°. (From photograph at Lowell Observatory)

could be made. Clairaut, after a most laborious investigation, fixed upon April 13 for the perihelion passage, but remarked that this result might easily be a month out of the way on account of the uncertainty of the masses of the planets (Uranus and Neptune were then unknown). The comet actually came to perihelion on March 13. At this return it was best seen in the southern hemisphere, and had at one time a tail 50° long. At its next return in 1835 it came within two days of the predicted time. It did not appear extremely brilliant but was fairly conspicuous, with a tail about 10° in length.

Its latest return to perihelion occurred on April 20, 1910. Astronomers knew beforehand exactly where to look for the comet, and it was detected photographically by Wolf, at Heidelberg, on September 11, 1909, while still 310,000,000 miles from the sun and a little farther from the earth. It was found later that it appeared on a photograph taken at Helwan, Egypt, on August 24. On its return journey it was followed photographically till July 1, 1911, when it was 520,000,000 miles from the sun. It was lost to sight visually about a month earlier.

On May 19, 1910, a month after the perihelion passage, the comet passed directly between the earth and the sun, transiting the sun's disk (§ 506). Its minimum distance from the earth, 14,300,000 miles, was reached on the following day. During the early part of May it was a magnificent object in the morning sky, growing larger and brighter day by day as it approached the earth, until, a week before the transit, its head was as bright as the brightest stars, and its tail 60° long. After the sixteenth the head was too near the sun to be seen, but the tail continued to be visible in the morning sky, before the head had risen, as a great band of light, almost as broad and bright as the Milky Way, extending clear across the heavens to a distance of more than 120° from the head. For two days after the comet's head had passed to the opposite side of the line joining the earth and sun the tail remained visible in the east, showing that it must have been strongly curved and must have lagged far behind the comet's radius vector (Fig. 169).

It is probable that the earth grazed the edge of the tail, or perhaps passed through it, on May 21, 1910.

After this the comet appeared in the evening sky (as a conspicuous object but not nearly so fine as it was when visible in the morning), but rapidly grew smaller and fainter as it receded from the earth.

491. Past Returns and Variations of Period. The motion of Halley's comet in the past has been made the subject of extensive theoretical researches by Cowell and Crommelin of Greenwich. They computed the return of 1910 with great accuracy by starting with the known position in 1835 and working forward to 1910 and backward to 1759, to connect with the older observations of that year. They have determined the time of every perihelion passage of the comet back to 240 B.C.

For every one of the twenty-seven returns between 87 B.C. and A.D. 1910 there is a definite record, in the ancient chronicles, of the appearance of a comet at the proper season and in the right quarter of the sky. In some cases the records are detailed, and suffice to prove that the comet not only appeared at the right time but followed an orbit very similar to that of Halley's comet.

Some of the observations and records were made in Europe, others in China, and an account of the one missing apparition (A.D. 912) has been found in an old Japanese manuscript.

It is therefore evident that this comet must have been a conspicuous object at every one of its returns, and also that the old chronicles of the appearance of comets are surprisingly accurate and complete.

The period from one perihelion passage to the next has varied by nearly five years on account of perturbations, the interval ending A.D. 530 being the longest, and that ending in 1910 the shortest. The difference between the lengths of successive periods averages a year and a half, and in one case reaches three years, but the actual intervals are never more than two and a half years on either side of the mean period of 77 years.

492. Comet Families. The short-period comets are often described as belonging to "Jupiter's family." The statement appears in most textbooks that Saturn, Uranus, and Neptune also have less numerous families of comets with periods about half their own. But only one, out of the three comets assigned to Saturn and the two attributed to Uranus, comes within

50,000,000 miles of the orbit of either planet, and not one of the eight comets supposed to belong to Neptune can ever get within 350,000,000 miles of it if they follow their present orbits. It seems, therefore, very unlikely that any of these eight comets has ever been captured by Neptune. Saturn and Uranus may be responsible for a comet or two, but very few in comparison with Jupiter.

According to the capture theory the great majority of the comets captured by any planet should undergo much less than the maximum possible retardation. If their orbits were originally parabolic, their periods after capture would usually be longer than that of the planet, the very short periods representing exceptional cases. Since Jupiter's family appears actually to consist almost entirely of comets of very short period, it is not unlikely that the material of which they were composed was moving, before its capture, not in parabolic orbits but in elliptical orbits within the solar system.

493. Comet Groups. There have been several instances in which a number of comets, certainly distinct, have followed one another along almost exactly the same path, at intervals varying from a few months to many years. The existence of such groups was first pointed out by Hoek, of Utrecht, in 1865.

The most remarkable group of the sort is composed of the great comets of 1668, 1843, 1880, 1882, and 1887. These are noteworthy for their very small perihelion distances, — 510,000 to 720,000 miles. They passed within 300,000 miles of the sun's surface and *through the corona* with a velocity of about 500 kilometers per second. (Only one other comet, that of 1680, has ever come anything like so near the sun, and its orbit is entirely different from the others.) Their orbits are exceedingly elongated, so that the comets enter the solar system along nearly straight lines, and all from almost exactly the same direction in space, though their orbit planes are inclined a few degrees to one another.

Yet, although the elements of these five comets are almost identical, they cannot possibly all be appearances of the same body. The comets of 1843 and 1882 were very brilliant and were accurately observed. In both cases the orbits are unquestionably elliptical but of long period, that of the first being probably between 400 and 800 years, and that of the second, before it broke

up (see below), between 800 and 1000 years. Neither of them can possibly have been a return of the comet of 1668. The comets of 1880 and 1887 were much fainter, with enormous tails but inconspicuous heads, and the computed orbits are less accurate. Even if one of them should prove to be a return of the comet of 1668 (which cannot be definitely decided, in view of the inaccuracy of the early observations), there must be at least four different comets in the group. Kreutz, who has studied this group with great care, has shown that a bright comet seen, but never accurately observed, in 1702 may belong to it, and also the little comet photographed at the solar eclipse of May 16, 1882, but never seen again.

The comet of 1882 (by far the greatest of the group) showed, before the perihelion passage, a single nucleus; this afterwards split up into four parts, which gradually drew away from one another along the line of the orbit. Kreutz has computed orbits for each of these portions, and finds periods of 664, 769, 875, and 959 years. It appears, therefore, that this comet will return as *four* great comets, about a century apart. This suggests that the existing comets of the group have been similarly formed in the past by the division of a single huge comet.

Many other groups of comets with more or less similar elements have been noticed, and in eight or ten cases the likeness is really striking; but no other group is as remarkable as that already described.

The distinction between comet families and comet groups must be carefully noted. In the former the orbits agree only in passing close to that of the capturing planet; in the latter the orbits are nearly identical, at least in the part near the sun.

494. The Parabolic Comets. Coming now to the majority of those comets with orbits which differ little, and often imperceptibly, from parabolas, it is found that their *orbit planes* are distributed almost at random, and that their perihelion distances vary greatly. Of the comets for which good orbits have been computed, 63 per cent have perihelion distances less than 1 astronomical unit; 32 per cent, perihelion distances between 1 and 2 astronomical units; 4 per cent, between 2 and 3; and only 1 per cent, greater than 3. For bodies moving at random the

numbers in the first three intervals should be equal. The observed differences probably mean that remote comets are not likely to be discovered. If allowance is made for the fact that comets of smaller perihelion distances may be unfavorably placed and so miss discovery, it is probable that not more than 1/8 of those which come within 5 astronomical units of the sun are picked up. The greatest observed perihelion distances are 4.05 for the comet of 1729, and 4.18 for the comet 1925*a*.

495. The Hyperbolic Comets. There are about twenty comets whose orbits appear to be more or less certainly hyperbolic, and this number is steadily increasing as more comets are accurately observed.

With regard to their inclinations, perihelion distances, and the like, they resemble the majority of parabolic comets. The greatest deviation from a parabola occurs in the case of comet 1886 III, which has an eccentricity of 1.0130; but this comet was observed for only 33 days, and its orbit cannot therefore be determined with the highest degree of precision. According to Strömgren there are only eight cases (up to 1914) in which the deviation from a parabola is so great, in comparison with the observational uncertainty, as to be unquestionably real. These hyperbolic orbits are, however, *osculating* orbits (§ 332), and the computations of Strömgren and Fayet show that in every one of the eight cases the comet's velocity had been increased by the attraction of Jupiter and Saturn while it was approaching the sun, and that the hyperbolic character of the observed orbits was due to this cause alone. In most cases the original orbits at a great distance from the sun were distinctly elliptical, and in the others the assumption that the original orbit was an ellipse of very long period was consistent with the observations, though not demanded by them. There remains, therefore, no conclusive evidence that any comet has ever *approached* the sun along a hyperbolic orbit. The observed hyperbolas have all been produced by planetary perturbations within a very few years before the perihelion passage. Unless, however, the attraction of the planets retards the motions of these comets, while they are retreating from the sun, to almost the same extent that they have previously been accelerated, they will *leave* the

solar system in really hyperbolic orbits, and depart into interstellar space, never to return. Fayet's calculations indicate that on the outward journey they are more likely to be retarded than to be accelerated (on the inward journey the reverse is the case). But nevertheless it seems probable that many of the hyperbolic comets will never be seen again.

496. Comets are Members of our System. There has been much discussion of the question whether comets have originated in our system or have come as visitors from interstellar space. Recent investigations point very strongly to the former conclusion.

The solar system is traveling through space at the rate of about twenty kilometers per second (§ 740). A body moving independently in space, if it came near the sun, would usually have a conspicuously hyperbolic orbit, quite unlike that of any known comet.

Strömgren's work, showing that there is not a single unquestionable case of *approach* along a hyperbolic orbit, leads to the conclusion that the comets so far observed have always been associated with our system and are not visitors from the depths of space.

The period between successive returns of the average comet must be many thousands of years, although, owing to perturbations, the successive revolutions will not be even roughly of equal length. The present rate of discovery of "new" comets is about 300 per century. Probably 2500 per century come to perihelion at distances less than that of Jupiter. The total number of comets in the solar system must be extremely great, — probably several hundred thousand.

Our system must, however, be gradually but steadily losing its comets, for in every century some are diverted into hyperbolic orbits by perturbations, and never return. The chances that such a wandering comet would pass near another star, or, conversely, that a comet, escaped from some other system, would pass near enough to the sun to be seen, are vanishingly small. If this exceedingly unlikely thing should happen, the time of passage from one star to another would probably be many millions of years.

THE COMETS THEMSELVES

497. Physical Characteristics of Comets. The orbits of these bodies are now thoroughly understood, and their motions are calculable with as much accuracy as the nature of the observations of these hazy bodies permits; but we find in their physical constitution and behavior some problems which have not yet received a satisfactory explanation.

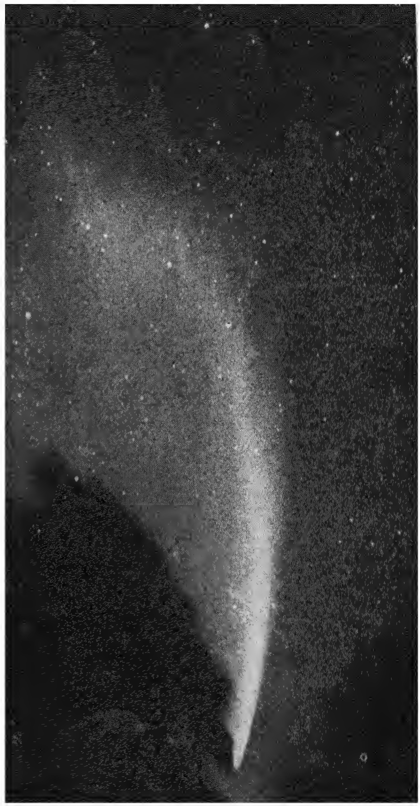


FIG. 168. Comet 1910*a*

Photographed January 26, 1910; exposure, ten minutes. The sharp curvature and intricate structure of the outer part of the tail are unusual. The dark objects in the lower part of the picture are pine trees, blurred by the motion of the camera in following the comet. (From photograph by C. O. Lampland, Lowell Observatory)

While comets are evidently subject to the attraction of gravitation, as shown by their orbits, they also exhibit evidence of being acted upon by powerful *repulsive* forces emanating from the sun. While they shine partly by reflected light, they are also certainly *self-luminous*, their light being generated in a way not yet thoroughly explained. They are the *bulkiest* bodies known, except the nebulæ, in some cases thousands of times larger than the sun or stars; but in mass they are "airy nothings," and one of the smaller asteroids probably rivals the largest of them in weight.

498. Photography of Comets. Much of our recent advance in knowledge regarding the physical characteristics of comets is due

to the application of photography, which often reveals details wholly invisible to the eye.

The first photograph of a comet was obtained by Bond in 1858. The next was in 1881, and the great comet of 1882 was well photographed by Gill in South Africa. Since then every comet of any account has been extensively photographed. Doublet lenses, giving a wide field, must be used, and the comet's head kept near one corner of the plate, to give room for the tail. Since the guiding telescope attached to the camera is kept pointed at the comet's head, which is moving more or less rapidly among the stars, the star images during a long exposure are drawn out into streaks parallel to the direction of the comet's motion, as may be seen in several of the illustrations in this chapter.

In the study of the *spectra* of comets, and of the spectra of all other faint objects, photography has a great advantage over visual observations.

499. The Constituent Parts of a Comet. (1) The essential part of a comet (that which is always present and gives it its name) is the *coma*, or nebulosity, — a hazy cloud of faintly luminous transparent matter, which is usually roughly circular or oval in outline, but not always so (Fig. 166).

(2) There is often a *nucleus*, — a bright, more or less star-like point near the center of the coma, which, when present, is the object pointed upon in determining the comet's place by observation.

Some comets, however, show no nucleus, and in others its place is taken by a more or less diffuse "condensation" of the light of the coma.

In most cases the nucleus makes its appearance only when the comet is near the sun, though some recently discovered comets have shown sharp nuclei when at great distances. In a few rare cases the nucleus is double, or even multiple as in the great comet of 1882.

(3) The *tail* is a stream of light which ordinarily accompanies a bright comet and is often found even in connection with a telescopic one. The tail follows the comet as it approaches the sun, but precedes the comet as it recedes from the sun. The broad statement may be made that the tail is always *directed*

away from the sun, although its precise form and position are determined partly by the comet's motion. It is practically certain that the tail consists of extremely rarefied matter, which is thrown off by the comet and then powerfully repelled by the sun.

Thus the tail may make any angle whatever with the direction of the comet's motion, just as the smoke-trail of a slow steamer at sea, consisting of fine sooty particles emitted from the funnels and carried by the wind, stretches always to leeward but may make any angle with the vessel's course.

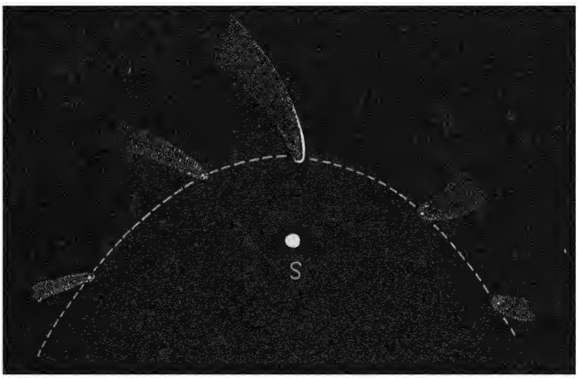


FIG. 169. The Tail of a Comet is directed away from the Sun

There is no sharp distinction between the coma and the part of the tail nearest the comet; one forms the continuation of the other.

(4) The head of a brilliant comet is often veined by *jets* of light, which appear to be continually emitted by the nucleus; and sometimes it throws off a series of concentric *envelopes* like hollow shells, one within the other (Fig. 173). These phenomena, however, are seldom observed in any but bright comets.

500. Dimensions of Comets. The volume of a comet is often enormous. As a general rule the *head*, or coma, is from 30,000 to 150,000 miles in diameter. The average diameter of a large number tabulated by Holetschek is 80,000 miles.

A comet less than 10,000 miles in diameter is very unusual; in fact, such a comet would be almost sure to escape observation. Some are much larger than 150,000 miles. The head of

the great comet of 1811 was at one time fully a million miles in diameter, — considerably larger than the sun itself.

The faint outer nebulosity surrounding the head of Halley's comet in December, 1909, was 550,000 miles in diameter, and that accompanying Holmes's comet of 1892 was at one time no less than 1,400,000 miles across.

501. Variation in Size of Coma. It is a remarkable fact that the head of a comet continually changes in diameter. Usually these changes bear some relation to the comet's distance from the sun, the coma being smaller near perihelion than at a moderately greater distance on either side. At great distances from the sun the diameter of the visible portion is usually small.

Thus, Halley's comet, when first seen in September, 1909, at a distance of 290,000,000 miles from the sun, was about 14,000 miles in diameter, with a diffuse nucleus and no tail. In December, when 180,000,000 miles from the sun, the diameter of the head was 220,000 miles. At perihelion, in April, 1910, 55,000,000 miles from the sun, the diameter was 120,000 miles, while in June, at double the distance, it had increased to 320,000 miles. In April, 1911, when 400,000,000 miles from the sun, the diameter had decreased again to 30,000 miles.

502. Dimensions of the Nucleus. The nuclei, even of great comets, when sharply defined, are small, often appearing as mere starlike points of light in very powerful telescopes. The nucleus of Halley's comet when near perihelion was only 500 miles in diameter; that of Brooks's comet of 1911, 750 miles; that of Donati's comet of 1858, about 900 miles. The great comet of 1882, one of the largest on record, had a nucleus 1800 miles in diameter.

503. Dimensions of a Comet's Tail. The tail of a comet, as regards magnitude, is by far the most imposing feature. A tail visible to the naked eye is seldom less than 5,000,000 or 10,000,000 miles long; lengths of from 30,000,000 to 50,000,000 miles are not uncommon among bright comets; and there are several cases in which the observed length has been nearly if not quite 100,000,000 miles. The tail is usually more or less fan-shaped, so that at the outer extremity it is millions of miles across, shaped and bent like a horn (Fig. 168).

504. Changes in Size and Form. Certain comets have changed in diameter, as well as in brightness, in an extraordinary fashion.

Holmes's comet (1892 III), when discovered, on November 6 of that year, was visible to the naked eye and about 200,000 miles in diameter, with an outer nebulosity 700,000 miles across, and with a central condensation but no nucleus. A month later it had doubled in diameter; but then it grew so faint and transparent that it could hardly be observed even with large telescopes. In the middle of January it suddenly contracted into a mere hazy star, with a strong nucleus, and with a coma 30,000 miles across (greatly increasing in brightness at the same time), and then gradually expanded to a diameter of 300,000 miles and faded out. At its returns in 1899 and 1906 it was barely visible in the most powerful telescopes. The cause of these remarkable changes is quite unknown.

Biela's comet, which has a period of $6\frac{3}{4}$ years, was observed in 1772, 1806, 1826, and 1832, and presented no unusual features. In 1846, while under observation, it became pear-shaped and divided into two parts. The twin comets traveled along side by side for more than three months at a distance of about 160,000 miles. Each developed a nucleus, and a tail about half a degree in length. When the comet returned in 1852 the twins were both seen, separated by about 1,500,000 miles, and were observed for a month, sometimes one and sometimes the other being the brighter.

Neither of them has ever been seen again, although they must have returned to perihelion ten times, and more than once under favorable conditions for visibility. Their invisibility cannot be accounted for by perturbations, for they did not come near Jupiter till 1889, and not very near it then. They must simply have *stopped shining*, — not partially, like Holmes's comet, but completely.

Taylor's comet (1916 I) also divided into two parts while under observation. The component which was brighter at first faded out some time before the other. This comet, like Biela's, has a short period, — about $6\frac{1}{2}$ years.

505. Masses of Comets. While the volumes of comets are enormous, their masses are apparently insignificant, — in no case comparable with those of the smallest of the planets.

The evidence on this point, however, is almost entirely negative; that is, while in several cases we are able to say positively that the mass of a particular comet cannot have exceeded a limit which can be named, there is no case in which we can with certainty fix a lower limit which we know it must have reached.

Comets have frequently come so near the earth or some other planet that their orbits have been considerably altered, or even entirely transformed, by the effects of the planet's attraction; yet no perceptible alteration of the motion of these planets has been produced by the attraction of the comets.

For example, Lexell's comet of 1770 came so near the earth that the comet's period was shortened by more than $2\frac{1}{2}$ days by the earth's attraction; but the length of the year was not altered by so much as a single second, and it would have been changed by this amount if the comet's mass had been 1/13,000 that of the earth.

Again, Brooks's comet, in 1886, had its period changed from 29 years to 7 years by an encounter with Jupiter; but the period of the planet was certainly not altered by more than two or three minutes, and probably by much less, and from this it follows again that the mass of the comet could not have been more than 1/10,000 that of the earth.

All that can be said with certainty at present is that it appears probable that the mass of even a large comet is less than a millionth part of the earth's mass. It should be remembered, however, that if its mass were still a million times less than this, a comet would nevertheless weigh 6000 millions of tons. The quantity of matter in a comet, therefore, while vanishingly small by planetary standards, may be, and in all probability is, large from the standpoint of ordinary terrestrial affairs (§ 520).

506. Density of Comets. The mean density of a comet is certainly extremely low, the mass being so small and the volume so great. If a comet of average size (80,000 miles in diameter) had a mass equal to a millionth of that of the earth, its mean density would be 1/230,000 of that of the air at the earth's surface, — a degree of rarefaction reached only by good air-pumps.

The extremely low density of comets is shown also by their *transparency*. Small stars are often seen through the head of a

comet 100,000 miles in diameter, and even very near its nucleus, with no perceptible diminution of their luster; and in at least one case this fact has been confirmed by exact photometric measurements.

Still more conclusive is the fact that the great comet of 1882, and Halley's comet in 1910, transited over the face of the sun, and that during the transits both comets were *absolutely invisible.* Even the nucleus was so transparent that no trace of it could be found, in spite of careful searching, — in the latter case by photography and with the spectroheliograph, as well as visually.

As for comets' tails, their density presumably is vastly less than that of the heads, and far below the best vacuum that we can yet produce by any artificial means. It is nearer to an airy nothing than anything else we know of. Schwarzschild, from the observed brightness of the tail of Halley's comet in 1910, has calculated that there could not have been more material in 2000 cubic miles of the tail than in a single cubic inch of ordinary air, and there may have been very much less!

We must bear in mind, however, that the low mean density of the comet does not necessarily imply that the density of its constituent parts is small. A comet may be in the main composed of small heavy bodies, and still have a very low mean density, provided they are widely enough separated. There is much reason, as we shall see later (§ 520) for supposing that this is really the case.

Another point should be referred to. Students often find it hard to conceive how such impalpable "dust clouds" can move in orbits like solid masses and with such enormous velocities; they forget that in a vacuum a feather falls as swiftly as a stone. Interplanetary space is a vacuum, far more perfect than anything we can produce by artificial means, and in it the lightest bodies move as freely and swiftly as the densest, since there is nothing to resist their motion. If the moon, and all the earth, were suddenly annihilated, except a single feather, the feather would keep on and pursue the same orbit, with the unchanged speed of $18\frac{1}{2}$ miles a second.

507. The Brightness of Comets. No other heavenly bodies differ so enormously in brightness as do comets. Some are barely

visible in great telescopes; others have been the most brilliant celestial objects ever observed, excepting only the sun and moon.

The great comet of 1882, just after its perihelion passage, was so bright that there was not the least difficulty in seeing it by simply shutting off the sun with the hand held at arm's length, though it was only three or four degrees from the sun. Yet, five months later, this great comet became invisible to the naked eye; and a year after its perihelion passage it could not be seen with powerful telescopes, though the observers knew just where to look for it. At this later date it could not have been as much as one-thousand-millionth part as bright as at perihelion. Variations in the distance of a comet from the earth account for a part of the change in brightness; but much the larger portion of the change is usually due to a real alteration in the amount of light emitted by the comet, which grows a great deal brighter as it draws near the sun, and fainter again as it recedes.

508. Law of Variation of Brightness. If a comet shone, like a planet, by reflected sunlight, its brightness, as seen from the earth (disregarding possible effects of phase), would be proportional to $1/R^2\Delta^2$, where R is its distance from the sun and Δ from the earth (§ 292).

For a few faint comets, and for some others at great distances from the sun, the actual variations in brightness approximately follow this law; but for the great majority, including all the brighter comets, the increase in brightness as the comet approaches the sun is far more rapid than is indicated by this assumption.

The light actually varies approximately as $1/R^n\Delta^2$, where the exponent n ranges from 3 to 5 or even 6, and averages about 4.

In some cases, however, the light of a comet changes greatly and almost capriciously, brightening or fading without apparent cause within a few days or even a few hours.

It is often the case, too, that a comet is decidedly brighter after its perihelion passage than it was at the same distance from the sun before perihelion. This was very conspicuous at the recent return of Halley's comet.

In spite of these irregularities the assumption that the brightness of a comet varies as $1/R^4\Delta^2$ will usually give a good general idea of its behavior. For purposes of prediction it is certainly far better than the assumption of variation proportional to $1/R^2\Delta^2$, which is still generally adopted by computers. This rapid falling off in a comet's brightness as it recedes from the sun explains why so few have been followed to great distances, and why the periodic comets can be seen only in the neighborhood of their perihelia.

509. Relative Brightness of Different Comets. From what has just been said it is evident that if we wish to obtain a real measure of the relative brightness of different comets, we must compare their brightness when at the same distances from the sun and the earth. When this is done, it is found that the differences are very great.

The most noteworthy comets of the last four centuries, measured by this standard, were those of 1577, 1744, 1811 I, and 1882 II. It may reasonably be estimated that each of these, at unit distance from the sun and earth, would have appeared

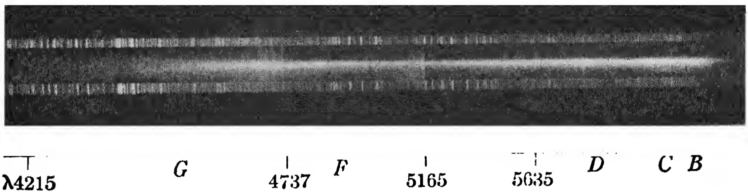


FIG. 170 *A*. Spectrum of Halley's Comet, May 6, 1910

Photographed with a one-prism spectrograph attached to the 24-inch Lowell refractor, and on a specially sensitized plate. Shows visual hydrocarbon emission bands (wave-lengths (§ 549) λλ4737, 5165, 5635), sodium bright lines (*D*), and also a strong solar spectrum (*C*, *G* etc.). The relative strength of the solar lines proves that this comet's brightness is due more to reflected sunlight than to emitted light of its own. (From photograph by Lowell Observatory)

about as bright as the brightest stars, such as Arcturus or Vega. Few other comets, at the same distances, would have been one tenth as bright as these.

The greatest comet yet recorded, however, was undoubtedly that of 1729, which was faintly visible to the naked eye, although its perihelion distance was more than four astronomical units. The four comets mentioned above would have appeared, on the average, only about one twentieth as bright at such a distance as the comet of 1729, and it seems certain that if the perihelion distance of this comet had been small, its brilliancy would have surpassed anything on record.

Most of the conspicuous comets owe their brightness to a relatively close approach to the sun or the earth.

510. Origin of the Light of a Comet. Spectroscopic study shows that the light of a comet's head arises partly from *reflected sunlight*, which gives a continuous spectrum, crossed by the familiar Fraunhofer lines (§ 570), and partly from the light emitted by

luminous gas (§ 558), which shows bright bands in the spectrum (identifying the presence of nitrogen and of various gaseous compounds of carbon, — carbon monoxide, hydrocarbons, and cyanogen).

The spectrum of the tail, when bright enough to be observed, shows reflected sunlight, and also includes bands which are given in the laboratory by carbon monoxide at an exceedingly low pressure.

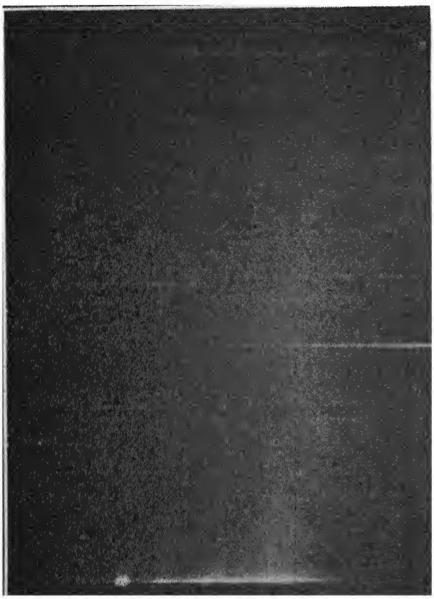


Fig. 170 *B*. Halley's Comet

Photographed May 6, 1910, with a prismatic camera, which spread the various colors out to right and left. (The tail runs up.) Carbon monoxide tail streamers are shown. The brightest image of the head is due to cyanogen. (From photograph by Lowell Observatory)

When a comet comes closer than 50,000,000 or 60,000,000 miles to the sun, light emitted by sodium vapor appears, and the emission of the other gases weakens. The great comet of 1882, when intensely heated near perihelion, gave evidence of the vapors of other metals, probably including iron. The proportion of reflected light is decidedly variable, both from one comet to another and in the same comet from time to time. The nucleus usually shines mainly by reflected light, but most of the light of the coma and tail comes from the luminous gas.

What stirs the gases up to shine is not fully understood. The latest work (Zanstra, 1926) makes it seem likely that the energy is derived from the absorption of sunlight by the gas. At any rate, there is no doubt that it comes originally from the sun and is only transformed by the cometary gases into the radiations that are observed. Strictly speaking, therefore, a comet is not self-luminous.

It is well to remark that while the bright bands in the spectrum prove the presence in the comet of carbon monoxide and nitrogen,

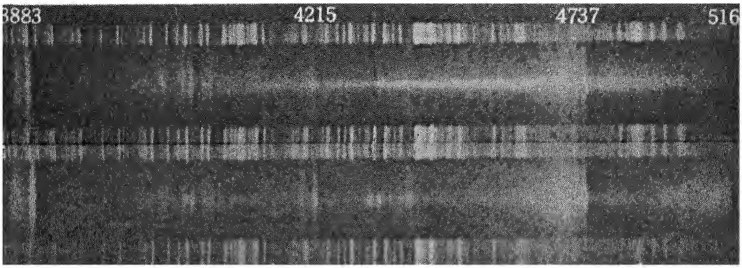


Fig. 171. Spectrum of Mellish's Comet (above), May 9, 1915, and of Zlatinsky's Comet, May 25, 1914

The cyanogen bands λ 3883 and λ 4215 are very different in these comets. The carbon band at λ 4737 is nearly the same

and often of hydrocarbons, they do not at all prove that the comet is mainly composed of these substances, or even that such gases constitute a considerable portion of its mass. It is much more likely that solid or liquid particles of various sizes form an overwhelmingly preponderant part of the whole.

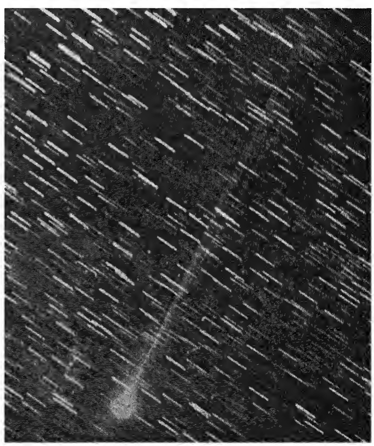


Fig. 172. Gale's Comet, May 5, 1894

511. Development of a Comet as it approaches the Sun. Even a large comet, if first seen at a great distance from the sun, appears as a mere roundish nebulosity, brighter in the middle but usually without a definite nucleus. As it approaches the sun it brightens rapidly, and the nucleus appears. The coma expands, and the tail begins to form, often at the start as a narrow streamer (Fig. 172). For great comets the distance from the sun at which the tail begins to form averages about 200,000,000 miles. As the activity increases, the nucleus, on its

sunward side, sends out jets of matter which spread out into a diffused, fan-shaped mass, from which the luminosity sweeps outward and backward into the tail. Usually these jets and streamers are irregular and subject to rapid changes, but in some cases the nucleus appears to throw off more or less symmetrical envelopes, several of which are sometimes visible at once (Fig. 173). The activity of the nucleus is usually greatest

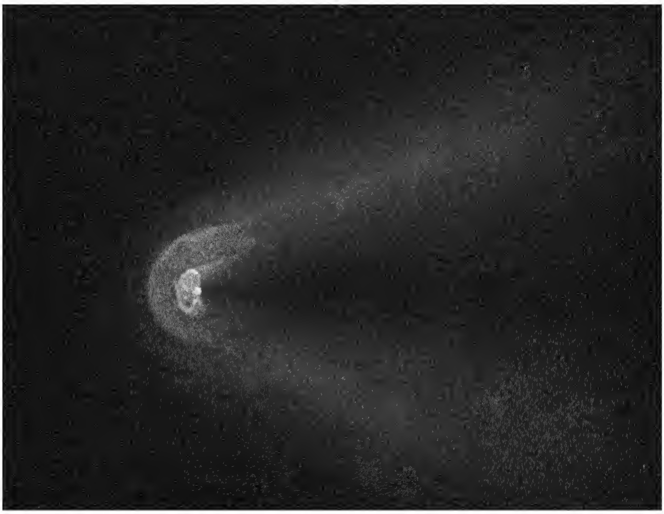


FIG. 173. Head of Donati's Comet

The envelopes of this comet were unusually symmetrical. (From drawing by Bond)

shortly after the perihelion passage, and then gradually dies down, the comet reversing the series of changes previously described, and appearing at last once more as a little patch of nebulosity before it fades wholly from sight.

Smaller comets (or larger ones which do not come so near the sun) run through only a part of the changes just sketched, and may reverse their development at any stage.

512. Formation of the Head and Envelopes. In Donati's comet of 1858 (Fig. 173) the envelopes were formed at intervals of from four to seven days, and remained visible for a fortnight or more, so that several concentric sheets of light were in sight at once.

In Morehouse's comet of 1908, however, the envelopes lasted but a few hours, and contracted as they grew older; but in this case also several of them were visible at the same time.

The envelopes appear to be formed by material projected from the nucleus on the sunward side, and repelled by the sun in the manner illustrated in Fig. 174 (in which, however, the downward curvature of the paths of the outer particles is greatly exaggerated).

It seems clear that such an envelope, like the jet of a fountain, must be continually composed of new material, though retaining its form unaltered. The only difficulty about this "fountain theory" of the formation of the envelopes is that in certain cases, notably in Morehouse's comet (as Eddington has shown), very high initial velocities and very great repulsive forces are demanded, to account for the rapid formation and changes of the envelopes; but no other theory thus far suggested appears to fit the facts nearly so well.

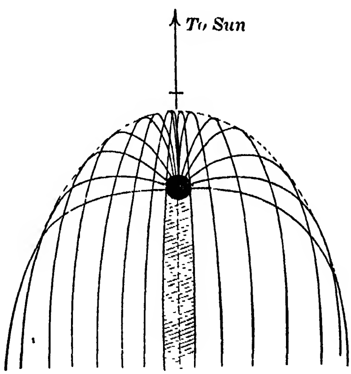


Fig. 174. Formation of a Comet's Tail by Matter expelled from the Head

513. Formation of the Tail. It may be supposed that the material ejected from the nucleus and repelled by the sun sweeps backward to form the tail, along which it streams, still luminous, until it becomes so widely diffused and so rarefied that it is no longer visible.

Comets' tails, therefore, are simply *streams of repelled particles*, each moving in its own hyperbolic orbit around the sun, the separate particles exerting no sensible influence upon one another, and (except perhaps at the very beginning of their course) being quite emancipated from the control of the comet's head.

This view of their nature accounts for the observed shapes and changes of the tails of different comets, and has been conclusively confirmed by direct observation since photography has made it possible to study details visible with difficulty, if at all, to the eye.

514. Shape of Comets' Tails. Since the initial velocity of projection of these particles from the comet is usually small, the extent to which they spread out laterally is usually small in comparison with the distance to which they are driven by the sun's repulsive action in the same interval of time. The tail, therefore, is in most cases a sort of flat, hollow, curved, horn-shaped cone, open at the large end. The edges of the tail, near the comet at least, appear, as a rule, much brighter than the central part.

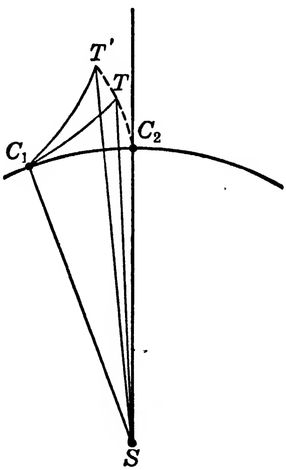


FIG. 175. Shape of Comets' Tails

Particles emitted from the comet at C_1, and repelled by the sun S, will travel in hyperbolic paths and reach T or T' when the comet reaches C_2. The lines joining all three with the sun sweep out equal areas C_1ST' and so on. This evidently demands that T must lag behind the line SC_2, and T' still farther

The tail is curved, because the repelled particles, after leaving the comet's head and receding from the sun, follow the law of areas, and hence lag behind the comet's radius vector, which, at their greater distance, would sweep out too large an area (Fig. 175). The stronger the repulsion, the less will be the curvature.

It should be noticed that this curvature is entirely *in the plane of the orbit.* A tail which, when seen from a direction at right angles to the orbit plane, appears strongly curved, like the longest one in Fig. 169, would appear to an observer situated in or close to the orbit plane to be straight, and much narrower because seen edgewise.

The theory of the forms of comets' tails was first worked out by Bessel, and has been greatly developed by Bredichin, who finds that the tails of almost all comets may be classified under three fairly distinct types.

In the first the repulsive force is 15 or 20 times as great as the sun's attraction, and the tail is almost straight; in the second the repulsion is only about twice the attraction, and the tail is decidedly curved; in the third the two forces are nearly equal, and the tail is short and stubby.

In some cases even greater repulsive forces appear to have existed, notably in Morehouse's comet, where detached masses in the tail themselves emitted subsidiary tails evidently composed of matter driven away from them even more rapidly than they themselves were moving.

The low-pressure spectrum of carbon monoxide has been found to be especially characteristic of the narrow, straight streamers, while the more featureless part of the tail, in Halley's comet at least, gave a nearly continuous spectrum.

515. Direct Evidence of Motion in the Tail. The most conclusive evidence that the tails of comets are composed of matter streaming away from the head is given by photography. In about half the brighter comets of the past twenty-five years, luminous knots, or condensations, have at times been observed in the tail. Whenever such an object has been photographed at intervals of even a few hours, it was found to be rapidly receding from the comet's head; and when the observations extended over a longer interval, they showed that its rate of recession was steadily increasing.

Thus, a detached mass in the tail of Halley's comet photographed by Curtis was receding from the head at an average rate of 70 kilometers per second between June 6 and 7, 1910, and at the rate of 91 kilometers per second on the following day.

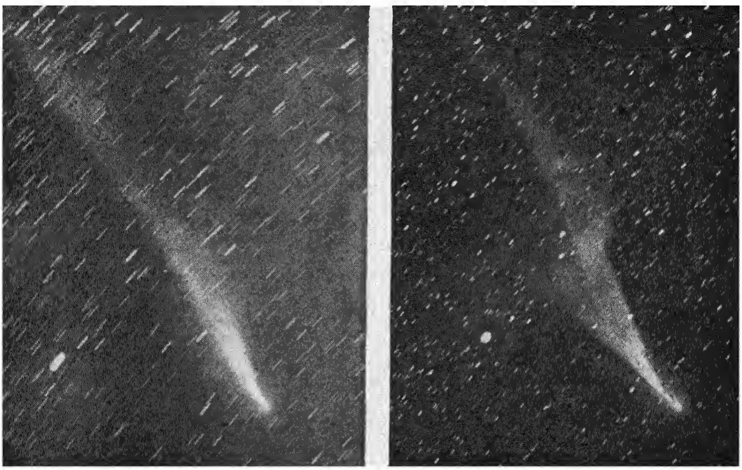
Velocities of the same order of magnitude have been observed in other comets. They indicate the existence of repulsive forces emanating from the sun (since the acceleration is almost entirely independent of the distance from the comet's head) and of magnitude varying in different cases between 30 and 100 times the sun's gravitational attraction.

In some cases, as in Morehouse's comet, the changes are very rapid (Fig. 176). Comparison of the motions of details at different distances from the head shows that the material is ejected from the nucleus with a relatively low velocity, and develops its high speed later under the action of the sun's repulsion. The end of a tail, many millions of miles long, would be reached by the outgoing material within a week.

From measures of the brightness of the tail of Halley's comet, Schwarzschild and Kron have shown that the gradual fading out

of the tail as it recedes from the head is not due to loss of light-emitting power by the luminous particles, but to the fact that they become spread out more and more widely as they proceed, — both laterally by the divergence of their paths and longitudinally by the steady increase of their speed.

516. Nature of the Repulsive Force. Various hypotheses have been proposed, from time to time, regarding the nature of the force which drives the material along the tail of a comet, but



17h 16m G.M.T. 20h 57m G.M.T.

FIG. 176. Changes in Morehouse's Comet

The two pictures were taken three and a half hours apart, September 30, 1908. (From photographs by E. E. Barnard, Yerkes Observatory)

it is now very generally believed that the principal agency is the radiation pressure (§ 320) of sunlight. This must exert, upon dust particles of the order of 1/100,000 inch in diameter, a repulsive force more than ten times as great as the sun's attraction. It is probably much more effective on isolated luminous molecules of gas which are absorbing energy from sunlight.

The particles, whatever their size, are so sparsely scattered in the tail that almost all the sunlight traversing it goes through between them, and no perceptible fraction of the whole beam suffers absorption of any kind.

Though the forces which cause the motion of the luminous matter along the tail appear now to be well understood, those which occasion its ejection from the nucleus are still obscure. The initial velocity is often several kilometers per second, — too great to be ascribed to the mere expansion of liberated gases into a vacuum, — and the appearance of the short-lived envelopes in Morehouse's comet suggests the occurrence of almost explosive emissions of gas, with velocities at first very high but dying down in the course of a few hours. The energy demanded by the emission may either be derived from the incident solar radiation or be latent in the material of the head and liberated in some way by sunlight.

517. Anomalous Phenomena. A number of comets have at times possessed anomalous tails (usually in addition to the normal tail, but sometimes substituted for it), sometimes directed straight toward the sun and sometimes nearly at right angles to that direction. Bredichin has shown that tails of the latter sort may be accounted for by changes in the velocity of projection of the particles from the nucleus, while those directed toward the sun may be composed of relatively large particles, emitted at low velocity, for which the light pressure is small in comparison with the gravitational attraction.

The great comet of 1882 also carried with it for a time a faintly luminous sheath, which seemed to envelop the comet itself and that portion of the tail near the head, projecting 2° or 3° toward the sun. For some days, moreover, it was accompanied by little clouds of cometary matter, which left the main comet, like smoke puffs from a bursting bomb, and traveled off at an angle until they faded away. It is natural to associate these unusual phenomena with the profound disturbances which the outer parts of the comet must have suffered during its precipitate flight around the sun at perihelion, when it went through 180° of orbital longitude in less than four hours.

518. Dissipation of the Cometary Material. *There is not the slightest reason to suppose that the matter driven off to form the tail is ever recovered by the comet.* The minute particles must fly off into space at very high velocities. A very small fraction of them may ultimately be picked up by dark bodies in space (though not by bright stars, whose light-pressure would repel them), but the overwhelming majority must wander alone in interstellar space indefinitely.

It follows that such comets as have tails lose a portion of their substance every time they visit the neighborhood of the sun. It

is quite conceivable also that the processes by which light is excited in the head of a comet may use up and render unfit for future shining a portion of its material, so that, as a periodic comet grows old, it may become both smaller and less luminous, until finally it ceases to be observable.

Some such process may account for the disappearance of Biela's comet (§ 504) and the probable loss of two or three other faint periodic comets which have not been seen at recent returns. On the other hand, Halley's comet, according to Holetschek, has lost very little, if at all, in brightness and length of tail in the past three centuries; nor has Encke's comet suffered serious diminution of brightness in the past century, in spite of thirty-one returns to perihelion. In these cases at least the disintegration has been slow.

519. Nature of Comets. The accepted view of the nature of comets (based partly on the phenomena already described and partly on those connected with meteors, described below) is that they are loose swarms of separate particles (probably of very different sizes) separated by distances great in comparison with their own diameters and accompanied by more or less dust and gas. The greater part of the mass is probably concentrated near the center of the cluster, but even here the open spaces must be exceedingly large in comparison with the particles. The whole swarm, though of very low mean density, moves freely through the perfect vacuum of interplanetary space, the slight mutual gravitation of its parts being just sufficient to keep the swarm together.

The phenomena attending a comet's apparition may then be explained as follows:

A comet like Halley's, when first seen at a great distance from the sun, is apparently little more than a cluster of meteoric particles shining by reflected light. As it approaches the sun and begins to be warmed by its rays, gases ooze out of the solid particles, carrying with them quantities of fine dust. These diffuse into space in all directions, and the coma expands. The dust reflects the sunlight and the gases are set shining by absorption of solar energy, so that the brightness of the comet grows very rapidly. As the activity increases, the finer particles, repelled by the sun's light-pressure, stream away visibly to form the tail,

which grows longer and brighter as the comet approaches perihelion. The emissions from the nucleus sometimes take the form of jets, or streams, and sometimes the outflow is more regular, resulting in the formation of envelopes.

As the comet withdraws from the sun, its history is reversed, except that the development of the activity lags a little behind the excitation which produced it, so that the maximum brightness is reached a little after the perihelion passage and the comet is brighter, at the same distance from the sun, after perihelion than before. Finally, as it recedes into the asteroid zone, the internal activity ceases; the remnants of gases and fine dust are blown away into space by radiation pressure; the coarser dust perhaps settles down or agglomerates to some degree; and the comet is left nearly in its original state during the long interval before the next perihelion passage.

520. Photometric Estimate of a Comet's Mass and Density. On this theory of a comet's constitution it sometimes becomes possible to make an estimate of the order of magnitude of its mass from measures of its brightness.

Take for example, Halley's comet. In September, 1909, it was still apparently inactive, and probably shone by reflected sunlight. Its measured light was about equal to that which would have been reflected from a single body, in the same position, of the albedo of the moon and 40 kilometers in diameter; but the comet itself was 22,000 kilometers in diameter and presented 300,000 times as large an area.

It follows that if we could have magnified the comet sufficiently to see the separate particles of which it was composed, we should have found them to be scattered very sparsely over the dark background of the sky, covering probably not more than 1/300,000 of the whole area within the comet's apparent boundary. It is no wonder that the comet was transparent.

We can do little more than guess what the size of the particles may be, but it is not at all probable that they average less than a centimeter in diameter, for if they were smaller the radiation pressure upon them would be a sensible, though small, fraction of the sun's attraction, and would produce perceptible modifications in the comet's orbital motion.

If the particles were all 1 centimeter in diameter and had the same albedo as the moon, it would require 16×10^{12} of them to reflect the observed amount of light. This is a large number; but the volume of the comet is $5\frac{1}{2} \times 10^{12}$ cubic kilometers, so that if the particles were uniformly distributed, there would be three of them per cubic kilometer. In more familiar terms, the observed brightness of the comet at this time can be explained by supposing that *in every cubic mile of its volume there were a dozen bodies about as big as small marbles, and nothing else.*

If the density of these particles was similar to that of ordinary rock, the *average mass per cubic mile would be a little over half an ounce.* The total mass of the comet would be about 25,000,000 tons, and the bulk of the particles, if collected in one heap, about 11,000,000 cubic yards, — only 5 per cent of the amount excavated in constructing the Panama Canal. With particles 100 times as large the calculated mass would be 100 times as great.

All these calculations are of course merely illustrative, for it is probable that the actual particles which compose the comet are of very different sizes; but the conclusion that the mass of the comet must be very small seems to be unescapable. The comet of 1729, which probably also shone by reflected sunlight and was 10,000 times as bright as Halley's comet would have been at the same distance, may possibly have had a mass as great as a millionth of that of the earth. The fainter comets, like Biela's and Encke's, are probably much less massive than Halley's comet.

The mass and density of the tail of a comet must be far smaller. Schwarzschild has calculated that if the tail of Halley's comet was composed of fine dust particles, there may have been a million tons of dust in the whole tail; but if the particles were gaseous molecules, the whole mass may have been no more than a hundred tons. The great difference of these results illustrates the outstanding uncertainty. If the estimate of the total mass which has just been made is anywhere near the truth, the lower figure for the mass of the tail appears the better.

521. Collisions with Comets. It is probable that the earth has undergone many collisions with comets during geological time.

It may readily be computed that a small, rapidly moving body which approaches the sun within one astronomical unit stands about one chance in 400,000,000 of hitting the earth. As about five comets come within this distance every year, the nucleus of a comet should hit the earth, on the average, once in about 80,000,000 years. Collisions with the outer parts of the head should be many times more frequent.

As to the consequences of such a collision it is impossible to speak positively for want of sure knowledge of the constitution of the comet. If the theory which has been presented is true, everything depends on the size of the separate particles which form the main portion of the comet's mass. If they weigh tons, the bombardment experienced by the earth when struck by the comet would be a serious matter, although it would probably fall very far short of producing a wholesale destruction of terrestrial life. If, as seems more likely in the case of the outer portions, they are for the most part as small as pinheads, the result would be simply a splendid shower of shooting stars.

A danger of a different sort has been suggested, — that if a comet were to hit the earth, our atmosphere would be poisoned by mixture with the gaseous components of the comet. Here again the probability is that on account of the low density of the cometary matter no sufficient amount would remain in the air to do any mischief at the earth's surface. Moreover, combination with the oxygen of our atmosphere would render quite harmless any of the gases whose presence has been detected in comets.

As for encounters with comets' tails, they are probably of frequent occurrence. It is certain that the earth passed through the tail of the great comet of 1861, and it is probable that it at least grazed that of Halley's comet in 1910. In neither case was any perceptible effect produced.

522. Effect of the Fall of a Comet into the Sun. As to this it may be stated that except in the case of Encke's comet there is no evidence of any action going on that might cause a periodic comet to strike the sun's surface; it is doubtless possible, however, that a comet may sometimes enter the system from a distance, so accurately aimed as to hit the sun.

It is not likely, however, that the least mischief would be done. If a great comet, with a mass equal to one millionth of the earth's mass, were to strike the sun's surface with the parabolic velocity of nearly 400 miles a second, the energy of impact would generate a little less heat than the sun radiates in an hour. If this were all instantly effective in producing increased radiation at the sun's surface, trouble would follow, of course; but it seems certain that nothing of the sort would happen. The density and quantity of the solar atmosphere are so small (§ 662) that the cometary particles would pierce the photosphere and liberate their kinetic energy as heat well below the sun's surface, adding to the sun's store of internal energy about as much as it ordinarily expends in an hour. There might be a brilliant flash at the solar surface as the shower of cometary particles struck it, but probably nothing that the astronomer would not take delight in watching.

If the directions of motion of the comets were absolutely at random, about one in two hundred of those which come inside the earth's orbit (§ 521) would strike the sun; that is, such collisions would occur, on the average, rather more than twice in a century. But no event of this kind has ever been observed, probably because, as we have seen, all known comets are returns of bodies which have passed perihelion before, and hence necessarily have orbits which take them clear of the sun. If comets with perihelion distances less than the sun's radius ever existed, they must have been eliminated at their first perihelion passages, and by this time it is not surprising that we find none.

METEORS

523. On any clear evening, starlike objects may frequently be seen which appear suddenly, dart swiftly across the sky, and vanish. They are known as *meteors*. The fainter ones, which are much the more frequent, and which usually disappear in less than a second, are commonly called *shooting stars*. The brighter ones are known as *fire-balls* (or *bolides*). They are sometimes bright enough to light up the landscape and cast strong shadows, and they are often in sight for several seconds. A bright meteor

is generally followed by a luminous train, which sometimes remains visible for many minutes after the meteor itself has disappeared. The motion is sometimes irregular, and here and there along its path the fire-ball throws off sparks and fragments. Sometimes it vanishes by simply fading out in the distance, sometimes by bursting like a rocket. By day the luminous appearances are mainly wanting, though sometimes a white cloud is seen, and even the train may be visible.

524. Nature of Meteors. The large apparent velocity with which meteors move indicates at once that they must be relatively very near us. This is fully confirmed when the same meteor has been observed at different stations and its parallax found. It thus is shown that visible meteors are within a hundred miles or so of the earth's surface.

They are small bodies which approach the earth from interplanetary space at high velocity, and shine only when they enter the upper atmosphere and become heated by friction in flying through the air. Most of them are soon consumed, burning completely away. Those which get through to the earth's surface are called *meteorites*. A number have been recovered which were actually seen to fall, and others have been identified by their resemblance to these.

525. Number. Meteors come from all parts of the sky, as well as in showers from small areas. The number of *sporadic* shooting stars is enormous. A single observer averages from four to eight an hour; if accustomed to such observation and well situated, he may see twice as many on a moonless night.

Estimated on the basis of individual counts, the total number which enter our atmosphere in twenty-four hours and are bright enough to be visible to the naked eye must be several millions, and in addition there is probably a still larger number so small as to be observable only with the telescope.

The average hourly number after midnight is about double the hourly number in the evening. In the morning we are on the front of the earth in respect to its orbital motion. We see at that time meteors which the earth overtakes, as well as those coming to meet it. The apparent velocity of the morning meteors averages high for the same reason. In the evening we see only

such as overtake us. There is also an annual variation in the number. They are most numerous, in the northern hemisphere, in autumn, when the point toward which the earth is moving is highest above the horizon at night.

526. Observation of Meteors. The object of the observer should be to obtain as accurate a record as possible of the path of the meteor (including direction of flight and the points of appearance and disappearance) and the length of time it is visible. If other observers several miles away are known to be on the watch,

FIG. 177. Trail of a Meteor

The trail was caught while this star-field was being photographed. The brightness of the meteor obviously varied rapidly. (From photograph at Harvard College Observatory)

too, the time of appearance also should be noted. When a meteor can be identified as having been observed at two or more stations, its path with reference to the surface of the earth can be computed (a problem of parallax).

If seen by night the path should be drawn among the stars on a chart or globe. It is usually of assistance to hold up a stick in line with the flight and notice what stars lie on or near this line. The observer must, of course, be familiar with the constellations and brighter stars. By day he must take advantage of natural objects and buildings to define the path of the meteor, noting carefully the exact spot where he stood. By taking a surveyor's transit to the place afterwards it is easy to translate such data

into altitude and azimuth. The time of flight is hard to estimate with any approach to accuracy. Some observers begin to repeat rapidly some familiar verse of doggerel when the meteor is first seen, reiterating it until the meteor disappears.

Sometimes trails of bright meteors are caught accidentally on photographic plates, and special apparatus has been devised and successfully used in photographing meteors. Such observations are of course much more accurate than visual ones.

527. Paths of Meteors. Visual observations are necessarily of low accuracy; but when several competent observers have seen the same meteor, it is possible to find the position of its path and the heights of appearance and disappearance within a few miles.

Such data show a dependence of the heights of appearance and disappearance, and of the length of flight, upon the apparent size of the body and its velocity. The smaller ones, or *shooting stars*, appear at an average height of about 70 miles and disappear about 50 miles from the earth's surface, after an average flight of 35 miles. For certain groups, of high velocity, the altitudes of appearance and disappearance are greater. The corresponding heights for *fire-balls* are 85 and 30 miles, and the length of path averages something like 200 miles. Some of them travel much farther.

Since large masses must suffer relatively less resistance from the air and be completely consumed less rapidly than small masses, it is reasonable to conclude that fire-balls and meteorites are much more massive than shooting stars.

The flight of an average shooting star occupies so brief a time that direct velocity determinations are difficult and inaccurate. The apparent paths of fire-balls and meteorites are usually much longer and the observations better. Velocities as low as 15 km./sec., and as high as 75 km./sec., have been observed. The velocities with which meteors enter the atmosphere must be rapidly reduced as they penetrate deeper into the denser layers. No appreciable slowing up toward the end of flight of shooting stars is observed (largely owing to the briefness of their visibility), but it is often distinctly apparent in the case of meteorites. By the time these come within a few miles or so of the earth's surface the air resistance is so great that they are

slowed down to velocities no greater than that of a spent cannon ball, and they do little harm when they land. In one case (a meteorite that fell near Upsala, Sweden, in January, 1869) several of the stones struck upon the ice of a lake and rebounded without breaking the ice or damaging themselves.

528. Explanation of Heat and Light. As the meteor enters the atmosphere the kinetic energy of its bodily motion is transformed into kinetic energy of the gas molecules which it hits and of the molecules of its surface. In other words, both the air and the surface of the meteor are very greatly heated by the friction. The surface layers liquefy and vaporize, and an envelope of incandescent gas and vapor many times as large as the meteor itself spreads around and is swept behind it. This process continues until the whole is vaporized or until the velocity falls below a certain value. It is from this envelope and not from the solid surface that most of the light comes. The spectra of a few meteors have been photographed by accident during observations of the stars, and show bright lines due to luminous gas.

There is not time for the heat to be conducted into the interior of the mass to any notable extent. As a general rule, therefore, meteorites are superficially hot if found soon after their fall, but the interior may still be cold. It is recorded that one of the fragments of the Dhurmsala (India) meteorite, which fell in 1860, was found in moist earth, half an hour or so after the fall, *coated with ice.*

The disturbance of the air by a large meteorite produces sounds like violent thunder, which are often heard at great distances. As sound travels only about 12 miles a minute, there is often an interval of several minutes between the flash of the meteor and the noise. Observations of this interval help to determine the path.

The hot vapors along the meteor's track expand laterally and form the *meteoric train.* Such trains often remain luminous for several seconds or minutes, sometimes as much as half an hour, and are carried by the wind like clouds. They usually appear at a certain mean altitude, often starting to form after the meteor has gone some distance, and sometimes ceasing to form before the meteor becomes invisible. The rarefied gas cannot, of course, remain hot for anything like this time. It is probable that part of the energy is stored up in the molecules (possibly by ionization, § 637) and is gradually released with emission of light.

529. Masses of Meteors. The luminous energy radiated by a meteor may be calculated from observations of its apparent brightness at a known distance and the duration of its visibility. If we knew what fraction of the kinetic energy of the meteor was transformed into luminous energy, we could find the kinetic energy and, knowing the velocity, the mass of the meteor. Only a rough guess at this fraction can be made; but, unless it is very much smaller than for other light sources of the same color, the mass of a typical shooting star is only a few milligrams.

In the case of large meteorites, which weigh many kilograms when found, in spite of the losses which they sustain in the atmosphere, sufficient energy is available to make the light radiated millions of times greater than from the average shooting star. This is entirely consistent with reports that large fire-balls have appeared much brighter than the full moon.

But while meteors are, on the average, very small, they are almost countless. The total annual amount of meteoric matter which falls on the earth, measured in tons, must be large; but as an addition to the earth's mass it is trivial, and its effects on the revolution and the rotation of the earth are too small to be measurable.

530. Meteorites. The number of instances in which meteorites have actually been seen to fall, and have been recovered, has averaged about four per year since 1850. But for one that is found, even of the fire-balls whose flight has been observed, a dozen are missed; and if we include all that presumably were not seen, or that fell unobserved into the ocean or in regions from which no report could come, the sum total must be very great. Their number is small, however, compared with that of shooting stars.

The mass that falls is sometimes a single piece, but more usually there are many pieces, sometimes to be counted by thousands. At the Pultusk fall, in 1869, the number of meteorites was estimated to exceed 100,000, most of them very small. The largest single mass, so far as known, is one of three brought to the American Museum of Natural History, in New York, from Melville Bay, Greenland, by Admiral Peary. Its weight is $36\frac{1}{2}$ tons, and its approximate dimensions are $10.9 \times 6.8 \times 5.2$ feet.

Up to 1923 about 850 specimens from separate falls had been collected, in all.

531. Appearance and Constitution of Meteorites. The most characteristic external feature of these celestial immigrants is a thin black crust, usually, but not always, glossy like varnish. It is formed by the fusion of the surface, in the meteor's swift motion through the air, and in some cases penetrates deep into

FIG. 178. Willamette Meteorite

The cavities in this iron meteorite were produced by oxidation while it lay in moist ground for an unknown time. Its present weight is $15\frac{1}{2}$ tons, and its length is a little over 10 feet. (By courtesy of American Museum of Natural History, New York)

FIG. 179. Modoc Meteorite

This specimen of a stony meteorite shows the characteristic pitting and crust. A patch of the crust has been artificially broken away, to show the interior. A scale of inches shows the size. (By courtesy of American Museum of Natural History, New York)

the mass through veins and fissures. The crusted surface usually exhibits pits and hollows called thumb marks because they look like prints produced by thrusting the thumb into a piece of putty. These cavities are formed by the burning out of certain more fusible substances.

A large majority of meteorites are stones, — masses of crystalline rock. Some, however, are masses of metallic iron alloyed with nickel and cobalt; others are mixtures of stone and iron. Only 10 out of about 350 meteorites which were seen to fall

(up to 1915) and were recovered for study were composed of iron. About half the specimens in museums, however, are iron meteorites, since these, even though they may have fallen centuries ago, are recognizable at once as remarkable objects, while stony meteorites, except to the trained observer, look like any other stones.

About thirty of the chemical elements, including argon and helium, have been found in meteorites. Some of the elements are combined to form minerals not found in terrestrial rocks, but no otherwise unknown chemical elements have been found in meteorites. Of those present, iron, oxygen, nickel, silicon, and magnesium are by far the most abundant, in the order given; then sulphur, calcium, cobalt, aluminium, and sodium.

When heated, meteorites give out gases, including hydrogen, nitrogen, carbon monoxide, and hydrocarbons. Some meteoric minerals, notably a phosphide of iron and nickel, could not have been formed in presence of free oxygen. The iron meteorites usually show a crystalline structure, and the stone meteorites a structure of rounded crystalline grains, both of which are distinctive of these bodies. These structures indicate that the material must once have cooled from a melted condition,—rapidly for most stone meteorites and slowly for most of the irons.

532. Groups of Meteorites. Meteorites are small compared with their apparent size when rushing through the air. Then the surrounding luminous shells of intensely heated vapor and air contribute to the apparent size. But this is perhaps not all. Sometimes the pieces picked up near the same locality show fresh, unfused fracture surfaces, evidently resulting from the bursting of a single body. More often, and especially in cases where a great many stones fall, they all have the same characteristic surface resulting from fusion, showing that they entered the atmosphere as a group of separate bodies. In such a case the detonation heard at the end of the visible flight is not the result of explosive bursting of the meteorite, but rather of the sudden equalization of air pressure.

The hypothesis, first advanced by Haidinger and Galle, that a meteor is usually not a single body, finds further confirmation in the observation of groups of meteors, particularly in the

spectacle of February 9, 1913, when a stream of from ten to twenty groups, each containing from thirty to forty meteors, was observed from Canada to beyond Bermuda, traversing a visible path of 6000 miles.

533. The Meteor Crater in Arizona. In northeastern Arizona, near Cañon Diablo, is a remarkable crater in the desert (Fig. 181), about 4000 feet in diameter, with walls rising 150 feet above the surrounding plain and descending 600 feet precipitously to the

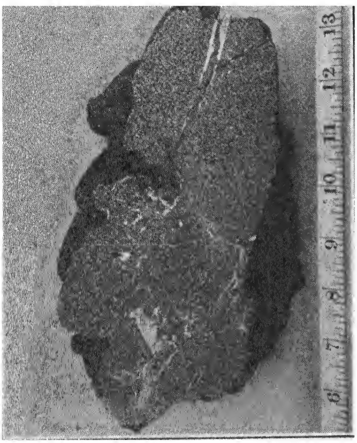


FIG. 180 *A*. Rose City Meteorite

This iron meteorite has been cut through and the surface polished to show the interesting internal structure. (This photograph, as well as that of Fig. 180 *B*, was furnished by the American Museum of Natural History, New York)

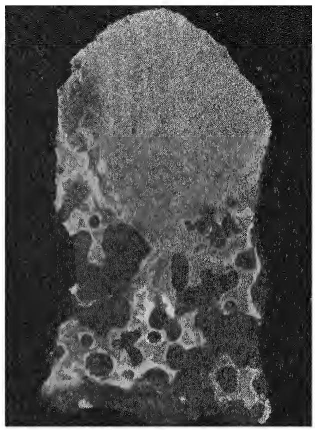


FIG. 180 *B*. A Brenham Meteorite

The surface of a section has been etched to bring out the crystalline structure of the metallic portion of the mass. A portion of this meteorite is almost entirely metallic; in the rest the metal surrounds large masses of stone.

floor. There is no evidence of volcanic activity within many miles. The walls are formed of tumbled masses of limestone and sandstone ejected from the interior.

Within five miles of the crater thousands of iron meteorites have been picked up on the surface, and a few have been found buried in the ejected material of the walls. Artificial borings show that the rocks of the crater's bottom have been crushed to a depth of several hundred feet. These rocks show also signs of great heating and reveal many specks of mixed oxides of

iron and nickel, a combination that is rarely found elsewhere than in meteoric material.

The evidence appears very strong that this crater has been produced within modern times, geologically speaking, by the impact of a great mass of meteoric material, perhaps a swarm and not a single body, which, too heavy to be stopped by air resistance, carried energy enough to excavate a hole more than half a mile across and a thousand feet deep (counting to the undisturbed rocks below the crushed material). Trees growing on the rims are as much as seven hundred years old, and it is probable that the impact occurred a few thousand years ago.

FIG. 181. Meteor Crater, Arizona
From photograph by D. M. Barringer

The swarm which produced this crater must have been so compact as to have been invisible at planetary distances. It is noteworthy that nearly every one of the large iron meteorites found on the earth's surface lies on the side of our planet which, at the time of the great impact, faced in the direction from which this swarm came. They *may* all have belonged to one large swarm.

534. Meteoric Showers; Radiants. There are occasions when the shooting stars, instead of appearing here and there in the sky at intervals of several minutes, appear in *showers* of thousands. Members of such showers do not move at random, but all their paths diverge, or radiate, from a single point in the sky, known as the *radiant*; that is, their paths produced backward all pass through or near that point, though they do not usually start there (Fig. 182).

The radiant keeps its place among the stars almost unchanged during the whole continuance of the shower, — for hours or days, it may be, — and the shower is named according to the place of the radiant among the constellations. Thus, we have the Leonids, or meteors whose radiant is in the constellation of Leo, the Andromedes, the Perseids, the Lyrids, etc.

The radiant is an effect of *perspective.* The meteors are all moving in nearly parallel lines when encountered by the earth,

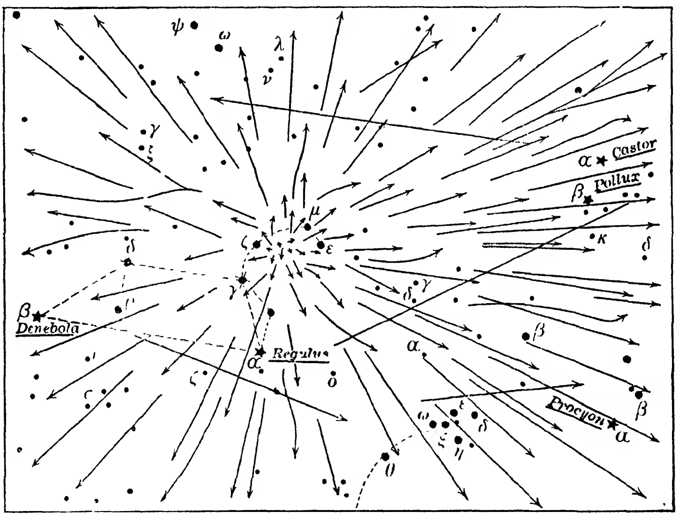


FIG. 182. Meteoric Radiant in Leo, November 13, 1866

All the tracks except four (which do not belong to the shower) appear to radiate from a small area near ζ Leonis

and the radiant is simply the perspective *vanishing point* of this system of parallels; their paths all appear to converge, like the rails of a railway track for an observer looking upon it from a bridge. Meteors which appear near the radiant, and which, therefore, are coming almost directly toward the observer, flash out for a few seconds as starlike points of light, or describe paths which are very short. Those in the more distant regions of the sky describe longer apparent paths.

The direction of the radiant is the apparent direction from which the meteors approach the earth. This is affected by the

orbital motion of the earth; but if the velocity of the meteors is known, their motion relative to the sun can be deduced.

Owing not only to errors of observation but also to the effect of the earth's attraction in changing the paths of the meteors, and probably also to lack of exact parallelism of the paths, the observed radiant is usually a small area of the sky rather than a point.

Probably the most remarkable of all the meteoric showers that have been recorded was that of the Leonids, on November 12, 1833. The number seen at some stations was estimated as high as 200,000 an hour for five or six hours. "The sky was as full of them as it ever is of snowflakes in a storm," and, as an old lady described it, looked "like a gigantic umbrella."

535. Dates of Meteoric Showers. Meteoric showers from the same radiant habitually recur on or about the same day of the year. Some of the radiants, notably that of the Perseids, are about equally active every year. From other radiants come a big shower one year and a rapidly diminishing number in the immediately preceding and following years, but always on the same date. This proves that the meteoric swarms pursue regular orbits around the sun, and that the annual shower occurs at the point where the earth's orbit cuts the path of the particular swarm. The earth reaches this point at the same date every year (Fig. 183).

In some cases the meteors are distiibuted in a nearly uniform ring along the whole orbit; for such, as in the case of the Perseids, the shower recurs every year with about the same vigor. On the other hand, the flock may have a distinct concentration with only a few stragglers scattered along the orbit; then a notable shower will occur only in the year when the earth meets this aggregation at the orbit crossing. This is the case both with the Leonids and with the Andromedes, though the latter are already getting widely scattered.

Olivier lists, up to 1920, over 1200 radiants, and expresses confidence in the reality of at least half of these. The most conspicuous of them are the Draconids, January 2; the Lyrids, April 20; the Aquariids I, May 6; the Aquariids II, July 28; the Perseids, August 12; the Orionids, October 20; the Leonids, November 14; the Andromedes, November 24; the Geminids, December 10.

Some of the swarms are of small diameter — so small that it takes the earth but a few hours to pass through them. Other showers persist for days or even for weeks, as do the Perseids. In such a case it is to be expected that the radiant will gradually shift its position among the stars on account of the change in the direction of the earth's motion, and such shifts have indeed been observed. In other cases they are probably masked by errors of observation and by the activity of neighboring radiants.

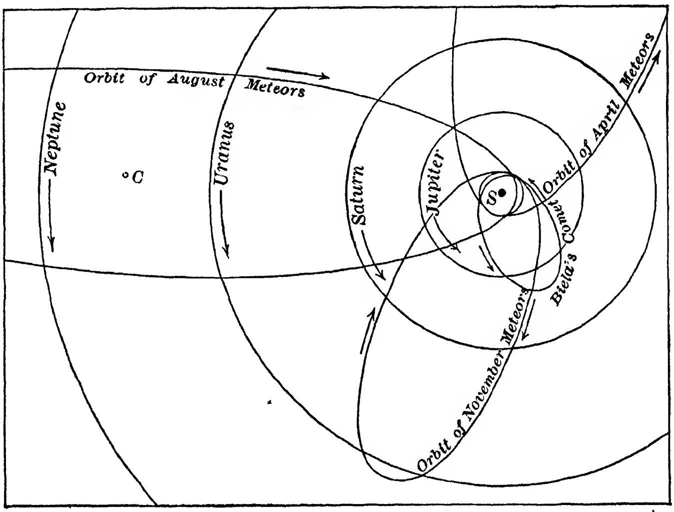


FIG. 183. Orbits of Meteoric Swarms

The meteors which belong to the same group have certain family resemblances. The Perseids are yellow and move with medium velocity. The Leonids are very swift (the earth meets them); they are of a bluish-green tint, with vivid trains. The Andromedes are sluggish (they overtake the earth); they are reddish, being less intensely heated than the others, and usually have only feeble trains.

536. Periodicity. In 1864 H. A. Newton showed by an examination of the old records that there had been a number of great meteoric showers in November, at intervals of thirty-three or thirty-four years, and he confidently predicted a repetition of the shower on November 13 or 14, 1866. The shower occurred as

predicted and was observed in Europe; and it was followed in 1867 by another, which was visible in America, the meteoric swarm being extended in so long a procession as to require more than two years to cross the earth's orbit. Neither of these showers, however, was equal to the shower of 1833. The researches of H. A. Newton, supplemented by those of J. C. Adams, the discoverer of Neptune, showed that the swarm moves in a long ellipse with a thirty-three-year period.

A return of the shower was expected in 1899 or 1900, but failed to appear, although on November 14–15, 1898, a considerable number of meteors were seen, and in the early morning of November 14–15, 1901, a well-marked shower occurred, visible over the whole extent of the United States, but best seen west of the Mississippi, and especially on the Pacific coast. At a number of stations several hundred Leonids were observed by the eye or by photography, and the total number that fell must be estimated by tens of thousands. The display, however, seems nowhere to have rivaled the showers of 1866–1867, and these were not to be compared with that of 1833. Very few meteors were seen in 1902, but in 1903 a large number were observed in Greece and in England.

The calculations of Downing and Stoney show that the failure of the Leonids to appear in 1900 was probably due to perturbations of the meteors by the action of Jupiter, Saturn, and Uranus, causing the main condensation to pass at a distance of nearly 2,000,000 miles from the orbit of the earth. The dates of some other showers have shown a gradual change, which is doubtless due to perturbations of the longitude of the node.

537. Identification of Meteoric Orbits with Cometary Orbits. The researches of H. A. Newton and J. C. Adams had awakened lively interest in the subject, and Schiaparelli, a few weeks after the Leonid shower, published a paper on the Perseids, or August meteors, in which he brought out the remarkable fact that they are moving in the same orbit as that of the bright comet of 1862, known as Tuttle's comet. Shortly after this Leverrier published his orbit of the Leonid meteors, derived from the observed position of the radiant in connection with the periodic time assigned by Adams. Almost simultaneously, but without any idea of a

connection between the orbits, Oppolzer published his orbit of Tempel's comet of 1866, which was at once seen to be almost identical with that of the Leonids. A single such coincidence might be accidental, but hardly two.

Five years later came the shower of the Andromedes, following in the track of Biela's comet; and careful comparison has shown three other similar relations between meteor swarms and comets. The Aquariids I of early May are connected with Halley's comet, moving in parallel paths, though several million miles distant from its orbit; and a shower in 1916, with Winnecke's comet. This shower, and likewise the Andromedes, are lost again, their orbits having been shifted away by perturbations.

538. The Origin of Meteors. The high velocity with which nearly all meteors enter our atmosphere shows that they must previously have been moving around the sun in independent orbits. The regularly recurring showers must arise from swarms moving in elliptical orbits, for one moving in a parabolic or a hyperbolic orbit would go by and never return. A swarm moving in an elliptical orbit would tend to spread out, in the course of time, into a continuous ring along the orbit, since the periods of the various members of the swarm would not be quite identical. It is probable that swarms which are almost uniformly distributed along their tracks have been pursuing their present orbits longest.

In several cases, as mentioned in the previous section, there is an obvious connection between a meteoric swarm and a known comet. Although a comet's density is very low, it is probably much greater than that of a meteoric swarm. (Even in the great shower of 1833 there was probably not more than one meteor per thousand cubic miles.) It appears almost certain that these swarms have been formed by the partial disintegration of the comets. How many of the other swarms are thus related to comets is unknown. It is of course by no means necessary that a meteor swarm should ever have been dense enough or have had enough gas and dust associated with it to form a visible comet.

Whether the sporadic meteors are members of the solar system or come in from stellar regions can only be settled by a considera-

tion of their velocities relative to the sun. The best way of getting at this is by studying the daily variation in the number observed. The greater the average velocity of the meteors, the less, evidently, will be the difference between the number that the earth overtakes and the number that overtake the earth. The weight of the evidence, and especially that recently presented by Hoffmeister, favors an *average* velocity which is strongly hyperbolic.

Meteorite falls and great fire-balls are more often observed tween noon and midnight than in the morning hours, and more often in the spring than in the autumn. This is exactly opposite to the behavior of shooting stars, and indicates that most meteorites overtake the earth and are moving in direct orbits about the sun. Bodies moving in retrograde orbits would have a much higher relative velocity and would be much more likely to be consumed in the earth's upper atmosphere. This may suffice to explain the observed facts.

Direct observations of velocity, which are notoriously hard to make with accuracy, indicate decidedly hyperbolic paths for most fire-balls and meteorites. On the other hand, fire-balls have often been observed during meteoric showers, coming from the same radiant. What proportion of shooting stars and meteorites belong to the solar system, and what proportion are visitors from interstellar space, can only be decisively settled by further study, and in particular by some more accurate method of measuring velocity, if such can be devised.

THE ORIGIN OF THE SOLAR SYSTEM

539. Regularities in the Solar System. The solar system is clearly no accidental aggregation of bodies. The remotest planet is hardly more than one ten-thousandth part as far away as the nearest known star, and all the planets share the rapid motion of the sun through interstellar space (§ 740). It is obvious, therefore, that the sun, the planets, and their satellites — together, probably, with the comets — must have had a common origin. Moreover, the planetary system presents numerous regularities of arrangement, for which the mind demands an explanation,

and which are not, like Kepler's laws, necessary consequences of gravitation. The most noteworthy of these are as follows:

As regards the orbits of the planets,

(1) The direction of revolution of all the planets about the sun is the same.

(2) Their orbits all lie nearly in the same plane (except those of some of the asteroids).

(3) Their orbits are nearly circular (except, again, for some asteroids).

(4) There is a curious and regular progression of their distances, expressed roughly by Bode's law (§ 269).

(5) The sun rotates in the direction in which the planets revolve, and its equator is but little inclined to their orbits.

As regards the planets themselves,

(6) The more massive planets (as a group) are of lower density.

(7) The larger planets rotate more rapidly.

(8) Among the four major planets the inclination of the equator to the orbit decreases with decreasing distance from the sun.

As regards the satellites,

(9) The satellites revolve about the planets in the direction in which the planets themselves rotate.

(10) Their orbits are nearly circular and nearly in the plane of the planet's equator. (These last two propositions are not true of the remote outer satellites of Jupiter and Saturn.)

It is not reasonable to suppose that such relations as these are due to chance, and it is in accordance with the whole trend of science to believe that they have originated by some orderly process.

540. The Nebular Hypothesis. The earliest attempt to explain these facts originated in the eighteenth century with Swedenborg and Kant, and was put into scientific form by Laplace. Briefly stated, this theory assumes that, at a remote epoch in the past, the matter which now forms the solar system composed one vast, rarefied, slowly rotating mass — a nebula — which gradually cooled, and contracted under the influence of its own gravitation. As it contracted, its rotation became more rapid (since angular momentum had to be conserved); in time the centrifugal force at the equator became equal to gravity, and a ring of diffuse matter was detached from the periphery.

This happened several times, leaving the shrinking nebula surrounded by rings, the smaller of which revolved more rapidly. Finally, the material of each ring somehow collected into a single body, forming a planet; and the central mass condensed to form the sun.

This theory accounts for many of the facts and can be modified to explain more, but it meets with two fatal difficulties. First, it can be proved that an extended tenuous ring would not condense into a single body, but into many bodies, like the asteroids or the rings of Saturn. Second, almost all the angular momentum of the solar system — 98 per cent of the total — is at present associated with the orbital motions of the major planets. The sun's rotation provides almost all the rest, the four terrestrial planets contributing less than 0.1 per cent of the whole. The total angular momentum cannot be altered by any internal changes within the system, and no process has ever been imagined by which 98 per cent of it could have been segregated in less than 1/700 of the total mass. Furthermore, it has been proved that if the outer parts of the nebula had had so much angular momentum, they could not have condensed at all — even into asteroids.

It appears, indeed, that no orderly process of evolution under the action of internal forces could have produced the existing distribution of angular momentum; and it follows that the angular momentum of the planets must have been *put into the system from outside.* Here, therefore, it seems necessary, for once, to abandon the "uniformitarian" hypothesis of gradual evolution and to adopt a "catastrophic" hypothesis of sudden change.

541. The Hypothesis of Dynamic Encounter. An alternative theory, which appears at present to be much more satisfactory than the "nebular hypothesis," was proposed about twenty years ago by Chamberlin and Moulton, of Chicago. According to this theory the sun was once an isolated star, without planets. At some remote epoch another star, in its motion through space, happened to pass very near the sun. The two bodies swung about one another in hyperbolic orbits, and separated again. Their minimum distance was so small that the tide-raising forces due to the star's attraction of the sun — aided by the expansive force of the solar gases — became great enough to counteract the sun's attraction at some points, so that great quantities of matter were ejected from the sun.

The initial motion of the ejected masses was straight away from the sun, and in the absence of disturbing forces they would all have fallen back again into it; but (as Moulton has shown) the disturbing force due to the star's attraction would, in general, tend to impart to these masses a lateral motion, and

set them revolving about the sun in the same direction in which the star was moving. After the star receded, some of the ejected matter must have fallen back into the sun; other parts may have flown off into space in parabolic or hyperbolic orbits; but the remaining portions were left circulating about the sun in elliptic orbits, all in the same direction and in planes not much inclined to that of the star's orbit about the sun. From this circulating material the planetary system was formed.

This hypothesis has the great advantage that it accounts for the angular momentum of the planetary motions, which is not supposed to have been primitive but to have been imparted to the ejected matter by the attraction of the passing star at the expense of a small diminution of the far greater angular momentum of the star's motion about the sun.

It also accounts for the sun's rotation. The material which fell back upon the sun would itself have acquired a certain, though small, angular momentum in the same direction as that of the planets, and this, communicated to the general mass of the sun, would set it rotating in this direction. Whatever rotation the sun previously possessed would have been "drowned out" by this effect, leaving perhaps a trace in the fact that the sun's equator is inclined 7° to the invariable plane of the solar system. Most of the ejected material would fall back near the sun's equator, setting this part of its surface into more rapid rotation than the rest. (The present equatorial acceleration (§ 225) may perhaps be a survival of this effect, which fluid friction has not yet been able to eliminate entirely.)

542. The Planetesimal and Tidal Theories. With respect to the further development of the solar system, the original views of Chamberlin and Moulton differ somewhat from those more recently propounded by Jeans and Jeffreys. All agree that the ejected material cooled down rapidly, and that it then contained (1) large masses, great enough to form at least the nuclei of the present planets, (2) numerous smaller solid particles, and (3) uncondensed gas.

According to the older, or "planetesimal," form of the theory most of the matter was in the form of small solid particles, revolving about the sun, like infinitesimal planets (whence the

name), and the present planets have grown from much smaller original nuclei by gradual accretion of these small bodies. The newer, or "tidal," form of the theory concludes that the planets must have possessed nearly their present masses from the beginning and that the diffuse material was largely gaseous. The difference is thus rather one of degree than of kind, and the two forms of the theory of dynamic encounter may well be discussed together.

543. Formation of the Planets. The ejected material must originally have been gaseous and exceedingly hot, and would have liquefied and solidified only after it had cooled by expansion and radiation. During this process the lighter and more volatile constituents would escape from all but the larger masses — just as the moon's atmosphere has escaped from the moon. The difference in density between the terrestrial and major planets may be due to differences in composition which have arisen in this way.

Chamberlin believes that the planets, except perhaps the largest, were solid throughout practically from the start. Jeffreys concludes that they must have passed through an intermediate liquid phase, but that the earth, for example, must have solidified within about 15,000 years of its birth. After this the surface began to cool, an ocean formed upon it, and geological history commenced.

544. Eccentricity of the Orbits. The dispersed material which did not condense into the primeval planets plays an important rôle in both forms of the theory. The particles, whether meteorites or molecules, must have been circulating about the sun in the same direction as the planets. Such a revolving swarm, or atmosphere, would have little influence upon the forward orbital motion of a planet (which must have been moving at about the same rate as the swarm in its neighborhood) but would in the long run slow down, as if by frictional resistance, the outward and inward motions arising from the eccentricity of the planet's orbit. The planetary orbits, which were probably at first highly eccentric, would thus become more nearly circular.

The dispersed matter would gradually tend to disappear. The separate planetesimals would be picked up by the planets or

(much more probably) collide with one another and be reduced to fine dust or to gas. The gaseous matter, as Jeffreys has shown, would either slowly diffuse away into space or settle back into the sun, leaving interplanetary space clear. The zodiacal light (§ 424) may represent the last remnants of it, while meteors may be surviving stray planetesimals.

The time required for this process would be very long. Jeffreys estimates it roughly as 7000 million years. He suggests that the asteroids, which have highly eccentric orbits, may have been produced by the disruption of a larger body or bodies, rather late in the history of the system. (Compare § 422.)

545. Rotation of the Planets. A planet formed by the infall of planetesimals upon a non-rotating nucleus might be set into either direct or retrograde rotation, according to the particular circumstances of accumulation; but in either case the rotation would probably be much slower than that of the major planets or even of the earth. It appears very probable, therefore, that most of the rotational momentum of the planets was possessed by their original nuclei immediately after their eruption from the sun. During this eruption, which was doubtless exceedingly turbulent, it may well have been imparted to them.

The regular progression of the inclinations of the equators of the major planets, 151° for Neptune, 98° for Uranus, 27° for Saturn, and 3° for Jupiter, suggests that the rotations of all were originally retrograde and that their equatorial planes have in some way been tipped over, fastest for the planets nearest the sun. Stratton has shown that the combined influence of the tides due to the sun and the satellites might, under certain circumstances, produce such an effect, but it is doubtful if the solar system is old enough to allow time for such exceedingly slow changes to act.

546. Origin of Satellites. The satellite systems of the great planets are solar systems in miniature, so similar in constitution as to suggest that they had a similar origin; but the details of their development are obscure. The satellites may have accompanied the planets at the time of the original eruption or may have been produced shortly afterwards, at the first perihelion passage of the planet, by the sun's tidal action. The frictional influence of the dispersed solid and gaseous matter would slowly modify the original orbits of the satellites, making

them more nearly circular and probably diminishing their size; but how they became so very nearly circular and so close to the planets' equatorial planes is not yet clear. The outer, retrograde satellites of Jupiter and Saturn may once have been asteroids, and may have been captured with the aid of the "resisting medium," but this is uncertain.

547. Origin of the Moon. The moon presents a special problem, being far larger, in proportion to its primary, than any other satellite, or than any planet compared with the sun. If the earth and moon ever formed one mass, it must have rotated in about four hours and have been greatly flattened at the poles. This rapid rotation would not, by itself, have been sufficient to cause the mass to break up; but if, as Darwin suggested, the natural period of free vibration of the mass was, at some stage in its contraction, the same as that of the tides produced by the sun, these tides would gradually rise to enormous heights, distorting the body and ultimately causing it to divide into two parts. It is not improbable (as Jeffreys has shown) that this may have happened, but the alternative hypothesis that the earth and moon were produced together at the time of the great catastrophe cannot be disproved.

There is a good deal in favor of the theory that some, if not all, of the lunar craters have been produced by the impact of planetesimal bodies of moderate size, relatively late in the moon's career, — though long ago, even as geological time is measured. Similar craters, if produced on the earth, would have been gradually destroyed by denudation, and this may suffice to explain their absence.

548. Present State of the Problem. The cosmogonic hypotheses which have here been summarily sketched are far from resting upon the secure bases of exact calculation which support most of the conclusions of astronomy. The mathematical difficulties of a detailed or precise treatment of problems like these much exceed those of the relatively simple "problem of three bodies" (§ 321), and only rough, qualitative conclusions are possible.

The hypothesis that the planetary system owes its origin to an encounter between the sun and a passing star appears to be

much the best that has yet been suggested, but considerable difficulties still remain. It is hard to see how matter erupted from a sun smaller than Mercury's orbit could be left traveling with such angular momentum as belongs to Uranus or Neptune. Furthermore, the satellite systems are imperfectly accounted for, and comets practically not at all. Future researches may clear up these problems.

It is noteworthy that the rough estimates of the time required for the suggested process of evolution (§ 544) are not far from those given by radioactive evidence for the age of the earth (§ 155), and the belief that the great catastrophe out of which the planetary system was born occurred five or ten billions of years ago stands on a fairly good foundation. In a very small fraction of this interval the star which caused the disturbance would have been lost in the depths of space, and would now be indistinguishable among the millions of others.

The stars are so far apart that such close encounters between them must be extremely rare; and, so far as it is possible to estimate, planetary systems, even if produced by every such encounter, should be infrequent among the stars. The multitude of stars is, however, so great that the actual number of such systems may nevertheless be large. Whether, among them, other planets are habitable by forms of life such as we know, and whether, if so, life actually exists upon them, are at present matters of pure speculation.

EXERCISES

1. What would be the mean density of the head of Halley's comet, on the assumption of § 520, if the particles were of density 2.7 times that of water? *Ans.* 4.2 grams per cubic kilometer, or 4.2×10^{-15} gm./cm.^{3}, which is 3.3×10^{-12} that of air under standard conditions.

2. Can the dimensions of a comet's tail be determined with much accuracy? If not, why not?

3. How can it happen that comets whose orbits nearly coincide within a distance of 100,000,000 miles from the sun may have periods differing by hundreds of years? For example, the comets of 1880 and 1882, of which the first has a computed period of only 33 years, and the other of more than 600.

4. The perihelion distance of the great comet of 1882 was 0.00775 astronomical unit. What was the velocity V at perihelion of the component which had a period of 769 years?

Ans. For this component, $a = (769)^{\frac{2}{3}} = 83.9$ astronomical units. By equation (7), p. 271, and § 315,

$$V^2 = (29.76)^2 \times (2/0.00775 - 1/83.9) = (29.76)^2 \times (258 - 0.012).$$

Hence $$V = 29.76\sqrt{257.99} = 477 \text{ km./sec.}$$

5. How much greater was the velocity at perihelion for the component with a period of 875 years if the perihelion distance was exactly the same?

Ans. For this component, $a = (875)^{\frac{2}{3}} = 91.4$. If V_2 is the velocity of this component and V_1 that of the other, then

$$V_1^2 = (29.76)^2 \times (2/r - 1/83.9),$$
$$V_2^2 = (29.76)^2 \times (2/r - 1/91.4),$$

whence $V_2^2 - V_1^2 = (29.76)^2 \times (1/83.9 - 1/91.4) = 886/83.9 - 886/91.4$
$= 10.57 - 9.71 = 0.86,$

or $$(V_1 + V_2)(V_1 - V_2) = 0.86.$$

But $V_1 + V_2 = 954$; $V_2 - V_1 = 0.0009$ km./sec., or 90 centimeters per second.

6. Will a given comet (say Encke's) have precisely the same orbit on successive returns?

7. Why can we not infer with certainty that two comets which have orbits practically identical are themselves identical?

8. Can we, from spectroscopic observations of a comet, infer the relative proportions of the luminous and non-luminous substances present in the comet?

9. Is it probable that a comet can continue permanently in the solar system as a comet? If not, why not, and what will become of it?

10. If a compact swarm of meteors were now to enter the solar system and be deflected by the attraction of some planet into an elliptical orbit around the sun, would the swarm continue to be compact? If not, what would be the ultimate distribution of the meteors?

11. Assuming that the earth encounters 20,000,000 meteors every 24 hours, what is the average number in a cubic space of 1,000,000,000 cubic miles (that is, a cube 1000 miles on each edge)? *Ans.* About 250.

12. If space were occupied by meteors uniformly distributed 100 miles apart on three sets of lines perpendicular to each other, how many would be encountered by the earth in a day? *Ans.* 78,700,000.

NOTE. In this cubical arrangement the *average* distance between the meteors much exceeds 100 miles. If they were packed as closely as possible, consistently with the condition that the distance between two neighbors *should nowhere be less than 100 miles*, the number would be increased by nearly 40 per cent.

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APPENDIX

Table II. Dimensions of the Terrestrial Spheroid

(Hayford's Spheroid of 1909)

Equatorial radius $a = 6378.388$ km. $= 3963.34$ miles.
Polar radius $b = 6356.909$ km. $= 3949.99$ miles.
Mean semidiameter, $\frac{1}{3}(2a + b), = 6.37123 \times 10^8$ cm. $= 6371.23$ km. $= 3958.89$ miles
Oblateness $\dfrac{a-b}{a} = \dfrac{1}{297.0}\cdot$ (See section 138.)
1° of latitude, ϕ, (in statute miles) $= 69.0569 - 0.3494 \cos 2\phi + 0.0007 \cos 4\phi$.
1° of longitude (in statute miles) $= 69.2316 \cos \phi - 0.0584 \cos 3\phi + 0.0001 \cos 5\phi$.

Table III. Astronomical Constants

Length of the day:
Sidereal $= 23^h\ 56^m\ 4^s.091$ of mean solar time.
Mean solar $= 24^h\ 3^m\ 56^s.555$ of sidereal time.
(For conversion of time-intervals see section 44.)
Length of the year (in mean solar units), 1900, Newcomb:
Tropical $= 365^d.24219879 = 365^d\ 5^h\ 48^m\ 45^s.98$.
Sidereal $= 365^d.25636042 = 365^d\ 6^h\ 9^m\ 9^s.54$.
Anomalistic $= 365^d.25964134 = 365^d\ 6^h\ 13^m\ 53^s.01$.
The length of the sidereal year is 31,558,149.5 sec. $= 3.1558 \times 10^7$ sec.
Length of the month (in mean solar units), according to Brown:
Synodical $= 29^d.530588 = 29^d\ 12^h\ 44^m\ 2^s.8$.
Sidereal $= 27^d.321661 = 27^d\ 7^h\ 43^m\ 11^s.5$.
Nodical $= 27^d.212220 = 27^d\ 5^h\ 5^m\ 35^s.8$.
Obliquity of the ecliptic $= 23°\ 27'\ 8''.26 - 0''.4684\ (t - 1900)$ } Newcomb.
General precession $= 50''.2564 + 0.000222\ (t - 1900)$ }
Constant of nutation $= 9''.21$ }
Constant of aberration $= 20''.47$ } Adopted for ephemeris purposes, Paris Conference (1911).
Solar parallax $= 8''.80$ }
Velocity of light $= 299{,}796$ km./sec. $= 186{,}285$ mi./sec. (Michelson, 1926.)
Constant of gravitation $G = 6.673 \times 10^{-8}$ c.g.s. units.
Acceleration of gravity g (in meters) $= 9.8060 - 0.0260 \cos 2\phi - \dfrac{2h}{R} g$ (Helmert),
in which h is the elevation above sea-level in meters, and $\log R = 6.80416$.
Earth's mass $= (5.974 \pm 0.005) \times 10^{27}$ grams.
Sun's mass $= 1.983 \times 10^{33}$ grams.
Sun's mean radius $= 6.953 \times 10^{10}$ cm.
1 astronomical unit (A.U.) $= 1.4945 \times 10^8$ km. $= 9.2870 \times 10^7$ miles.
1 light-year $= 6.3310 \times 10^4$ A.U. $= 9.463 \times 10^{12}$ km. $= 5.88 \times 10^{12}$ miles. (§ 714.)
1 parsec $= 3.258$ light-years $= 2.06265 \times 10^5$ A.U. $= 3.084 \times 10^{13}$ km.
$= 1.92 \times 10^{13}$ miles. (§ 714, Vol. II.)

TABLE IV. PRINCIPAL ELEMENTS

(For the Epoch 1920,

	Name	Symbol	Semi-Major Axis of Orbit	Mean Dist. (Millions of Km.)	Sidereal Period (Mean Solar Days)	Period in Sidereal Years
Terrestrial Planets	Mercury	☿	0.387099	57.85	87.96926	0.2408
	Venus	♀	0.723331	108.10	224.7008	0.6152
	Earth	⊕	1.000000	149.45	365.2564	1.0000
	Mars	♂	1.523688	227.72	686.9797	1.8808
	Ceres	(1)	2.767303	413.58	1,681.449	4.6035
	Eros	(433)	1.458296	217.94	643.230	1.7610
Major Planets	Jupiter	♃	5.202803	777.6	4,332.588	11.862
	Saturn	♄	9.538843	1425.6	10,759.201	29.457
	Uranus	♅	19.190978	2868.1	30,685.93	84.013
	Neptune	♆	30.070672	4494.1	60,187.64	164.783

	Name	Symbol	Apparent Angular Diameter (Equatorial)	Mean Diameter		Mass		Volume ⊕=1
				Km.	⊕=1	☉=1	⊕=1	
	Sun	☉	31′ 59″.3 (mean)	1 390 600	109.1	1.000	331.950	1 300 000
	Moon	☾	31′ 5″ (mean)	3 476	0.273	$\frac{1}{27\,070\,000}$	$\frac{1}{81.56}$	0.0203
Terrestrial Planets	Mercury	☿	4″.7 to 12″.9	5 000	0.39	$\frac{1}{8\,000\,000}$	0.04	0.06
	Venus	♀	9″.9 to 64″.0	12 400	0.973	$\frac{1}{410\,000}$	0.81	0.92
	Earth	⊕		12 742	1.000	$\frac{1}{331\,950}$	1.000	1.000
	Mars	♂	3″.5 to 25″.1	6 770	0.531	$\frac{1}{3\,085\,000}$	0.108	0.150
	Ceres	(1)	0″.27 to 0″.69	770	0.060	$\frac{1}{2.5 \times 10^9}$?	$\frac{1}{8000}$?	0.0002
	Eros	(433)	0″.02? to 0″.25?	25?	0.002?			8×10^{-9}?
Major Planets	Jupiter	♃	30″.5 to 49″.8	139 560	10.95	$\frac{1}{1047.40}$	316.94	1312
	Saturn	♄	14″.7 to 20″.5	115 100	9.02	$\frac{1}{3499}$	94.9	734
	Uranus	♅	3″.4 to 4″.2	51 000	4.00	$\frac{1}{22\,650}$	14.66	64
	Neptune	♆	2″.2 to 2″.4	50 000	3.92	$\frac{1}{19\,850}$	17.16	60

OF THE SOLAR SYSTEM

January $0^d.0$ G.M.T.)

Mean Orbital Velocity (Km./Sec.)	Eccentricity	Inclination to Ecliptic	Longitude of Ascending Node	Longitude of Perihelion	Mean Longitude at Epoch
47.83	0.20562	7° 00′ 12″	47° 22′ 59″	76° 12′ 39″	165° 14′ 43″
34.99	0.00681	3 23 38	75 57 35	130 26 44	166 36 34
29.76	0.01674	0 00 00		101 33 53	99 51 02
24.11	0.09333	1 51 01	48 56 25	334 35 12	162 05 15
17.89	0.07653	10 36 56	80 45 39	149 26 12	223 19 21
24.64	0.22297	10 49 40	303 35 09	121 25 32	28 36 33
13.05	0.04837	1 18 28	99 38 24	13 02 01	125 18 37
9.64	0.05582	2 29 29	112 57 29	91 28 50	151 16 01
6.80	0.04710	0 46 22	73 35 27	169 22 07	329 20 35
5.43	0.00855	1 46 38	130 53 56	43 55 50	128 59 54

Mean Density		Axial Rotation	Inclination of Equator to Orbit	Oblateness	Mean Surface Gravity	Albedo	Stellar Magnitude at Mean Opposition	Velocity of Escape Km./Sec.
⊕ = 1	Water = 1				⊕ = 1			
0.256	1.41	$24^d.65$ (equatorial)	7° 10′.5	0.0000	27.89		−26.72	617.0
0.604	3.33	$27^d\ 7^h\ 43^m\ 11^s.5$	6°40′.7	0.0006	0.165	0.07	−12.55	2.4
0.70	3.8	$88^d.0$		0.00	0.27	0.07	0.16 *	3.6
0.88	4.86			0.00	0.85	0.59	[illegible].07 * 25	10.2
1.000	5.52	$23^h\ 56^m\ 4^s.09$	23° 26′ 59″	$\frac{1}{296}$	1.00	0.45?	3.5 [illegible] †	11.2
0.72	3.96	$24^h\ 37^m\ 22^s.58$	25° 10′.	$\frac{1}{192}$	0.38	0.15	−1.85	5.0
0.6?	3.3?				0.037 ?	0.06	7.15	0.5?
0.6?	3.3?	$5^h\ 16^m$			0.001 ?		9.7	0.02?
0.242	1.34	$9^h\ 50^m$ to $9^h\ 55^m$	3° 6′.9	$\frac{1}{15.4}$	2.64	0.44	−2.23	60.
0.13	0.71	$10^h\ 14^m$ to $10^h\ 38^m.5$	26° 44′.7	$\frac{1}{9.5}$	1.17	0.42	+0.89 to −0.18	36.
0.23	1.27	$10^h.7$	98°.0	$\frac{1}{14}$	0.92	0.45?	5.74	21.
0.29	1.58	15^h?	151°	$\frac{1}{40}$?	1.12	0.52?	7.65	23.

* At elongation. † As seen from sun.

Table V. The Satellites of

	Name	Discovery		Mean Dist. in Equatorial Radii of Planet	Apparent Distance as seen from Sun	Mean Distance Km.	Mean Distance Miles	Sidereal Period
	Moon .			60.2673	530″.5	384 403	238 857	27ᵈ 7ʰ 43ᵐ 11ˢ.5
								SATELLITES OF
1	Phobos .	Hall,	1877	2.79	8.52	9 380	5 826	0ᵈ 7ʰ 39ᵐ 13ˢ.851
2	Deimos .	Hall,	1877	6.96	21″.25	23 460	14 580	1 6 17 54 .9
								SATELLITES OF
5	Nameless	Barnard,	1892	2.540	48″.06	181 200	112 600	11ʰ 57ᵐ 22ˢ.70
1	Io . . .	Galileo,	1610	5.905	111 .78	421 300	261 800	1ᵈ 18 27 33 .51
2	Europa .	Galileo,	1610	9.401	177 .86	670 500	416 600	3 13 13 42 .05
3	Ganymede	Galileo,	1610	14.995	283 .70	1 069 300	664 200	7 3 42 33 .35
4	Callisto .	Galileo,	1610	26.379	498 .99	1 881 000	1 168 700	16 16 32 11 .21
6	Nameless	Perrine,	1904	160.6	3037	11 450 000	7 114 000	250ᵈ.68
7	Nameless	Perrine,	1905	164.6	3113	11 730 000	7 292 000	260 .06
8	Nameless	Melotte,	1908	330	1° 44′	23 500 000	14 600 000	738 .9
9	Nameless	Nicholson,	1914	338	1° 46′	24 100 000	15 000 000	745 .0
								SATELLITES OF
7	Mimas .	W. Herschel,	1789	3.11	26″.83	185 700	115 300	22ʰ 37ᵐ 5ˢ.25
6	Enceladus	W. Herschel,	1789	3.99	34 .42	237 900	147 800	1ᵈ 8 53 6 .82
5	Tethys .	J. D. Cassini,	1684	4.94	42 .60	294 500	183 000	1 21 18 26 .14
4	Dione .	J. D. Cassini,	1684	6.33	54 .57	377 200	234 400	2 17 41 9 .53
2	Rhea .	J. D. Cassini,	1672	8.84	76 .20	526 700	327 300	4 12 25 12 .23
1	Titan .	[illegible]ns,	1655	20.48	176 .67	1 220 000	758 800	15 22 41 26 .82
8	Hyperion	G. [illegible]nd,	1848	24.82	214 .13	1 480 000	919 700	21 6 38 24 .0
3	Iapetus .	J. D. Cassini,	1671	59.68	514 .73	3 558 000	2 210 000	79 7 56 24 .4
9	Phoebe .	W. Pickering,	1898	216.8	1870 .4	12 930 000	8 034 000	550ᵈ.44
								SATELLITES OF
1	Ariel . .	Lassell,	1851	7.35	13″.78	191 700	119 100	2ᵈ 12ʰ 29ᵐ 20ˢ.8
2	Umbriel	Lassell,	1851	10.2	19 .20	267 000	165 900	4 3 27 36 .7
3	Titania .	W. Herschel,	1787	16.8	31 .50	438 000	272 200	8 16 56 26 .7
4	Oberon .	W. Herschel,	1787	22.4	42 .12	586 000	364 000	13 11 7 3 .5
								SATELLITE OF
1	Nameless	Lassell,	1846	14.1	16″.23	353 700	219 800	5ᵈ 21ʰ 2ᵐ 38ˢ.1

THE SOLAR SYSTEM

INC. OF ORBIT TO PLANET'S EQUATOR OR TO "PROPER PLANE" *	MEAN INC. OF ORBIT OR PROPER PLANE TO PLANET'S ORBIT	ECCENTRICITY	STELLAR MAG. AT MEAN OPPOSITION	DIAMETER KM.	MASS	
					Primary = 1	Moon = 1
28° 35′ to 18° 19′ †	5° 8′ 33″	0.05490	−12.3	3476	$\frac{1}{81.56}$	1.00
MARS						
0° 57′.5	25° 19′.6	0.017	11.5	15?		
1 44 .0	24 14 .7	0.003	13.0	8?		
JUPITER						
0° 27′.3	3° 6′.9	0.0028	13.0	160?		
0 1 .6	3 6 .7	0.0000	5.5	3730	0.000042	1.09
0 28 .1	3 5 .8	0.0003	5.7	3150	0.000025	0.65
0 11 .0	3 2 .3	0.0015	5.1	5150	0.000081	2.10
0 15 .2	2 42 .7	0.0075	6.3	5180	0.000022	0.58
180 53 ‡	28 45 (1900)	0.155	13.7	130?	Perturbations large	
243 0 ‡	27 58 (1900)	0.207	16	40?		
208 ‡	148 4 (1910)	0.378	16	25?	(1910–1916)	Perturbations enormous
31 ‡	156 (1914)	0.25	18	25?	(1914–1916)	
SATURN						
1° 36′.5	26° 44′.7	0.0190	12.1	650?	$\frac{1}{16\,840\,000}$	$\frac{1}{2120}$
0 1 .4	26 44 .7	0.0001	11.6	800?	$\frac{1}{4\,000\,000}$?	$\frac{1}{520}$?
1 4 .4	26 44 .7	0.0000	10.5	1300?	$\frac{1}{921\,500}$	$\frac{1}{119}$
0 4 .0	26 44 .7	0.0020	10.7	1200?	$\frac{1}{586\,000}$	$\frac{1}{69}$
0 19 .8	26 41 .9	0.0009	10.0	1750?	$\frac{1}{250\,000}$?	$\frac{1}{30}$?
0 16 .9	26 7 .1	0.0289	8.3	4200	$\frac{1}{4150}$	1.86
17′ to 56′	26 0 .0	0.1043	13.0	500?	$< \frac{1}{4\,500\,000}$	$< \frac{1}{600}$
13° 51′.3	16 18 .1	0.0284	10.1 to 11.9	1800?	$< \frac{1}{100\,000}$	$< \frac{1}{13}$
148°.9 †	174 .7	0.1659	14.5	250?		
URANUS						
0°	97° 59′	0.007	15.2?	900?		
0	97 59	0.008	15.8?	700?		
0 0′	97 59	0.0023	14.0	1700?		
0 0	97 59	0.0010	14.2	1500?		
NEPTUNE						
20° †	139 49′ (1900)	0.000	13.6	5000?		

* See note at top of page vi. † To planet's equator. ‡ Longitude of ascending node.

NOTE TO TABLE V

The first and second columns on page v require some explanation. For most of the satellites the orbit plane is inclined at substantially a fixed angle to a "proper plane" on which the nodes regress. In these cases the inclinations of the orbit plane to this plane, and of this plane to that of the planet's orbit, are given in the first and second columns. For the moon, the four outer satellites of Jupiter, Phœbe, and the satellite of Neptune the second column gives the inclination of the orbit plane to the planet's orbit, at the given date, while the first column is used to give other data, as indicated by the footnotes.

TABLE VI. MEAN REFRACTION

(Corresponding to temperature of 50° F., and to a barometric pressure of 29.6 inches)

ALTITUDE	REFRACTION	ALTITUDE	REFRACTION	ALTITUDE	REFRACTION
0°	34′ 50″	11°	4′ 47″.7	30°	1′ 39″.5
1	24 22	12	4 24 .5	35	1 22 .1
2	18 06	13	4 04 .4	40	1 08 .6
3	14 13	14	3 47 .0	45	57 .6
4	11 37	16	3 18 .2	50	48 .3
5	9 45	18	2 55 .5	55	40 .3
6	8 23	20	2 37 .0	60	33 .2
7	7 19	22	2 21 .6	65	26 .8
8	6 29	24	2 08 .6	70	20 .9
9	5 49	26	1 57 .6	80	10 .2
10	5 16	28	1 48 .0	90	0 .0

For every 5° F. by which the temperature is *less* than 50° F., *add 1 per cent* to the tabular refraction, and decrease it in the same ratio for temperatures above 50° F.

Increase the tabular refraction by *3½ per cent* for every inch of barometric pressure above 29.6 inches, and decrease it in the same ratio below that point. These corrections for temperature and pressure, though only approximate, will give a result correct within 2″, except in extreme cases.

COMSTOCK'S FORMULA FOR REFRACTION

$$r'' = \frac{983\, b}{460 + t} \tan \zeta.$$

b is the barometer reading in *inches*; *t*, the temperature in degrees *Fahrenheit*; ζ, the apparent zenith distance. The error of the formula is less than 1″ for zenith distances under 75°, except in extreme conditions of temperature and pressure.

NAME INDEX

SUBJECT INDEX